Information and strategic interaction

Assumptions of *perfect competition*:

(i) agents (believe they) cannot influence the market price
(ii) agents have all relevant information

What happens when neither (i) nor (ii) holds?

Strategic interaction among a group of firms where some or all are incompletely informed

In particular: What happens when a firm knows more than the others about demand, own costs, etc.?

Equilibrium outcome is now also determined by incompletely informed firms' beliefs. These beliefs are represented by subjective probabilities.

(i) Incomplete information in a static model
  - how beliefs determine the equilibrium

(ii) … in a dynamic model
  - how beliefs are formed

Games with incomplete information

Perfect Bayesian Equilibrium:
Both strategies and beliefs are in equilibrium.

- Given the strategies in equilibrium, which revised beliefs are consistent with these strategies?

- Given the beliefs in equilibrium, which strategies are in equilibrium?

Two different kinds of problem:

- *Asymmetric information* – and the importance of the uninformed firm observing the informed firm's actions.
- *Symmetric, incomplete information* – and how there still may be a lot of action even though firms cannot observe each other's actions.

Signalling

A typical signalling game:

Stage 1: The *informed* player chooses an action (*signals*)

Stage 2: The uninformed player observes stage 1, revises his beliefs about the informed player, and chooses an action himself.

The informed player's private information – his type

$\theta \in$ {High, Low}

The uninformed player's beliefs about the other's type:

Initial beliefs

Pr(High) = p_H
Pr(Low) = $p_L = 1 - p_H$

Stage 2: *revised* beliefs

Equilibrium: actions and revised beliefs

Separating equilibrium: the action taken by the informed player at stage 1 depends on his type.

Pooling equilibrium: the action taken by the informed player at stage 1 is independent of his type.

In a pooling equilibrium, the uninformed player learns nothing about the other player's type from observing his stage-1 action. Beliefs cannot be updated based on that action.

In a separating equilibrium, on the other hand, the stage-1 action reveals the informed player's type, and so, based on that action, the uninformed player can update his beliefs about the other player's type and act accordingly.

First – a static model:
Price competition with asymmetric information

Two firms. Product differentiation. Price competition.

Product differentiation: A slight increase in a firm's price causes a slight decrease in its demand and a slight increase in the other firm's demand.

$$D_1 = D_1(\underset{-}{p_1}, \underset{+}{p_2}); \quad D_2 = D_2(\underset{-}{p_2}, \underset{+}{p_1})$$

Firm 1 has private information about own costs.
Both firms know firm 2's costs.

Firm 1's unit costs:

$c_1 = c_1^L$, with probability x

$c_1 = c_1^H$, with probability $(1 – x)$

$c_1^L < c_1^H$

Firm 2 only knows the probability distribution (c_1^L, c_1^H, x)

Firm 1 knows both c_1 and the probability distribution.

In the case of *complete information*:

$\pi_1 = (p_1 - c_1)D_1(p_1, p_2)$

$$\frac{\partial \pi_1}{\partial p_1} = D_1(p_1, p_2) + (p_1 - c_1)\frac{\partial D_1(p_1, p_2)}{\partial p_1} = 0$$

Best response of firm 1: $R_1(p_2)$.

