Lecture 1: Ricardian Theory of Trade*

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1 Simple Ricardian Model

- Countries engage in international trade for two basic reasons: They are different from each other in terms of climate, land, capital, labor, and technology.
- They also try to achieve scale economies in production.
- The Ricardian model is based on technological differences across countries. These technological differences are reflected in differences in the productivity of labor. This model can be used also to explain wage disparities across countries.

Assumption of the model

- Labor is the only factor of production.
- Only two goods (say Wine and Cloth) are produced.
- The supply of labor is fixed in each country.
- The productivity of labor in each good is fixed.
- Perfect competition prevails in all markets.

1.1 Preferences

Suppose that the preference in each country (of the representative consumer) is of a Cobb Douglas type. Consumer will choose consumption to maximize

$$\sum_{i=1}^{I} \beta_i \log c_i$$

subject to the budget constraint

$$\sum_{i=1}^{I} p_i c_i \leq w_n L_n$$

where p_i is the price of good i, w_n is the wage in country n, and L_n the country size.

Cobb Douglas preference implies that expenditure of good i is a constant share β_i of income, ie

$$p_i c_i = \beta_i w_n L_n$$

1.2 Technologies

- Ricardo assume constant return to scale technologies.
- With only one factor of production, technology can be described in terms of the number of workers required to produce one unit of commodity i in country n as a_n^i.
- Suppose we have two countries, H and F, and two goods C and W.
- We say that country H has *absolute advantage* in producing C if

$$a_H^C < a_F^C$$

- We say that country H has comparative *advantage* in producing Cif

$$\frac{a_H^C}{a_H^W} < \frac{a_F^C}{a_F^W}$$

- Equivalently, a *country has a comparative advantage in producing a good if the opportunity cost of producing that good in terms of other goods is lower in that country than it is in other countries.*

Production possibility frontier

- Let define Q_H^W the quantity of wine produced at home, and Q_H^C the amount of cloth produced. Hence, the total labor allocated to the production of wine is $a_H^W Q_H^W$ and the total labor used in producting cloth is $a_H^C Q_H^C$. Because the total endowment of labor for home country is L_H, the limits on production are defined by the inequality

$$a_H^C Q_H^C + a_H^W Q_H^W \leq L_H$$

- The graph below to the left shows the PPF for home. Notice the axis of the graph, where in the horizontal axis we have cloth production and wine production in the vertical axis. To get the equation you see in the graph solve to the quantity of wine Q_W from the above equation to get

$$Q_H^W \leq \frac{L}{a_H^W} - \frac{a_H^C}{a_H^W} Q_H^C$$

- The oportunity cost in the production of cloth (notice that we have cloth in the horizontal axis) is the slope $\frac{a_H^C}{a_H^W}$ of the PPF.

1.3 Autarky equilibrium

- Ricardo assumed perfect competition.
- Hence, constant return to scale implies zero profits. Therefore, goods will cost what it costs to pay the workers who make them.

1.3.1 Equilibrium conditions

An equilibrium in each country is a set of commodity prices p_n^i, wage w_n, production amounts Q_n^i, and consumption c_n^i, that satistfy the following conditions

Zero profit conditions : 4 complementary slackness conditions

$$\begin{array}{llll} p^C \leq a_H^C w_H & 0 \leq Q_H^C & \qquad p^C \leq a_F^C w_F & 0 \leq Q_F^C \\ p^W \leq a_H^W w_H & 0 \leq Q_H^W & \qquad p^W \leq a_F^W w_F & 0 \leq Q_F^W \end{array}$$

Labor marker clearing condition: 2 complementary slackness conditions for country n

$$\begin{array}{cc} a_H^C Q_H^C + a_H^W Q_H^W \leq L_H & \qquad a_F^C Q_F^C + a_F^W Q_F^W \leq L_F \\ 0 \leq w_H & \qquad 0 \leq w_F \end{array}$$

*Good market clearing conditions.*2 condition by Walras Laws.

$$Q_H^C = c_H^C = \frac{\beta_C}{p_H^C} w_H L_H \qquad Q_F^C = c_F^C = \frac{\beta_C}{p_F^C} w_F L_F$$

Numeraire

- In general equilibrium analysis, prices correspond to the rate at which one commodity of factor is trade in terms of another, we need to choose a unit of account.
- For this reason let wine to be the numeraire (ie: p^W = 1), then w and p^C is measure in terms of its purchasing power of units of wine.

