# Exam ECON3620/4620: Public Economics – Taxation, 2nd exam 2025

## 1. True/false questions (20% weight)

You need to explain your answer, just a true false or "it depends" statement without any arguments is not rewarded.

(a) The altruistic inheritance model suggests that inheritance taxation leads to an efficiency loss.

(b) When taxation of a commodity is changed, the consumers (and not the suppliers) bear the burden of the tax change when the consumer demand is inelastic.

(c) A lump sum tax is distortionary.

(d) The Gini coefficient builds on the Lorenz-curve.

## Solutions

(a) Yes, this is correct. Parents treat their children's utility as part of their own, so a bequest tax reduces the overall resources available to the family, distorting the parent's consumption-saving decision and creating inefficiency.

(b) Yes, this is correct. That means even if the price goes up due to a tax, they will continue to buy roughly the same amount.

(c) False. A lump-sum tax is generally considered non-distortionary as it does not depend on any economic behavior such as labor supply, saving, or investment. Since everyone pays the same fixed amount regardless of their choices, it does not create incentives to alter behavior in order to reduce tax liability.

(d) Correct. The Gini coefficient measures inequality by calculating the area between the Lorenz curve and the line of perfect equality.

## 2. Elasticities and deadweight loss (40% weight)

Suppose a consumer has utility given by:

$$u(x,y) = a\ln(x) + y$$

and faces the budget constraint:

$$p_x x + p_y y = M$$

(a) Derive the Marshallian demand functions for $x$ and $y$.

(b) Does good $x$ have an income effect? Explain.

(c) Derive the price elasticity of demand for $x$. Is this the uncompensated (Marshallian) or compensated (Hicksian) elasticity?

(d) Suppose the price elasticity of supply for good $x$ is 0.5. How will a unit tax of size $t$ on good $x$ be split between demand and supply?

(e) Derive the expenditure function and calculate the compensating variation for a tax increase from $p_x$ to $p_x + t$.

(f) Derive the tax revenue generated from the tax.

(g) Derive the deadweight loss (efficiency loss) from the tax.

(h) Why does the deadweight loss increase in $a$?

## Solutions

(a) **Marshallian Demand**

Maximize $u(x,y) = a\ln(x) + y$ subject to $p_x x + p_y y = M$.

First-order conditions yield:

$$x = \frac{ap_y}{p_x}, \qquad y = \frac{M - ap_y}{p_y}$$

(b) **Income Effect**

Demand for $x$ is:

$$x = \frac{ap_y}{p_x}$$

which is independent of income $M$. Therefore, good $x$ has **no income effect**.

(c) **Price Elasticity of Demand for $x$**

Demand for $x$ is:

$$x(p_x) = \frac{ap_y}{p_x}$$

Taking the derivative:

$$\frac{dx}{dp_x} = -\frac{ap_y}{p_x^2}$$

Elasticity formula:

$$\varepsilon_{x,p_x} = \frac{dx}{dp_x} \cdot \frac{p_x}{x} = -1$$

Since utility is quasi-linear, good $x$ has no income effect. Thus, this is both the Marshallian and Hicksian elasticity:

$$\boxed{\varepsilon_{x,p_x} = -1}$$

(d) **Tax Incidence**

With a unit tax of size $t$ on good $x$, and elasticities:

$$\varepsilon_d = -1, \quad \varepsilon_s = 0.5$$

The fraction of the tax borne by consumers is:

$$\text{Buyer's share} = \frac{\varepsilon_s}{\varepsilon_s - \varepsilon_d} = \frac{0.5}{0.5+1} = \frac{1}{3}$$

And by producers:

$$\text{Seller's share} = \frac{-\varepsilon_d}{\varepsilon_s - \varepsilon_d} = \frac{1}{1.5} = \frac{2}{3}$$

So the tax is split 1/3 on consumers and 2/3 on producers.

(e) **Expenditure Function and Compensating Variation**

The expenditure function is:

$$e(p_x, p_y, u) = p_y u + a p_y \left[1 - \ln\left(\frac{a p_y}{p_x}\right)\right]$$

The compensating variation for a price increase from $p_x$ to $p_x + t$ is:

$$CV = e(p_x + t, p_y, u_0) - e(p_x, p_y, u_0) = a p_y \ln\left(\frac{p_x + t}{p_x}\right)$$

(f) **Tax Revenue**

After-tax demand is:

$$x = \frac{a p_y}{p_x + t}$$

So tax revenue is:

$$T = t \cdot x = t \cdot \frac{a p_y}{p_x + t}$$

(g) **Deadweight Loss (DWL)**

The efficiency loss is:

$$DWL = CV - T = a p_y \ln\left(\frac{p_x + t}{p_x}\right) - t \cdot \frac{a p_y}{p_x + t}$$

Or:

$$\boxed{DWL = a p_y \left[\ln\left(1 + \frac{t}{p_x}\right) - \frac{t}{p_x + t}\right]}$$

(h) **Why Does DWL Increase in $a$?**

The parameter $a$ reflects how much utility the consumer derives from good $x$. When $a$ increases:

- The consumer places more weight on $x$ in their utility function.
- The marginal utility lost from distorting consumption of $x$ increases.
- Therefore, the welfare loss per unit of tax increases.

