General Equilibrium and Gains from Trade

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1 Basic concepts in general equilibrium

- Main building-blocks of competitive general equilibrium theory.
- These notes are meant as a refresher and are incomplete.

1.1 Firms

- Consider an economy where two goods $X_1$ and $X_2$ are produced by two firms.

- For simplicity suppose that there exist only two factors of production capital $K$ and labor $L$.

- Suppose that each good $i = 1, 2$ can be produced using the following technologies

$$X_i = F^i(K_i, L_i)$$

- Assume also full employment of resources, i.e.

$$\bar{K} = K_1 + K_2$$
$$\bar{L} = L_1 + L_2$$

Production possibility frontier

- Before we turn the problem of the firm, let us define the concept of technical efficiency in the allocation of factors.

- The production possibility frontier (PPF) measures the maximum amount of good 2 that can be produced given the production level of good 2.

- Mathematically, we need to solve the following problem

$$X_2(X_1) = \max_{K_2, L_2} F^2(K_2, L_2)$$

$$st :$$

$$F^1(\bar{K} - K_2, \bar{L} - L_2) = X_1$$

- Solving this problem gives an implicit function for $X_2(X_1) = F^2(K^*(X_1), L^*(X_1))$ which for a given $X_1$ finds the most efficient level of $X_2$. 

Using the envelope theorem it is possible to show that the slope of the PPF is the \textit{marginal rate of transformation} (MRT)\textsuperscript{1}.

\[ \frac{dX_1}{dX_2} = MRT = \frac{F^2_L}{F^1_L} = \frac{F^2_K}{F^1_K} \]

Intuitively, the PPF measures how much of good 2 the economy would have to sacrifice in order to increase production of good 1 by one unit, given technologies (F’s) and factors available (endowments).

\textit{The problem of the firm}

So far we have a characterization of the technology, but which combination of \((X_1, X_2)\) will be produced in this economy?

Suppose for that now good prices \(p_1, p_2\) are given, and consider the problem of the each firm

\[
\max_{K, L} p_j X_j - w L_j - r K_j \\
\text{st :} \\
X_j = F^j(K_j, L_j)
\]

Optimization in each firm implies

\[
p_1 F^1_{L_1} = w \quad p_2 F^2_{L_2} = w \\
p_1 F^1_{K_1} = r \quad p_2 F^2_{K_2} = r
\]

which implies

\[
\frac{p_1}{p_2} = \frac{F^2_{L_2}}{F^1_{L_1}} = \frac{F^2_{K_2}}{F^1_{K_1}}
\]

Now using the endowment equation we have \(dL_2 = -dL_1\) and \(dK_2 = -dK_1\) and

\[
\frac{p_1}{p_2} = \frac{F^2_{L_1}}{F^1_{L_1}} = \frac{F^2_{K_1}}{F^1_{K_1}} = \frac{dX_2}{dX_1} = MRT
\]

That is when markets are competitive and there are no distortions, the economy will produce where the MRT equals the price ratio \(p_1/p_2\).

\textsuperscript{1}Notice that the difference between the MRT and the MRTS. MRT measures ratios across firms for a given factors whereas the MRTS measures ratios across factors for a given firm.
1.2 Consumers

- Suppose there is a representative consumer that has preference $U(c_1, c_2)$ over the two goods produced in the economy.
- Suppose again (for the moment) that prices $p_1/p_2$ are given to the consumer.
- The consumer will choose consumption levels $c_1, c_2$ by maximizing utility subject to familiar budget constraints

$$\max_{c_1, c_2} U(c_1, c_2)$$

$$\text{st : } p_1 c_1 + p_2 c_2 = rK + wL$$

- Utility maximization implies

$$\frac{p_1}{p_2} = \frac{U_1}{U_2} = MRS$$

1.3 General equilibrium

- For a closed economy we need to determine endogenously the following variables 1) good prices $\{p_1, p_2\}$, 2) factor prices $\{w, r\}$, 3) demand for the representative consumer $c_1, c_2$, 4) factor demand in each sector $j$ $L_j, K_j$ and 5) output for each firm $j$ $X_j$.
- We now can define our competitive equilibrium for a closed economy as

**Definition**

- An set of variables $\{p_1, p_2, w, r, c_1, c_2, L_j, K_j, X_j\}$ is a general equilibrium if