1.3.2 Solving the autarky equilibrium

1. Find relative prices

- Recall, we have two countries H and F, and two good C and W. Let W be the numeraire.
- Without trade workers will have to buy at home.
- Preference implies that demand is strickly positive independent of the price, so each good will produced at home. This means that we need all Q's to be positive and therefore all ZCP become equalities.
- From the zero profit conditions of wine we have

$$1 = w_n a_n^W$$

and therefore wage is

$$w_n = \frac{1}{a_n^W}$$

and from the zero profit condition in the cloth sector we have

$$p_n^C = \frac{a_n^C}{a_n^W}$$

- *Labor theory of wages*: All prices in the economy (p$'s$ and w) are independent of the preference parameter β.

- In other words, knowledge of technology suffices to establish relative prices when there is CRS and a single factor of production.

2. *Find consumption and labor allocation*

- Using the good market clearing conditions we have that consumption and production of Cloth is

$$\begin{aligned} Q_n^C &= c_n^C = \frac{\beta^C}{p_n^C} w_n L_n \\ &= \frac{\beta^C}{a_n^C} L_n \end{aligned}$$

and production and consumption of wine is

$$Q_n^W = c_n^C = \frac{(1-\beta^C)}{p_n^W} w_n L_n$$

- Labor allocation: β^C of the total labor force is making cloth and the rest wine.

- Real Wages and Welfare: in terms of W the real wage is $\frac{1}{a_n^W}$.

1.4 Trade Equilibrium

- As before suppose there are two countries H and F and two goods C and W

- Assumption : suppose that Home country has a comparative advantage in the production of C, ie

$$\frac{a_H^C}{a_H^W} < \frac{a_F^C}{a_F^W}$$

Equivalently, we say that Home relative productivity in cloth is higher than it is in wine.

1.4.1 Solving the trade equilibrium

- By the autarky case, we have that under autarky $p_H^C \leq p_F^C$. So we expect trade to create incentive to produce more C in the Home country and then trade it for wine with the F country.

- An trade equilibrium in each country is a set of commodity prices p^i (notice prices are equalized across countries so we drop the n index), wage w_n, production amounts Q_n^i, and consumption c_n^i, that satistfy the following conditions

Zero profit conditions : 4 complementary slackness conditions

$$\begin{array}{llll} p^C \leq a_H^C w_H & 0 \leq Q_H^C & \qquad p^C \leq a_F^C w_F & 0 \leq Q_F^C \\ p^W \leq a_H^W w_H & 0 \leq Q_H^W & \qquad p^W \leq a_F^W w_F & 0 \leq Q_F^W \end{array}$$

Labor marker clearing conditions: 2 complementary slackness conditions for country n

$$\begin{array}{cc} a_H^C Q_H^C + a_H^W Q_H^W \leq L_H & \qquad a_F^C Q_F^C + a_F^W Q_F^W \leq L_F \\ 0 \leq w_H & \qquad 0 \leq w_F \end{array}$$

World good market clearing condition. condition by Walras Laws.

$$Q_H^C + Q_F^W = \frac{\beta_C}{p^C} w_H L_H + \frac{\beta_C}{p^C} w_F L_F = c_H + c_F$$

- It is important to notice that it is imposible to hold the four ZPC since we have only three independent variables p^C, w_H, w_F.

- Therefore, we need to consider three cases: complete specialization and two cases of incomplete specialization

Complete specialization

- Since H has comparative advantage, we could assume that H will produce only C and F will produce only W.