So:

DWL is directly proportional to $a$

## 3. On the ETI (40% weight)

(a) The change in taxable labor income for individual $i$ between income at time $t$, $z_{it}$, and income at time $t+3$, $z_{t+3}$, can be explained by the change in the net-of-tax rate, $1 - \tau_i$ over the same period, seen as

$$\log \frac{z_{it+3}}{z_{it}} = \alpha_t + e \log \frac{1 - \tau_{it+3}}{1 - \tau_{it}} + \varepsilon_i$$

where $\alpha_t$ is a time-specific effect and $\varepsilon_i$ is the error term.

The key parameter, $e$ in the equation measures the elasticity of taxable income (ETI). Explain why most authors use an instrument for the change in the net-of-tax rate in the identification of $e$ and why some representation of income in period $t$ is entered as an additional variable in the equation because of the instrument that is commonly used in the literature.

(b) Say that the marginal tax rate is the top rate of the scheme for taxation of earnings. The following formula places the elasticity of taxable, $e$, into an expression for the optimal tax, $\tau^*$:

$$\tau^* = \frac{1-g}{1-g+ae}$$

Describe how the optimal top marginal tax rate varies with $g$, $a$, and $e$. Does it reflect the top of the Laffer-curve for $g = 0$?

(c) An alternative empirical strategy to obtain information about how taxpayers respond to tax changes is defined by estimating a structural labor supply model and then use the estimated model to simulate effects of tax change. Explain briefly how the analyst may go forward to estimate a structural labor supply model how it can be used to simulate effects of changes in the tax system.

(d) Discuss the advantages and disadvantages of the two approaches—the quasi-experimental ETI approach and the structural labor supply model—for analyzing how taxes affect labor earnings.

## Solutions

(a) Because the explanatory variable is clearly endogenous, most authors use an instrument for the change in the tax variable. Typically, both marginal tax rates—$\tau_{it+1}$ and $\tau_{it+3}$—are calculated using first-period income, and an instrument is derived accordingly (in a 2SLS framework). However, this method of instrumenting based on initial income introduces a mean reversion problem. Since the literature often focuses on changes at high income levels, individuals with temporarily elevated incomes may revert to their usual income levels, leading to potential bias in the results. Students have been introduced to several approaches to address mean reversion. A strong answer should mention some of these methods: for instance, Auten & Carroll (1999) use first-period income directly, while Gruber & Saez (2002) employ income splines based on the first period.

(b) $\tau^*$ decreases with $g$, $a$, and $e$. Should mention that $g$ and $a$ are welfare weights and a measure of the thinness of the tail of the income distribution (large $a$ gives a thin tail), respectively. When the the social marginal value of a krone in consumption for top-income individuals is 0, the optimal tax reflects the top of the Laffer-curve.

(c) The students have been introduced to discrete choice modeling, choosing between jobs in different working hours intervals. When also specifying the utility function, estimation by maximum-likelihood gives parameters which can be used to determine the probabilities for the discrete choices. Estimation on cross-sectional data. The model can

in turn be used to simulate effects of changes in wages and taxes. Quasi-experimental approach: This method relies on econometric techniques that require fewer assumptions and are generally easier to implement than structural models. Can provide causal results when estimation is carried out using well-designed empirical strategies. It often yields estimates for intensive margin responses and produces local estimates, meaning the elasticity of taxable income (ETI) is typically identified based on the behavior of specific subgroups, such as high-income earners. This can limit the generalizability of the results to broader populations or new policy contexts. Nevertheless, since the ETI can serve as a sufficient statistic, this approach aligns closely with optimal tax theory by capturing the key elements needed for welfare evaluations of tax reforms.

Structural labor supply model: This approach is typically more econometrically demanding than the quasi-experimental method. Practitioners often employ a discrete choice random utility framework with error terms assumed to follow an extreme value distribution. The model can be criticized for its simplicity—individuals are assumed to optimize only over consumption and leisure, which may result in misspecification. However, its strength lies in its ability to simulate policy effects across the entire taxpayer population, accounting for both intensive and extensive margins. Once validated, a structural model allows for robust simulation of a wide range of prospective policy reforms.