ECON4351 - Final exam

2023

The exam consists of three parts. The first part is a question without any math (except, perhaps, stating an accounting equation), the second question is related to the Solow model, and the third is related to the endogenous-growth frameworks.

1 Rival and exclusionary goods (30 points)

(a) Describe Paul Romer's distinction between rival and non-rival goods, as well as exclusionary and non-exclusionary goods. Give examples of rival and exclusionary, rival and non-exclusionary, non-rival and exclusionary, non-rival and non-exclusionary goods (the examples need not be 100% perfect but you should write 1-2 sentences explaining why each example works).

(b) Why does Paul Romer introduce the distinction between rival and non-rival goods? Relate the distinction to *increasing returns to scale* and *ideas* as the engine of growth. Make sure to define increasing returns to scale in the process.

2 Solow model (40 points)

In this problem, we characterize the capital-output ratio in the Solow model. Assume the population growth is n ($L_t = (1+n)^t L_0$) and the labor-augmenting technological growth is g ($A_t = (1+g)^t A_0$). Given a savings rate s, production function $F(K, L)$, and depreciation rate δ, the law of motion for capital K_t is given by

$$K_{t+1} = sF(K_t, A_t L_t) + (1-\delta)K_t.$$

(a) Define capital per efficiency units of labor by $\tilde{k}_t = K_t/(A_t L_t)$. Show that the law of motion for $\tilde{k}_t$ is given by

$$\tilde{k}_{t+1} = \frac{1}{(1+g)(1+n)} \left[sf(\tilde{k}_t) + (1-\delta)\tilde{k}_t \right]$$

where $f(k) = F(k, 1)$. Be clear when you invoke an assumption about the production function F.

(b) Show that in the long run (you may assume a balanced-growth path exists and that the model converges to it),

$$K_t/Y_t = s/(g + n + gn + \delta)$$

(where $Y_t = F(K_t, L_t)$ denotes output). Again, be clear when you invoke an assumption about the production function F. *Hint: solve for $\tilde{k}_{ss}/f(\tilde{k}_{ss})$ by evaluating the law of motion for $\tilde{k}_t$ at its steady state.*

(c) In the long run, what is the growth rate of GDP? What is the growth rate of GDP per capita? (simply state the answers)

(d) Assume $s = 0.15$ and $\delta = 0.05$. Assume GDP growth is 0.025. Compute the long-run capital-to-output ratio.

(e) Now assume GDP growth falls to 0. Compute the long-run capital-to-output ratio.

3 Romer model (40 points)

In this exercise, we go through the simple product-variety model with lab equipment. Recall that in this environment, the household's Euler equation, $r = \rho + \epsilon g$ holds, where ϵ is the inverse of the household's elasticity of intertemporal substitution. Final output at time t is produced under perfect competition, using labor and a range of intermediate inputs, indexed by i in the interval $[0, M_t]$ where M_t is the measure of product varieties available at time t. Final good production at time t is perfectly competitive and given by

$$Y_t = L^{1-\alpha} \int_0^{M_t} x_{ti}^{\alpha} di$$

where Y_t is output and each x_{ti} is the amount of intermediate product i used as input. Labor input is always equal to the fixed supply L. The coefficient

α lies between zero and one. Each intermediate product is produced using the final good as input, one for one, and the patent for each intermediate product is owned by a monopolist.

(a) By computing the marginal product of x_{ti} in the final-goods production, show that the price of good x_{ti} (with the final good as numeraire) is $p_{ti} = \alpha L^{1-\alpha} x_{ti}^{\alpha-1}$.

(b) The monopolist for good i sells its good at price p_{ti} and produces it at cost 1. Write down its maximization problem and show that the profit maximization implies

$$x = L\alpha^{2/(1-\alpha)},$$

$$\Pi = \frac{1-\alpha}{\alpha} L\alpha^{2/(1-\alpha)}.$$

In particular, all intermediate-goods producer choose the same quantity and earn the same profit at all times.

(c) Gross domestic product is final-goods output net of intermediate-goods, $GDP_t = Y_t - \int_0^{M_t} x_{ti} di$. Show that GDP_t is proportional to M_t and thus that the growth rate of GDP equals $g = \frac{\dot{M}_t}{M_t}$.

(d) Product variety grows at a rate that depends on the amount R_t of final output that is used in research. I.e., we have

$$\dot{M}_t = \lambda R_t$$

where R_t is the amount of final output that is used in research. Assume that the research sector is perfectly competitive with free entry, such that the flow of profit in the research sector must be zero. Each blueprint is worth Π/r to its inventor, the net present value of the profit stream from owning the patent to a new blueprint. Show that the "research-arbitrage equation" is

$$r = \lambda \Pi.$$

(e) Combining the research-arbitrage equation with the Euler equation, show that the endogenous growth rate is given by

$$g = \frac{\lambda\Pi - \rho}{\epsilon}.$$

(f) If the population size L doubles, does the growth rate g change and if so in which direction? Explain your answer.