ECON4351 - Final exam

2024

The exam consists of three parts. The first part is a question without any math (except, perhaps, stating an accounting equation), the second question is related to the Solow model, and the third is related to the endogenous-growth frameworks.

1 Directed technical change (30 points)

This question asks you to explain how *directed technical change* can generate an increased skill premium in response to an increase in the supply of skilled workers, as in Acemoglu (2002).

(a) First, describe what the *skill premium* commonly refers to.

(b) Provide a supply-and-demand-based mechanism for why the skill premium may be expected to fall when the supply of skilled labor increases.

(c) Explain how the incentives for skill-biased technical change are affected by an increase in the supply of skilled labor and how the endogenous response of directed technical change can partially or even fully offset the supply-and-demand-based mechanism. Your answer may be helped by an appropriately drawn diagram.

2 Solow model (35 points)

In this problem, we characterize the capital-output ratio in the Solow model. Assume the population growth is n ($L_t = (1+n)^t L_0$) and the labor-augmenting technological growth is g ($A_t = (1+g)^t A_0$). Given a savings rate s, production function $F(K, L)$, and depreciation rate δ, the law of motion for capital K_t is given by

$$K_{t+1} = sF(K_t, A_t L_t) + (1-\delta)K_t.$$

(a) Define capital per efficiency units of labor by $\tilde{k}_t = K_t/(A_t L_t)$. Show that the law of motion for $\tilde{k}_t$ is given by

$$\tilde{k}_{t+1} = \frac{1}{(1+g)(1+n)}\left[sf(\tilde{k}_t) + (1-\delta)\tilde{k}_t\right]$$

where $f(k) = F(k, 1)$. Be clear when you invoke an assumption about the production function F.

(b) Show that in the long run (you may assume a balanced-growth path exists and that the model converges to it),

$$K_t/Y_t = s/(g + n + gn + \delta)$$

(where $Y_t = F(K_t, L_t)$ denotes output). Again, be clear when you invoke an assumption about the production function F. *Hint: solve for $\tilde{k}_{ss}/f(\tilde{k}_{ss})$ by evaluating the law of motion for $\tilde{k}_t$ at its steady state.*

(c) What is the steady state consumption-to-output ratio?

(d) In the long run, what is the growth rate of GDP? What is the growth rate of GDP per capita? (simply state the answers)

(e) Assume $s = 0.2$ and $\delta = 0.04$. Assume GDP growth is 0.01. Compute the long-run capital-to-output ratio.

3 Schumpeterian model (35 points)

In this exercise, we go through the basic Schumpeterian growth model. Final-good production is given by

$$Y_t = L^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^{\alpha} di$$

where each x_{it} is the quantity of intermediate product i and A_{it} reflects the quality of the product. Each intermediate product has its own monopoly, and its price equals its marginal product in the final sector.

(a) Show that the price of good i is given by $p_{it} = \alpha(A_{it}L)^{1-\alpha} x_{it}^{\alpha-1}$.

(b) The intermediate goods are produced one-to-one using the final good. Write down intermediate-good monopolist i's problem and show that the solution to the monopolist's problem is given by

$$\begin{aligned} x_{it} &= \alpha^{2/(1-\alpha)} A_{it} L, \\ \Pi_{it} &= \pi A_{it} L \qquad\qquad \text{with } \pi = (1-\alpha)\alpha^{(1+\alpha)/(1-\alpha)}. \end{aligned}$$

(c) In each sector, there is a single entrepreneur who spends final output on research and innovates with probability $\phi(n_{it})$ (with ϕ a concave function), where $n_{it} = R_{it}/A_{it}^*$ and $A_{it}^* = \gamma A_{i,t-1}$. If the entrepreneur succeeds, she becomes the monopolist for one period and $A_{it} = \gamma A_{i,t-1}$. Write down the entrepreneur's maximization problem and show that the research arbitrage equation is

$$\phi'(n_{it})\pi L = 1.$$

In particular, the probability of a successful innovation, $\phi(n_{it})$, is equal across sectors.

(d) It can be shown that the growth rate of GDP is equal to the growth rate of aggregate technology $A_t = \int_0^1 A_{it} di$. Argue heuristically that the growth rate of GDP equals $g = (\gamma - 1)\phi(n)$ where n solves the research-arbitrage equation.

(e) Now, assume that the government chooses to tax the profits of the monopolist at rate τ, using the tax revenue to finance consumption of the household. Adjust the entrepreneur's maximization problem and the research arbitrage condition to reflect this.

(f) Finally, assume that $\phi(n) = \lambda\sqrt{n}$. Show that the growth rate of the economy depends linearly on the tax rate. What is the effect of the tax on growth? *Hint: Use that $\phi(n) = \lambda\sqrt{n}$ to explicitly solve for n in the research arbitrage equation and then plug this into the growth rate formula to get the growth rate as a function of τ.*