- In this case we have

$$\begin{array}{llll} p^C = a_H^C w_H & 0 < Q_H^C & \quad p^C \leq a_F^C w_F & 0 = Q_F^C \\ 1 \leq a_H^W w_H & 0 = Q_H^W & \quad 1 = a_F^W w_F & 0 < Q_F^W \end{array}$$

- In other words C will be procuded at home if the cost of producing it in country H is lower than producing it in country F, ie:

$$\frac{w_H}{w_F} \leq \frac{a_F^C}{a_H^C}$$

- To find p^C we have to use the labor market condition (or PPF) since we know that

$$a_H^C Q_H^C \leq L_H \qquad a_F^W Q_F^W \leq L_F$$

- Now use the world market equilibrium to find the demand for C in the world as

$$Q^C = \beta^C \left(\frac{L_H}{a_H^C} + \frac{L_F}{p^C a_F^W} \right)$$

- Finally using the world supply and demand we have

$$p^C = \frac{\beta^C}{1 - \beta^C} \left(\frac{L_F / a_F^W}{L_H / a_H^C} \right)$$

- Notice that the relative price of C is bigger, 1) the bigger the share of utility, 2) the bigger the labor force in the C importing country F relative to country H, 3) the larger the relative productivity of producing C in the H relative to the production of W in F

- Relative wages can be found as

$$\begin{aligned} w_H &= \frac{p^C}{a_H^C} \\ w_F &= \frac{1}{a_F^W} \end{aligned}$$

- The final step of the check that there are no profits in the sectors that are producing zero quantities. That is $1 \leq a_H^W w_H$ and $p^C \leq a_F^C w_F$. Using the equilibrium conditions derived above we need to check that

$$\frac{\beta^C}{1-\beta^C}\left(\frac{L_F}{L_C}\right) \geq \frac{a_F^W}{a_H^W}$$
$$\frac{\beta^C}{1-\beta^C}\left(\frac{L_F}{L_C}\right) \leq \frac{a_F^C}{a_H^C}$$

- The first condition checks that there p^C is high enough so H workers in the C sectors do not have incentives to move to the W sector. Likewise, make sure that the relative price of W is low so they do not have incentive to make their own cloth.

1.5 The Gains from Trade

- If countries specialize according to their comparative advantage, they all gain from this specialization and trade.
- We will demonstrate these gains from trade in two ways. First, we can think of trade as a new way of producing goods and services (that is, a new technology).
- Another way to see the gains from trade is to consider how trade affects the consumption in each of the two countries.

Method 1:

- Recall that H has specialized in C. Home could produce it directly, but trade with F allows it to produce (or obtain) W by producing and then trade it for W. Consider two alternative ways to use one unit of labor at H. First, it could produce1$/a_H^W$ units of wine. Alternatively, it could produce $1/a_H^C$ units of cloth and trade it for P^C/P^W (recall, the relative barter price measure exhange units of wine by one units of cloth) gallons of wine, so you can obtain $1/a_H^C\left(P^C/P^W\right)$

units of wine. Therefore, you obtain more wine producing indirectly than directly as long as

$$\frac{1}{a_H^C}\left(P^C/P^W\right) > \frac{1}{a_H^W}$$

but this is exactly what they obtain by trading with country F.

Method 2:

- The consumption possibility frontier states the maximum amount of consumption of a good a country can obtain for any given amount of the other commodity.
- In the absence of trade, the consumption possibility curve is the same as the production possibility curve. Trade enlarges the consumption possibility for each of the two countries.

1.6 Exercise

Suppose that two countries H and F produce cheese and wine with the following worker requirements.

	Cheese (per pound)	Wine (per gallon)	Total Labor Force
Home	$a_H^C = 1$	$a_H^W = 2$	L_H
Foreign	$a_F^C = 6$	$a_F^W = 3$	L_F

Assume $L_H = L_F = 100$ workers. Preferences are Cobb Douglas with cheese having a share 3/4.

a. Which country has absolute advantage and which has comparative advantage.

b. Draw the production possibility frontier (PPF) for each country

c. Please state the autarky equilibrium conditions in both countries.

d. Solve for the equilibrium prices and quantities when countries are not allowed to trade. Where the relative price of cheese is greater?. Why?

e. State the trade equilibrium conditions.

f. Now suppose that countries are allowed to trade. Compute the world relative price of cheese and relative wages. Which is bigger?

g. Explain the pattern of trade using the PPFs for both countries.

h. Show that both countries benefit from trade.