1. Each firm $j$ maximizes profit

$$\pi_j = p_j X_j - w L_j - r K_j$$

$$\text{st : } X_j = F^j(K_j, L_j)$$

2. The representative consumer maximizes utility

$$\max_{c_j} u(c)$$

$$\text{st : } p_1 c_1 + p_2 c_2 \leq rK + wL$$
3. Factor markets clear

\[ \tilde{K} = K_1 + K_2 \]
\[ \tilde{L} = L_1 + L_2 \]

4. Good markets clear

\[ c_1 = X_1 \]
\[ c_2 = X_2 \]

1.3.1 Closed economy (autarky) equilibrium

- Let us recapitulate the main equilibrium conditions for our closed economy.
- Profit maximizing producers will pick their output such that

\[ \frac{p_1}{p_2} = MRT \]

- Consumers choose their consumption level such that

\[ \frac{p_1}{p_2} = MRS \]

- Finally, the market clearing requires the supply of each good to be equal to demand, i.e.:

\[ c_1 = X_1 \]
\[ c_2 = X_2 \]

- Figure 1 depicts the equilibrium for this closed economy that satisfies the three conditions.
- Notice that firms produce at point A where the slope of the PPF is equal to the price ratio \( p_1/p_2 \).
- Similarly, consumers consume optimally at point A where the slope of the indifference curve is equal to the price ratio.

\[ \Rightarrow MRT = \frac{p_1}{p_2} = MRS \]
1.3.2 Open economy equilibrium

- Now suppose that an economy can trade with the world at a fixed price $p^* = p_1^*/p_2^*$ (different from autarky to make things more interesting!).
- We retain the two first conditions: that is, consumer and firms behave optimally.
- However, with international trade the economy is no longer constrained to consume what it can produce.
- Instead, now we need a condition where the value a country sells on the world market be equal to what it buys $\implies$
- Trade balance condition requires that the value of all imports is equal to the value of all exports, i.e.: $p_1^*(c_1 - X_2) + p_2^*(c_2 - X_2) = 0$
- To gain some intuition, rearrange this equation as follows $p_1^*X_1 + p_2^*X_2 = p_1^*c_1 + p_2^*c_2$
- That is the value of production for a country must be equal to the value of consumption.
- The value of production represents the income of the country. Notice that the slope of this curve is $p_1^*/p_2^*$.
- Given this budget line for this country, it is free to choose any consumption point along this line.
- Summarizing, a general equilibrium for a open economy is given by

  - Firm maximization $\frac{p_1^*}{p_2^*} = MRT$
  - Consumer maximization $\frac{p_1^*}{p_2^*} = MRS$
  - Trade balance $p_1^*(X_1 - c_1) + p_2^*(X_2 - c_2) = 0$

- Figure 2 represents an equilibrium that satisfies all these conditions for a fixed relative price $p_1^*/p_2^*$.
- Producers optimize by choosing at a point B, where the price ratio $p_1^*/p_2^*$ equals the MRT (slope of the PPF).
• Consumers optimize by choosing a consumption level $C$, where the price ratio equals the MRS.

• For this particular example, the economy imports good 1 and export good 2.
2 Gains from trade

- *Free trade is preferred to autarky*, because a trading equilibrium allows for all allocations which would have been feasible without trade, plus some more.

- The free trade production/consumption bundle would not have been a choice given autarky prices

- Figure 3 illustrate that at trade prices the domestic consumption bundle is restricted by BC which includes the autarky consumption bundle but also other choices

2.1 The sources of gains from trade

1. Separation of consumption from production

2. Exploitation of comparative advantage

2.2 Proofing the gains from trade

- Trade can always produce a more efficient outcome than autarky.

- We state the result in a general form and then provide an example.

**Theorem 1** Assume 1) convex technologies and preferences, 2) perfect competition, and 3) one representative consumer in each country. Then every country gains from trade.

**Proof:**

- Competition implies $r(p, v) = \max px$

- $p^T x^T \geq p^T x^A$ because profits are maximzed

- Market clearing under autarky implies $c^A = x^A$.

- Recall the expenditure function, $e(p, u) = \min_c \{pc \mid u(c) \geq u\}$. 