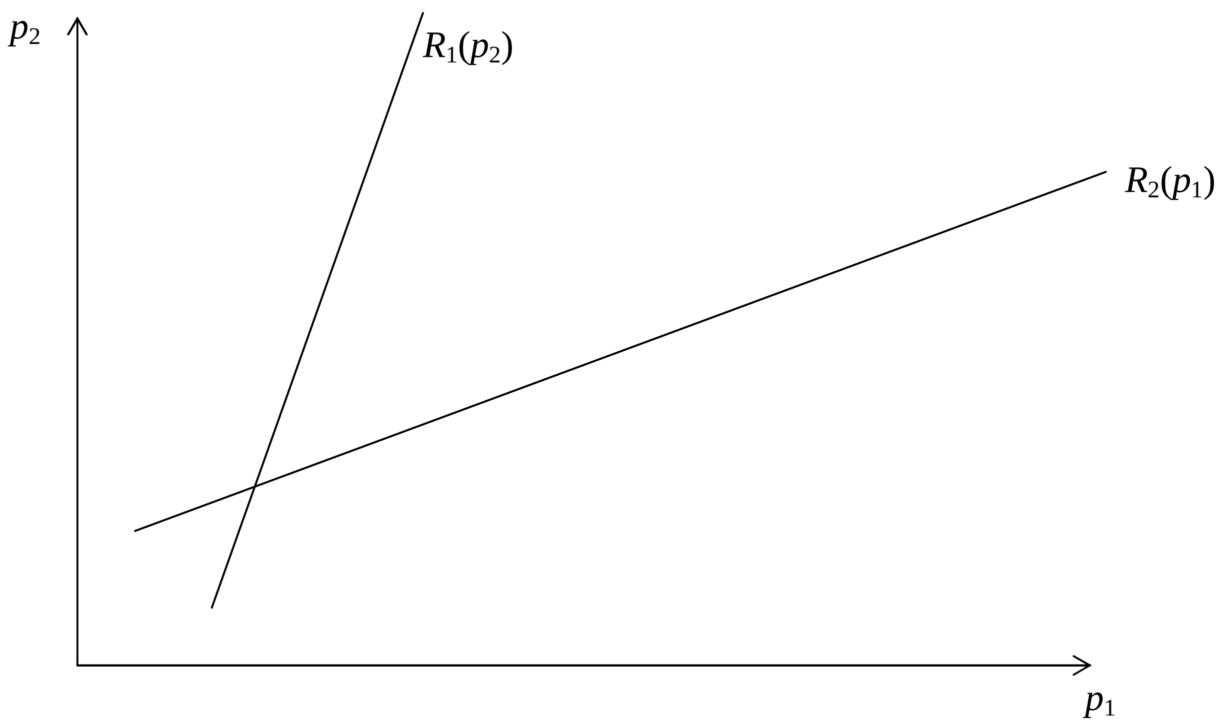
Slope of the best response:

$$\text{sign } R_1'(p_2) = \text{sign } \frac{\partial^2 \pi^1}{\partial p_1 \partial p_2}.$$

$$\frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} = \frac{\partial D_1(p_1, p_2)}{\partial p_2} + (p_1 - c_1)\frac{\partial^2 D_1(p_1, p_2)}{\partial p_1 \partial p_2}$$

- First term positive
- Slope of the best response positive unless $\frac{\partial^2 D_1}{\partial p_1 \partial p_2}$ very negative.

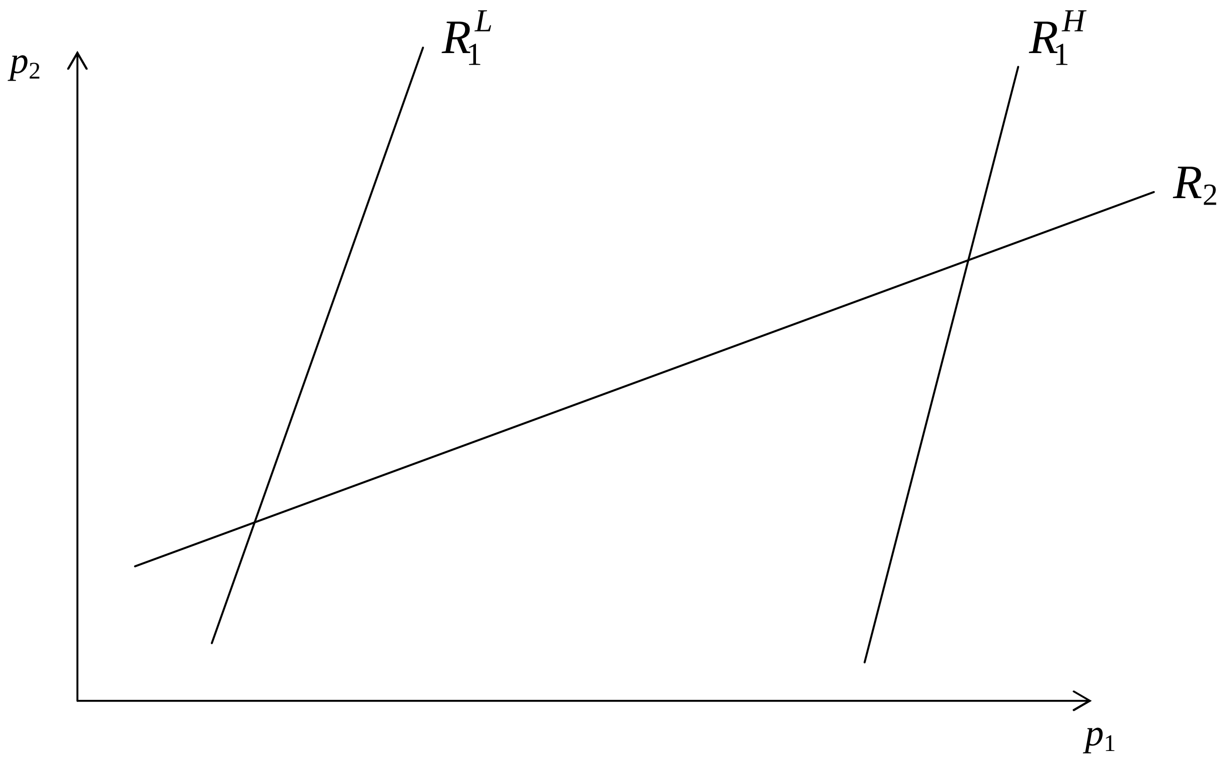
Equilibrium with complete information:



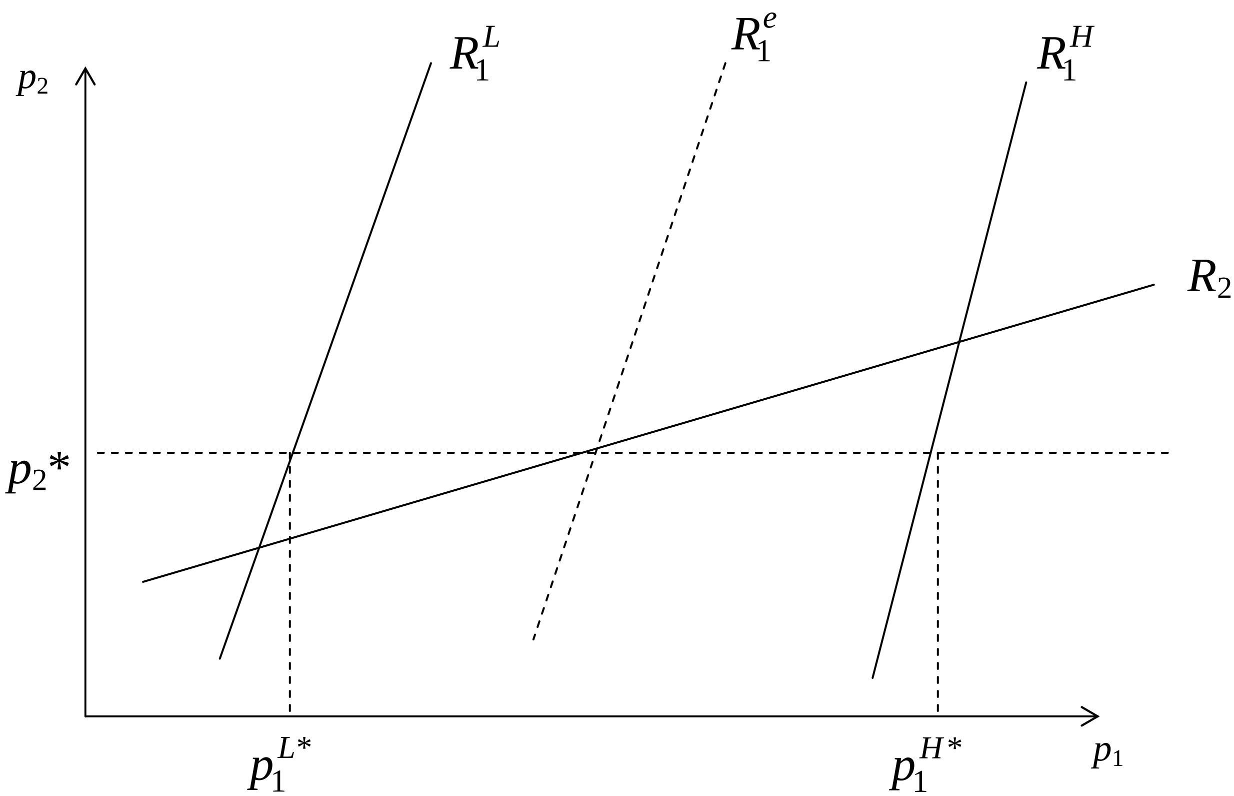
The optimum p_1 is increasing in c_1:

$$\frac{\partial^2 \pi_1}{\partial p_1^2} dp_1 + \frac{\partial^2 \pi_1}{\partial p_1 \partial c_1} dc_1 = 0$$

$$\frac{dp_1}{dc_1} = -\frac{\partial^2 \pi_1 / \partial p_1 \partial c_1}{\partial^2 \pi_1 / \partial p_1^2} = \frac{\partial D_1 / \partial p_1}{\partial^2 \pi_1 / \partial p_1^2} > 0$$



Firm 2 doesn't know firm 1's *type*. Firm 2 behaves as if confronting an *expected* firm 1.