2 Ricardian Model with a continuum of goods

- Dornbush, Fisher and Samuelson (1977) consider the case of trade in a Ricardian model between two countries with many commodities.
- They show that it is simpler to consider this case in a continuum of goods.
- To motivate the discussion, first consider the discrete case. As before we have two countries H and F. We can order the N goods according to country H comparative advantage, so that

$$\frac{a_H^1}{a_F^1} \leq \frac{a_H^2}{a_F^2} \leq .... \leq \frac{a_H^N}{a_F^N}$$

- Let $\omega = w_F/w_H$ be the foreign wage relative to home wage.
- As before, country H will produce 1,,...,n-1 goods and F will produce n+1,...,N.
- If we let good 1 to be the numeraire, then we have that goods 2,....,n-1 will have price p^i $= a_H^i/a_H^1$ while goods n+1,...,N will have prices .... and good c mus satisfy

$$\frac{a_H^{c-1}}{a_F^{c-1}} \leq \omega \leq .\frac{a_H^c}{a_F^c}$$

- If the first inequality then country H will produce c.

2.1 Technology in a continuum

- The key DFS insight was to think of the space of commodities in a continuum. Let i be a good where $i \in [0, 1]$.
- As in the discrete case, suppose that the unit labor requirement in the two countries are a$_H(i)$ and $a_F(i)$. Let us define the function $A(i) = a_F(i)/a_H(i)$.
- Order the good so that A'(i)<0, that is the comparative advantage of country H declines as i increases.

- Let w$_h$ and w$_f$ the wages in each country measure in any common unit. Define $\omega = w_H/w_F$

- We kwow that the H country will produce all goods for which the unit cost is the lowest that is good i will be produce by H if

$$a_H(i)w_H \leq a_F(i)w_F$$

or equivalently

$$\omega \leq A(i)$$

- H will produce goods in 0$\leq i \leq \bar{\imath}$ whereas F will produce goods in the interval $\bar{\imath} \leq i \leq 1$

- Notice that at $\bar{\imath}$ we have that $a_H(\bar{\imath})w_H = a_F(\bar{\imath})w_F$ so we can determine $\bar{\imath}$ as $\bar{\imath} = A^{-1}(\omega)$. Notice that $\bar{\imath}$ is decreasing in ω.

Relative good prices

- It is important to notice, that the structure of relative prices are determined by the minimum cost condition.

- Consider the relative price of two goods produced by H.Their relative prices can be written as

$$\begin{aligned} \frac{p(i)}{p(i')} &= \frac{w_H a_H(i)}{w_H a_H(i')} \\ &= a_H(i)/a_H(i') \end{aligned}$$

which is the ratio of unit labor costs.

- *Terms of trade*: The relative price of home good produce i in terms of a good produced at F i" is

$$\begin{aligned} \frac{p(i)}{p(i'')} &= \frac{w_H a_H(i)}{w_F a_F(i'')} \\ &= \omega a_H(i)/a_F(i'') \end{aligned}$$

2.2 Preferences

- We will maintain the usual identical homothetic preference assumption in the two countries.
- In a continuum of goods our consumer problem is written as follows[1]

$$\max \int_0^1 \beta(i) \log c_n(i)$$

subject to the budget constraint

$$\int_0^1 p(i)c_n(i) \leq w_n L_n = Y_n$$

where p_i is the price of good i, w_n is the wage in country n, L_n the country size and Y_n is by definition total income (or total output).

- Cobb Douglas preference implies that expenditure of good i is a constant share β_i of income, ie

  $$p(i)c(i) = \beta(i)w_n L_n$$

  so the share of expendire on good relative to total income is

  $$\beta(i) = \frac{p(i)c(i)}{w_n L_n}$$

- Notice that there is no country index here. Both countries have the same preference.
- Define the fraction of expenditure in each country for goods produced by country H as

  $$\varphi(\bar{\imath}) = \int_0^{\bar{\imath}} \beta(i)di$$

  with $\varphi'(\bar{\imath}) = \beta(i)$.

- This the fraction of each country's income spent on H produced goods. It implies that the fraction spent on F produced goods is $1 - \varphi(\bar{\imath})$.

[1] Notice that CRS implies $\int_0^1 \beta(i) = 1$.