Since market equilibrium requires total expenditure equal to total income, i.e. \( e(p^T, u^T) = r(p^T, v) \), then

\[
e(p^T, u^A) \leq p^T c^A \quad \text{by def. of expenditure function}
= p^T x^A \quad \text{by condition of the autarky equilibrium}
\leq p^T x^T \quad \text{by definition of the revenue function}
= e(p^T, u^T)
\]

Since \( e(p^T, u^A) \) is increasing in \( u \),

\[
\implies u^A \leq u^T.
\]

Therefore, every country gains from trade.

– first inequality\(\implies\) shows that consumers at trade prices can attain autarky utility more economically
– second inequality shows how producers can generate greater value of money
– the two together shows that consumers can use the higher income to achieve greater utility
2.3 Basic law of comparative advantage

- Countries gain from trade because they export goods whose prices are relatively higher in the trading equilibrium, and because they import goods whose prices are relatively lower, see Figure 4.
- This positive correlation between net imports and autarky prices is always present.

Theorem 2 Let $m_j = c_j - x_j$ be the vector of relative imports to country $j$. The following is true:

i. $p_j m_j \geq 0, \forall j$ (trade is preferred to autarky)

ii. $pm_j = 0, \forall j$, (trade balance)

iii. $\sum_j m_j = 0, \forall j$, (market clearing)

Proof: $c^T$ is weakly preferred to $c^A$; since $c^A$ is chosen at prices $p^A$ it must cost less at these prices (written as vectors), i.e.

$$p^A c^T \geq p^A c^A$$  \hspace{1cm} (1)

Profit maximization implies that

$$p^A x^A \geq p^A x^T$$  \hspace{1cm} (2)

and the definition of autarky means

$$x^A = c^A$$  \hspace{1cm} (3)

Together these imply

$$p^A (c^T - x^T) \geq 0$$  \hspace{1cm} (4)

$(c^T - x^T)$ are imports, so the budget constraint is

$$p^T (c^T - x^T) = 0$$  \hspace{1cm} (5)

Subtracting (5) from (4) gives

$$(p^A - p^T) (c^T - x^T) \geq 0$$  \hspace{1cm} (6)

which states that, on average, countries import goods which are relatively more expensive under autarky that under free trade.
At equilibrium the home import vector $m^h$ equals minus the foreign import vector

$$m^h = e^h - x^h = -(e^f - x^f)$$  \hspace{1cm} (7)

(omit superscripts T; all variables take their free trade values unless a superscript A is attached)

$$ (p^h)^A m^h \geq 0, \quad - (p^f)^A m^h \geq 0 $$  \hspace{1cm} (8)

adding, we obtain

$$ \left[ (p^h)^A - (p^f)^A \right] m^h \geq 0 $$  \hspace{1cm} (9)

which says that on average, a country will import those goods which in autarky are more expensive at home than abroad.

The relationship between the equilibrium world prices and autarky prices

$$ (p^h)^A m^h \geq p m^h = 0 \geq (p^f)^A m^h $$  \hspace{1cm} (10)

The inner product between the vector of world prices and the home country vector $p m^h$ equals zero.
2.4 What determines autarky prices?

- From the basic law of comparative advantage we put emphasis on the role of differences in autarky prices in the determination of trade patterns and trade volumes.
- In other words, if \( p_j^A = p^A, \ \forall j \), there is no trade.
- There are three fundamental sources of autarky price differences across countries:
  1. Differences in tastes.
  2. Differences in technologies.
  3. Differences in endowments.
- In what follows of the course we will concentrate mainly on parts 1 and 2.
- Let us review point 1 briefly before we move on.

**Difference in preferences**

- Preferences can influence comparative advantage if 1) preferences differ across countries or if 2) they are identical but not homothetic.
- Consider the example of Figure 5.
- Here both countries have the same endowments and technologies (why?), but country \( H \) has a higher relative preference for good 1 and therefore \( p_1^A / p_2^A \) is higher in country \( H \).
- Opening for trade, country H will import good 1, which is relatively expensive at home.