Analytically, we find three prices:

The price of the uninformed firm.
The price of the informed firm if it has high costs.
The price of the informed firm if it has low costs.

How is the equilibrium affected by incomplete information?

If firm 1 is *low-cost*, then incomplete information *increases* the equilibrium prices.

If firm 1 is *high-cost*, then incomplete information *reduces* the equilibrium prices.

Probability of firm 1 being low-cost: x

An increase in x reduces equilibrium prices, whether firm 1 is low-cost or high-cost.

If firm 1 could choose x, it would want x to be low, whether the firm actually is low-cost or high-cost.

- The informed firm would like to be believed to have high costs, because that would keep prices high.

Dynamic model

Stage 1: An action by firm 1 that may potentially influence firm 2's subjective probability that firm 1 is low-cost.

Stage 2: Price competition with asymmetric information

What action?

(i) Verifying costs – external audit
Verification is good for firm 1 if it is high-cost, but not if it is low-cost.

(ii) Verification not possible

Model: Two-period price competition between two firms

Period 1: Price competition

Period 2: Price competition

Is it possible for firm 2 to infer firm 1's cost from firm 1's price in stage 1?

In period 1, a high-cost firm 1 would want to set a price that reveals its cost, while a low-cost firm 1 would not want to reveal its cost.

Signalling game.

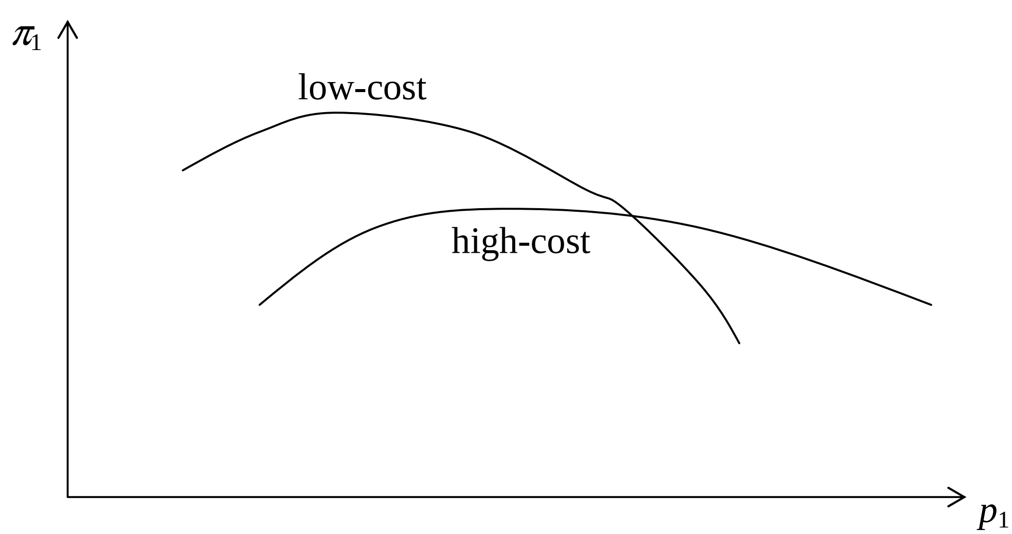
Could it be possible for a high-cost firm 1 to set a price in period 1 that is so high that a low-cost firm 1 would not want to mimic it?

