ECON4820 – Strategic Competition – Postponed exam, spring 2022

Problem 1 – the Bertrand paradox (25%)

In class we have discussed the *Bertrand paradox*, which is about the unique Nash equilibrium in the classical model where two firms who both sell a perfectly homogeneous good (perfect substitutes) and compete in prices. Assume that the two firms share the same marginal cost.

a)

Explain the paradox in your own words. What is the equilibrium, and why is it unique?

Answer:

The unique equilibrium is that both firms charge the common marginal cost. Whenever one firm charges a price above mc, the other should charge a price just below that, steal the entire market, and make a lot of profits. But then the other firm should charge slightly below that again, and so on. If a firm charges a price below mc, it will sell a lot of units, but incur a loss on every one of them.

In class we have discussed three different extensions of the model, that all lead to a different price equilibrium. The three extensions are i) capacity constraints, ii) imperfect substitutes, and iii) search costs/imperfectly informed consumers.

b)

Explain why the equilibrium is no longer the same as in problem a) when firms are capacity constrained.

c)

Explain why the equilibrium is no longer the same as in problem a) if the goods provided by the firms are imperfect substitutes.

d)

Explain why the equilibrium is no longer the same as in problem a) if there are search costs/the consumers are imperfectly informed.

e)

What is the common theme connecting problems b, c and d?

Answer:

With capacity constraints there is no incentive to lower the price beyond the point at which you sell out your capacity.

When goods are imperfect substitutes, some consumers will prefer your good, even if you charge a higher price than your competitor.

When consumers have to pay a search cost to learn about the competitors' prices, each store holds a little market power, and can charge a price a little above its competitors without losing the sale.

In each case the incentive to compete/lower your price is weakened, and the equilibrium implies some positive margin to each firm.

Problem 2 – Vertical relationships (25%)

In real life, we observe many contracts that do not rely exclusively on linear prices (a constant per-unit price). In class, we argued that one reason for this is the poor performance of the linear price contracts in certain vertical relationships.

a)

Explain the equilibrium when a monopolist manufacturer sells his goods wholesale to a monopolist retailer, using a linear price contract. Why is the outcome undesirable?

Answer:

Both firms mark up their price in order to make some profits. That is the only way they can make money. But in marking up their price, they fail to take into account the effect this has on the other firm. The price therefore gets marked up twice, the price is higher, and aggregate profits are lower, than under a vertically integrated monopolist. This also means that aggregate welfare is lower than under vertical integration.

b)

Explain how a two-part tariff (the franchise model) solves this problem.

Answer:

With a two-part tariff, we decouple the link between the manufacturer's profits and the wholesale price. This allows the firms to make profits without distorting the price, and the retailer gets the right incentives to set the retail price. (It could also be the other way around, that the manufacturer sets a high wholesale price, which is then passed on without markup to consumers. Then the manufacturer would make all the profits, and could transfer profits to the retailer through the fixed fee.)

c)

We are still in a model with a monopolist manufacturer and a monopolist retailer. Explain what we mean by downstream service provision. *What additional problems arise when sales of the good depend on such service provision? Does a two-part tariff solve this new problem as well?*

Answer:

When sales depend on downstream service provision, like advertisements, advice to consumers, demonstration facilities, etc, the manufacturer has to figure out how to

provide the retailer with the right incentives to provide such services. With a linear price, the manufacturer captures some of the marginal profits, and the retailer has too weak incentives to provide services. With a two-part tariff, the retailer is the residual claimant, and makes 100 per cent of the marginal profits. He therefore has the right incentives to provide services.

d)

Explain what additional problems arise for the manufacturer when he sells his goods wholesale to more than one retailer, but sales still depend on downstream service provision. Can you think of any interventions the manufacturer can undertake in order to improve on the outcome under linear price contracts?

Answer:

When there are more retailers, a two-part tariff isn't sufficient to provide them with the right incentives to provide services. The reason is that when one retailer provides services, the increased demand could leak to the other retailers also, hence the retailer who provided the service doesn't capture all the marginal profits, even with a two-part tariff. This is a problem for the manufacturer, who does benefit irrespective of where the final sale actually happens. One way to solve this could be by implementing local monopolies, in which each retailer is guaranteed not to have any competing retailers close by. In a brick-and-mortar world, this reduces the leakage.

Problem 3 – Horizontal product differentiation (50%)

In this problem we assume that products and consumers are located at different points along the real line [0,1]. Assume further that if a consumer of type x buys product i at location l_i, then she derives utility $v_{i(x)} = r - \tau(x - l_i)^2 - p_i$. Further, assume that firm i has a constant marginal production cost c_i, which might differ between the firms.

a) *Explain what the parameters r and τ measure, and how we should interpret them.*

Answer: r measures the willingness to pay for a product of this kind, the utility a consumer gets irrespective of which variety he buys. It is the utility he gets when he buys a perfect match to his preferences (location). The parameter τ is proportional to the marginal cost of compromising with your true preferences. The higher is τ, the more the willingness to pay declines with distance to the product offered.

b) *Explain what we mean by the indifferent consumer and show that she will be located at $\hat{x} = \frac{l_1+l_2}{2} - \frac{p_1-p_2}{2\tau(l_2-l_1)}$.*

Answer: The indifferent consumer is the one who, given the prices and locations, would be indifferent between the two goods. Since preferences are monotone in the location of the consumer, all consumers to the left (right) of the indifferent one would buy from firm 1 (2).