**Non-homothetic preferences**

- Consider the case of identical but non-homothetic preferences.
- Suppose that country \( H \) is richer (higher income).
- Because \( H \) is richer, its demand is tilted towards good 1 and therefore \( p_1^A / p_2^A \) is higher in country \( H \).
3 Optimal trade policy

- Trade policy under perfect competition: Partial equilibrium
- Focus on one single sector
- All prices except this sector’s output price are fixed
- Marginal utility of income fixed

Notation:
- World price: $p^W$
- Price: $p = p^W + t$
- Consumer surplus is given by indirect utility function, $v(p)$
- Roy’s identity gives consumers’ demand, $d(p) = -v'(p)$
- Profits, $py - c(y)$
- Profit maximization, $p = c'(y)$
- Import demand, $m(p) = d(p) - y$

Optimal tariff argument
What level of tariffs is welfare maximizing?
Depends on the extent to which the country can change its terms of trade (ToT).

$$W = v(p) + tm(p) + py - c(y)$$

where is import demand function.

$$\frac{dW}{dt} = -d(p) \frac{dp}{dt} + m + t \frac{dm}{dt} \frac{dp}{dt} + \frac{dp}{dt} y + (p - c'(y))dy$$

$$= (1 - \frac{dp}{dt})m + t \frac{dm}{dt} \frac{dp}{dt}$$

$$= t \frac{dm}{dp} \frac{dp}{dt} - \frac{dp^W}{dt} m$$

To get from line one to line two we use that the fact that since $y$ maximizes profits at prices $p$,
then a small change in the production vector must have zero effect on profits, i.e. $(p + c')dy = 0$,
and that $c = m + y$. To get from line two to line three, we use that $[1 - (dp/dt)] = -dp^W/dt$.)
Small country case:
World prices are fixed, $\frac{dp}{dt} = 1$, $\frac{dp^W}{dt} = 0$

\[ \Rightarrow \frac{dW}{dt} = t \frac{dm}{dp} \] (13)

\[ \Rightarrow \frac{dW}{dt} \bigg|_{t=0} = 0 \] (14)

\[ \Rightarrow t^* = 0 \] (15)

(see Figure 5)

Large country case:
Import demand of large country is "large enough" to have an impact on world prices, $\frac{dp}{dt} \neq 1$, $\frac{dp^W}{dt} < 0$

\[ \Rightarrow \frac{dW}{dt} = t \frac{dm}{dp} \frac{dp}{dt} - \frac{dp^W}{dt} m \] (16)

\[ \Rightarrow \frac{dW}{dt} \bigg|_{t=0} = - \frac{dp^W}{dt} m > 0 \] (17)

Optimal tariff
\[ \Rightarrow \frac{dW}{dt} = 0 \Rightarrow \frac{t^*}{p^W} = \left( \frac{dp^W}{dt} \frac{m}{p^W} \right) / \left( \frac{dm}{dp} \frac{dp}{dt} \right) \] (18)

use that domestic import demand = foreign export supply:

\[ m = x \Rightarrow \frac{dm}{dp} \frac{dp}{dt} = \frac{dx}{dt} \] (19)

\[ \Rightarrow \frac{t^*}{p^W} = \left( \frac{dp^W}{dt} \frac{x}{p^W} \right) / \frac{dx}{dt} = 1 / \left( \frac{dx}{dt} \frac{p^W}{x} \right) \] (20)

Optimal percentage tariff ($\frac{t^*}{p^W}$) equals the inverse of the elasticity of foreign export supply
- a small country: the elasticity of foreign export supply is infinite $\Rightarrow$ the optimal tariff = 0
- a large country: the elasticity of foreign export supply is finite and positive $\Rightarrow$ the optimal tariff $> 0$

But how "large" does an importing country have to be before it is reasonable to treat the elasticity of foreign export supply as less than infinite?

- rewrite (16) to provide a second interpretation of optimal tariff more suitable to answer this question:

\[ \frac{t^*}{p} = \left( \frac{dm}{dp} \frac{p}{m} \right)^{-1} \left( \frac{dp^W}{dt} / \frac{dp}{dt} \right) \] (21)
Optimal percentage tariff \((t^*/p)\) equals the inverse of the elasticity of import demand times the ratio of the change in the relative foreign and domestic price of imports. Since the elasticity of import demand is negative, and assuming that \(dp/dt > 0 \Rightarrow \) the optimal tariff is positive provided that \(dp^W/dt < 0\).

When exporting firms face a tariff, how will they adjust the net of tariff price \((p^W)\) that they receive?

- Will they absorb part of the tariff; i.e. \(dp^W/dt < 0\) and \(dp/dt < 1\) ?
- \(dp/dt = \)"pass-through" of the tariff. If domestic prices rise by less than the tariffs \((dp/dt < 1)\) it means that foreign exporters have absorbed part of the tariff so that \(dp^W/dt < 0\). The smaller the pass-through of the tariff, the larger the optimal tariff. (see Figure 6).