Final exam – retake

2024

The exam consists of three parts. The first part is a question without any math (except, perhaps, stating an accounting equation), the second question is related to the Solow model, and the third is related to the endogenous-growth frameworks.

1 Convergence (30 points)

(a) Describe the assumptions under which the Solow model predicts "convergence". Explain what convergence means in this context.

(b) Describe a regression that you can run to test whether there is convergence in GDP across regions. Explain the regression, in particular which regression coefficient is of interest and how do we interpret it?

(c) In broad strokes, do we see convergence across countries? Give an example of a set of regions (e.g., countries) for which we have seen convergence.

2 Solow model (35 points)

In this problem, we characterize factor prices in the Solow model.

(a) Assume no population growth ($n = 0$) and no technological growth ($g = 0$). Write down the law of motion for capital given a savings rate s, production function $F(K, L)$, and depreciation rate δ.

(b) Assume that the markets for labor and capital are perfectly competitive. Write down the problem of a profit-maximizing representative firm which has access to the technology $F(K, L)$.

(c) Show that, in equilibrium, factors (labor and capital) are paid their *marginal product*.

(d) Write down an expression for the labor share, in terms of the production function F, its derivative(s), as well as K and/or L.

(e) Finally, assume that $F(K, L) = K^{\alpha} L^{1-\alpha}$. Show that the labor share equals $1 - \alpha$.

3 Schumpeterian model (35 points)

In this exercise, we go through the basic Schumpeterian growth model. Final-good production is given by

$$Y_t = L^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^{\alpha} di$$

where each x_{it} is the quantity of intermediate product i and A_{it} reflects the quality of the product. Each intermediate product has its own monopoly, and its price equals its marginal product in the final sector.

(a) Show that the price of good i is given by $p_{it} = \alpha(A_{it}L)^{1-\alpha} x_{it}^{\alpha-1}$.

(b) The intermediate goods are produced one-to-one using the final good. Write down intermediate-good monopolist i's problem and show that the solution to the monopolist's problem is given by

$$\begin{aligned} x_{it} &= \alpha^{2/(1-\alpha)} A_{it} L, \\ \Pi_{it} &= \pi A_{it} L \qquad \text{with } \pi = (1-\alpha)\alpha^{(1+\alpha)/(1-\alpha)}. \end{aligned}$$

(c) In each sector, there is a single entrepreneur who spends final output in research and innovates with probability $\phi(n_{it}) = \lambda n_{it}^{\sigma}$ (with $0 < \sigma < 1$) where $n_{it} = R_{it}/A_{it}^*$ and $A_{it}^* = \gamma A_{i,t-1}$. If the entrepreneur succeeds, she becomes the monopolist for one period and $A_{it} = \gamma A_{i,t-1}$. Write down the entrepreneur's maximization problem and show that the research arbitrage equation is

$$\phi'(n_{it})\pi L = 1.$$

In particular, the probability of a successful innovation, $\phi(n_{it})$, is equal across sectors.

(d) It can be shown that the growth rate of GDP is equal to the growth rate of aggregate technology $A_t = \int_0^1 A_{it} di$. Argue heuristically that the growth rate of GDP equals $g = (\gamma - 1)\phi(n)$ where n solves the research-arbitrage equation.

(e) Now, assume that when an entrepreneur makes a successful innovation, with probability χ they fail to get a monopoly (e.g., someone else sees their recipe, runs to the patent office and gets the patent before them). Adjust the entrepreneur's maximization problem and the research arbitrage condition to reflect this. Qualitatively, will there be more or less research when $\chi > 0$? Qualitatively, how does the growth rate depend on χ?