The indifferent consumer has $\mathbf{r - p_1 - \tau(\hat{x} - l_1)^2 = r - p_2 - \tau(\hat{x} - l_2)^2}$. Solve for $\hat{\mathbf{x}}$.

c) We imagine a game taking place in two stages; first the firms choose locations on the line $[0,1]$, then the locations become common knowledge and the firms compete in prices. In this exam, we are only concerned with the second-stage price equilibrium as a function of the pair of locations. Derive this price equilibrium!

Answer:

$$\mathbf{q_1 = \hat{x} = \frac{l_1 + l_2}{2} - \frac{p_1 - p_2}{2\tau(l_2 - l_1)}}$$

$$\mathbf{q_2 = 1 - \hat{x} = \frac{2 - l_1 - l_2}{2} + \frac{p_1 - p_2}{2\tau(l_2 - l_1)}}$$

Firm 1 solves $\max\limits_{p_1}(p_1 - c_1)q_1 \Rightarrow 0 = \hat{x} + \frac{(p_1-c_1)(-1)}{2\tau(l_2-l_1)} \Rightarrow p_1^{BR} = \frac{\tau(l_2-l_1)(l_2+l_1)}{2} + \frac{c_1+p_2}{2} \equiv \frac{A_1}{2} + \frac{c_1}{2} + \frac{p_2}{2}$, **where** $A_1 = \tau(l_2 - l_1)(l_2 + l_1)$.

Firm 2 solves $\max\limits_{p_2}(p_2 - c_2)q_2 \Rightarrow 0 = 1 - \hat{x} + \frac{(p_2-c_2)(-1)}{2\tau(l_2-l_1)} \Rightarrow p_2^{BR} = \frac{\tau(l_2-l_1)(2-l_2-l_1)}{2} + \frac{c_2+p_1}{2} \equiv \frac{A_2}{2} + \frac{c_2}{2} + \frac{p_1}{2}$, **where** $A_2 = \tau(l_2 - l_1)(2 - l_2 - l_1)$.

When we solve this system using the last two expressions, by for instance inserting for p_2^{BR} in the expression for p_1^{BR}, we get

$$p_1^{NE} = \frac{2A_1 + A_2}{3} + \frac{2c_1 + c_2}{3}, p_2^{NE} = \frac{A_1 + 2A_2}{3} + \frac{c_1 + 2c_2}{3}$$

d) What will the equilibrium price be if $c_1 = c_2 = c$, and either $l_1 = l_2$ or $\tau = 0$? What kind of an equilibrium is this?

Answer: For $c_1 = c_2 = c$, the last fraction collapses to c. We see that both for $l_1 = l_2$ and for $\tau = 0$, $A_1 = A_2 = 0$. Therefore $p_1^{NE} = p_2^{NE} = c$ in this case. This is the extreme case where the goods are perfect substitutes.

e) Consider again this second-stage price equilibrium and general locations l_1 and l_2. If firm 1 gets a lower marginal cost, what happens to the price that firm 1 and firm 2 charges, respectively? Does the response depend on the locations of the two firms? Derive and explain!

Answer: In the equilibrium derived in c), we have that $\frac{d\,p_1^{NE}}{dc_1} = \frac{2}{3}$ and $\frac{dp_2^{NE}}{dc_2} = \frac{1}{3}$. So, we see that when one firm gets a lower marginal cost, both firms end up reducing their price. A good answer here connects this to the best response curves derived in c). When c_1 is reduced, the best response curve of firm 1 shifts down. Furthermore, since the best response curve of firm 2 is upward sloping (i.e. the prices are strategic complements), firm 2 will respond to the shift by lowering its price. The final outcome is a reduced price for both firms, but a bigger reduction for the firm with the initial cost reduction, since the slope of the reaction function is only $\frac{1}{2}$.

Finally, we can see that the answer is independent of the locations, and this also traces back to the response curves; neither the initial shift nor the slopes depend on the locations.

f) In this final problem assume that the two firms are located symmetrically, i.e. equally far from their respective extremes. This means that if $l_1 = a$, then $l_2 = 1 - a$, and *consequentially that $l_1 + l_2 = 1$. Define the distance between them to be L, i.e. $L = l_2 - l_1 \in [0,1]$. When firm 1 gets a lower marginal cost, how does that move the equilibrium location of the indifferent consumer? How does the answer depend on τ and on the distance between the firms?*

Answer: Recall that $\hat{x} = \frac{l_1+l_2}{2} - \frac{p_1-p_2}{2\tau(l_2-l_1)}$, which with the provided assumptions is equal to $\frac{1}{2} + \frac{p_2-p_1}{2\tau L}$. The difference $p_2 - p_1$ can be solved for by inserting from the equilibrium derived in c). We find that (recall the relations between l_1 and l_2) $p_2 - p_1 = \frac{A_2-A_1}{3} + \frac{c_2-c_1}{3} = \frac{2\tau(l_2-l_1)(1-l_1-l_2)}{3} + \frac{c_2-c_1}{3} = \frac{2\tau L(1-1)}{3} + \frac{c_2-c_1}{3} = \frac{c_2-c_1}{3}$. So, when the firms are symmetrically placed on the line, their equilibrium prices will only be different to the extent that their costs are different. This should be no surprise. Insert this back into $\hat{x}$ to get $\hat{x} = \frac{1}{2} + \frac{c_2-c_1}{6\tau L}$. Finally, we get $\frac{d\hat{x}}{dc_1} = -\frac{1}{6\tau L} < 0$. So a reduction in c_1 shifts $\hat{x}$ up, i.e. to the right, meaning that more consumers buy from the leftmost firm 1. But the effect is lower when either τ or L is high, i.e. either when the transportation costs are really high or when the products are far apart. In both cases, the price change plays a smaller role for the consumers' choices.