2.3 Trade equilibrium

- We need to close the factor and good markets
- We provide two alternative ways of close the model. First, notice that world demand must equation total output, which equals world labor income (recall that we only have labor as an input)

$$C_W = Y_W = w_H L_H + w_F L_F$$

- Therefore, clearing the H good market requires domestic labor income equal world spending on domestically produced goods

$$Y_H = w_H L_H = \varphi(\bar{\imath}) \left(w_H L_H + w_F L_F \right)$$

where the LHS represents the world demand on H goods.

- Solving for $\omega = w_H / w_F$ we have

$$\begin{aligned} \omega &= \frac{\varphi(\bar{\imath})}{1 - \varphi(\bar{\imath})} \frac{L_F}{L_H} \\ &= B\left(\bar{\imath}, \frac{L_F}{L_H}\right) \end{aligned}$$

- Interpretation of the B(schedule): If the range of domestically produced goods were to increase (a rise in $\bar{\imath}$) for a constant ω, then the demand for domestic labor (goods) would increase and the demand for F labor would fall. A rise in the relative demand for H goods increase the relative wage (recall that relative supply is fixed and there is not labor mobility.)
- An alternative representation is trade balance. This states that imports must be equal to exports in a balanced two country world economy. That is

$$\begin{aligned} (1 - \varphi(\bar{\imath})) Y_H &= \varphi(\bar{\imath}) Y_F \\ (1 - \varphi(\bar{\imath})) w_H L_H &= \varphi(\bar{\imath}) w_F L_F \end{aligned}$$

- On this interpretation, the B-schedule is upward sloping because an increase in the range of good produced by H (keeping ω constant) lowers H imports and raises exports. This trade imbalance is corrected by an increase in the relative wage that raises import demand and reduce export to restore the balance.

The equilibrium of factor and goods market

- Upward sloping demand schedule

$$\omega = B\left(\bar{\imath}, \frac{L_F}{L_H}\right)$$

- Downward sloping supply schedule

$$\omega = A(\bar{\imath})$$

  which pin down values for $\bar{\imath}$ and ω.

- Notice that this equilibrium depends of 1) technology parameters, A(), 2) tastes $\varphi()$ and 3) relative size $\frac{L_F}{L_H}$.

- Quantities $q_H(i)$ and $q_F(i)$ and labor employment can be determined from the demand structure and unit labor requirements.

- The prices $p(i)$ can be determined from the demand equation.

2.4 Analysis

- Having determine the relative wage and the pattern of specialization $\bar{\imath}$, we move on to perform comparative statics experiments.

2.4.1 The effect of an increase in relative size L_F/L_H

- Consider the effect of an increase in the relative size of the rest of the world.

Equilibrium effect

- First, notive that of the two equations, only the B schedule is a function of L_F/L_H. Hence, the B() curve shift upward in proportion to the change in relative size.
- As a result, the relative equilibrium wage raises and reduces the range of goods produced domestically.
- Intuition: at the initial equilibrium, the relative increase in foreign labor creates an excess labor supply, and an excess demand at home.
- This translates into a trade surplus, since for the home country there is more and cheaper labor to produce its goods and this implies more exports.
- To eliminate the trade surplus, we need an increase in the relative wage and an increase in the unit labor cost.
- The increase in the relative unit labor cost, implies a loss for marginal industries, and therefore a reduction in the range of goods is needed as an adjustment.

Welfare implications

- Foreign and home wages remain constant in terms of their own goods.
- Home real wage in terms of the goods that F produces rises.
- A similar argument can be used to show that Foreign, real wages must fall in terms of the initial goods produced at home i<i'.
- This conclusion is rather striking: a rise in F relative labor supply lower its real wage and improves Home's. Surprinsingly, H is better off besides the migration of industries from Home to Foreign.

2.4.2 The effect of technological progress

- Consider a uniform reduction in F unit labor requirement $a_F(i)$.

Equilibrium effect

- This change produces a downward shift in the A schedule.
- At the initial equilibrium a loss of comparative advantage, due to a reduction in foreign labor cost, implies a loss of some industries at home, and therefore a trade deficit.
- The resulting decline in relative wage, serves to restore trade balance equilibrium.

Welfare implications

- Both domestic and foreign real wage increase. Show it!.