– Yes, because increasing the price is less costly for a high-cost firm than for a low-cost firm.

$$\pi_1 = (p_1 - c_1)D_1(p_1, p_2)$$

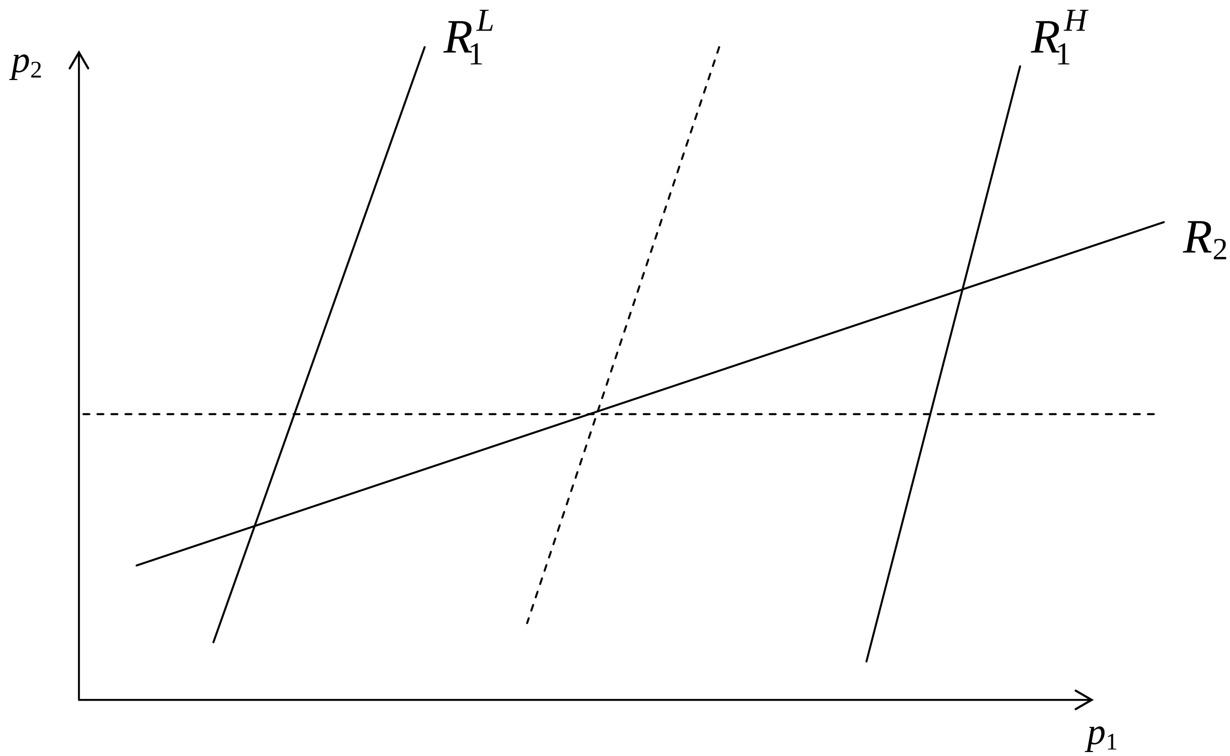
$$\frac{\partial^2 \pi_1}{\partial p_1 \partial c_1} = -\frac{\partial D_1}{\partial p_1} > 0$$

The effect on firm 1's profit of a price increase depends on the firm's costs. The higher costs are, the stronger is the effect if it is positive, and the weaker is the effect if it is negative.



A *separating equilibrium* is one where firm 1's price in period 1 depends on its costs.

A *pooling equilibrium* is one where firm 1's price in period 1 is the same whether it is low-cost or high-cost.



If firm 1's price in period 1 reveals its costs, then there is complete information in period 2.

