Vidar Christiansen

**Econ 4620 Public Economics S2012: Seminar assignment for week 10**

**Problem 1**

Denote the income of a person by y and let T(y) be the income tax function, which is progressive.

Give examples of measures of progressivity that can be used.

Let x be the demand for a commodity which is taxed at a constant rate t. For a fixed price we write the demand function as x(y-T(y)). The commodity tax paid can be expressed as $\theta(y) = tx((y - T(y))$. Denote the average commodity tax liability $\bar{\theta}(y) = \theta(y) / y = tx((y - T(y)) / y$.

Discuss conditions for the commodity tax being progressive or regressive wrt income y.

**Problem 2**

Consider the following optimality condition from Atkinson and Stiglitz .

$$\frac{HU_G}{\alpha} = \frac{\lambda}{\alpha} p_G - \frac{\lambda}{\alpha} tH \frac{\partial X}{\partial G}$$

What is the underlying model (described in words)?

Which is the optimum characterised by the condition?

What is the interpretation if *t=0*, $\frac{\lambda}{\alpha} = 1$?

What is the interpretation of $\frac{\lambda}{\alpha}$?

What is the interpretation if $\frac{\partial X}{\partial G} = 0$ and $\frac{\lambda}{\alpha} > 1$?

What is the interpretation of $tH \frac{\partial X}{\partial G} (\neq 0)$?

Why is $tH \frac{\partial X}{\partial G}$ multiplied by $\frac{\lambda}{\alpha}$?

**Problem 3**

Assume that an amount X of a good is produced by means of labour, *H*. The productivity of the economy depends on the supply of a publicly provided good, g, such that the production function can be written

$X=A(g)H$

where $A'(g)>0$.

*g* is produced by means of the X good and the required amount of input is *kg*, such that what is left for consumption of the good is

$C=X-kg$

where *k* is a positive constant.

Assume that one household represents many consumers and let the labour supply of the household be *N*. The balance equation for labour is then

$H=N$.

The utility function of the household is

$U(C,N)$.

a) Show that the first-best condition for production efficiency is

$NA'(g)=k$.

b) Explain the contents of the condition.

Assume there is a market where X is the numeraire, and *w* is the wage rate.

c) Explain why the equilibrium wage rate is equal to $A(g)$.

Suppose that *g* is financed by a tax on earnings with tax rate *t*.

d) Explain why the labour supply can be written as $L((1-t)w)$ where *L* is the supply function.

e) Explain why the tax revenue is $twL((1-t)w)$ and must equal *kg*.

f) State the reason why it is desirable to maximise $(1-t)w$ to achieve a social optimum provided that a tax on earnings is used.

g) Analyse if production efficiency is achieved at the social optimum when *g* is tax-financed and the budget constraint in e) must hold.

**Problem 4**

Consider fireworks on New Year's Eve as a public good.

Characterise the (first best) optimum amount of fireworks.

Assume that the amount of fireworks is determined by voluntary contributions as a Nash equilibrium in a non-co-operative game.

Assuming there are *n* agents, set up a simple model and characterise an agent's choice of contribution.

Compare the equilibrium with the first best allocation.

**Problem 5**

Consider an excludable public good.

In which circumstances may exclusion be first best efficient?

**Problem 6**

Assume that a public good is publicly provided and funded by a tax system where individualised lump-sum taxes are excluded. To increase the provision of a public good, taxes are raised. The tax increase has substitution and income effects.

To what extent would you be concerned with income and substitution effects, respectively, in order to allow for the efficiency effects of the tax increase?

**Problem 7**

Assume there is a population of heterogeneous agents supplying labour (h) at fixed but different wage rates (w) and paying a linear income tax. The tax liability of a person is given by $ty+T$ where $y=wh$ is income and t and T are strictly positive parameters of the tax function. Further provision of a public good can be financed by increasing t and/or T. Compare the welfare effects of increasing T and t, respectively.