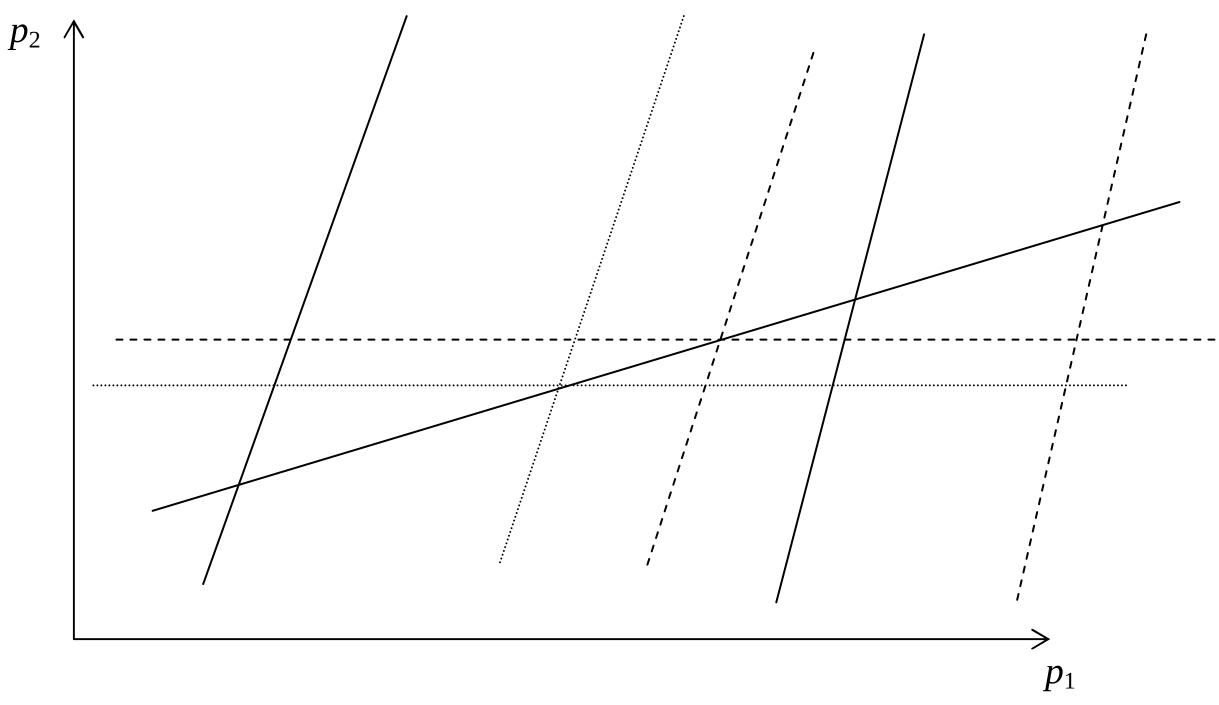
If firm 1's price in period 1 is uninformative of its costs, then the period-2 game is as in the static model.

Firm 1 would want firm 2 to believe it is high-cost, whether this is true or not.

Firm 2 will only believe firm 1 is a high-cost firm if it sets a price in period 1 that is so high that a low-cost firm would never set it – even though, by doing so, it would be considered a high-cost firm in period 2.

Thus, in a separating equilibrium, the high-cost best-response curve in period 1 is further to the right than in the static model.

Therefore, the expected best-response curve shifts to the right, and all prices are higher in period 1 of the two-period model than in the static model.



An extension: each firm has private information about own costs. The result that prices are higher still holds.
[Mailath, ”Simultaneous Signaling in an Oligopoly Model”, *Quart J Econ* 1989]

High-cost firm sets high price today in order to induce a high price tomorrow. → Puppy Dog strategy

Entry deterrence

Top Dog strategy

Two periods. Firm 1 has private information about own costs.

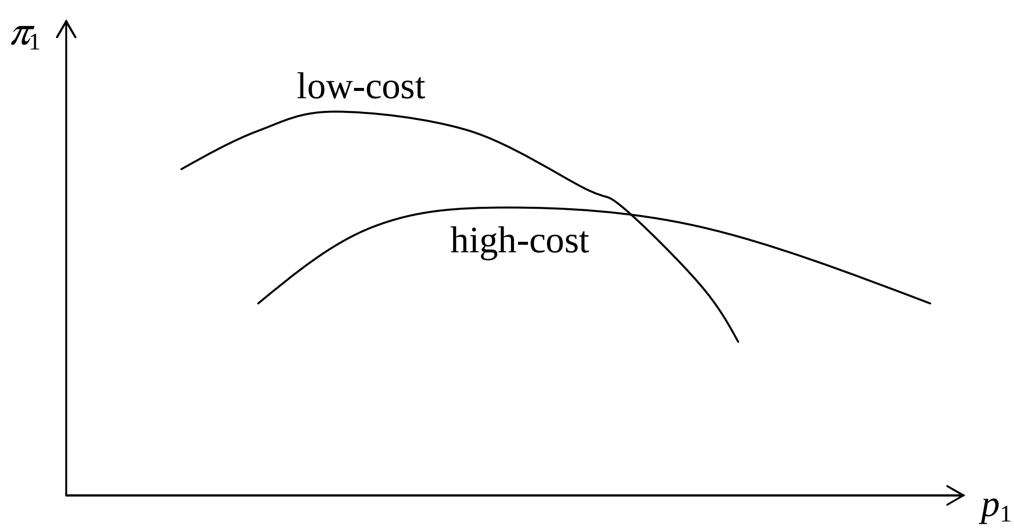
Period 1: Firm 1 is monopolist. It cannot deter entry through capacity investments, etc. Can it deter entry through its period-1 price?

Firm 1 wants firm 2 to believe its costs are low.

$$\frac{\partial \pi_2}{\partial c_1} > 0, \quad \frac{\partial E\pi_2}{\partial x} < 0$$

The interesting case: Entry is profitable for firm 2 if firm 1 has high costs but not if it has low costs.

Reducing the price is less costly for a low-cost firm than for a high-cost firm.



Complete information:
Period-1 price is the monopoly price.
Firm 2 enters if and only if firm 1 has high costs.

Incomplete information: One of two situations may occur.

(i) Low-cost firm 1 sets a price below its monopoly price, in order to signal its low costs.

- Separating equilibrium

(ii) Both types of firm 1 set the low-cost monopoly price.

- Pooling equilibrium
- Can only happen if firm 2, without any new information, is deterred from entry.

Limit pricing: Price reduction to deter entry.

Is limit pricing credible?

In case (i), it is. The price reduction in the separating equilibrium serves to inform the potential entrant that entry is not profitable because of the presence of a very potent incumbent.

In case (ii), it is not. However, the outside firm hasn't learned anything during period 1 and therefore chooses to stay out.

What are the *welfare consequences* of incomplete information?

In both cases: Expected price lower because of incomplete information.

In case (i) – separating equilibrium – entry behaviour is unaffected by incomplete information. Thus, with a separating equilibrium, incomplete information is good for welfare.

In case (ii) – pooling equilibrium – the high-cost firm 1 manages to deter entry by mimicking the low-cost type. Thus, incomplete information implies less entry. Total effect on welfare is unclear.

What if the entrant does not know its own costs? Suppose firms' costs are the same, but only firm 1 knows what they are.

$$\frac{\partial \pi_2}{\partial c} < 0$$

Firm 1 wants to signal high costs in order to deter entry. Now, the high-cost firm sets price above monopoly price in order to deter entry.

Puppy Dog as entry deterrence.

[Harrington, "Limit Pricing When the Potential Entrant Is Uncertain of Its Cost Function", *Econometrica* 1986]

Incomplete information and unobservable action

- Rival's price is unobservable
  (recall Green & Porter)

- Incomplete information about demand

- *Symmetric* information: Both firms incompletely informed

- Learning over time
  - Collecting information today in order to have more knowledge about demand tomorrow

- Strategic aspects of learning
  - A firm may try to disturb the other firm's learning today in order to affect future decisions

Model:

Two firms. Two periods.

Product differentiation. Price competition each period.
- Prices are strategic complements.

Firms do not observe each other's prices.

Firms do not know the market demand function.

$$q_i = a - p_i + bp_j$$

- Firm A wants firm B to set a high price in period 2.
- Firm B will only set a high price in period 2 if it believes demand is high.
- Firm B may think demand is high if it has high sales in period 1.
- Firm A may set a high price today in order for firm B to believe demand is high.
- But also firm B reasons the same way about firm A.
- And each firm also knows the other firm manipulates its learning.
- Both firms set high prices in period 1 in order to manipulate each other's learning.
- But each firm is able to see through the other firm's manipulation and learns the correct demand condition before period 2.
- *Signal-jamming*: manipulating others' learning
- In our case: signal-jamming increases period-1 prices.

Signal-jamming

$$\underbrace{s}_{\substack{\text{observed}\\ \text{by the other}}} = \underbrace{\alpha}_{\substack{\text{controlled}\\ \text{by the firm}}} + \underbrace{\varepsilon}_{\substack{\text{stochastic}\\ \text{term}}}$$

Other applications:
Organizational economics, corporate governance
– moral hazard

A specific model:

Firms: I and II
No costs.

Demand: $D_i(p_i, p_j) = a - p_i + p_j, \quad i \neq j.$

No firm knows a, only its expected value: $a^e = \mathrm{E}a$

The one-period case: (Benchmark)

Each firm solves:

$$\max_{p_i} E\pi_i = E\{(a - p_i + p_j)p_i\} = (a^e - p_i + p_j)p_i$$

Best-response function: $p_i = \dfrac{a^e + p_j}{2}$

Equilibrium: $p_I = p_{II} = a^e$.

The two-period case:

Learning about a if other firm's price is observable:

$$a = D_i + p_i - p_j$$

But other firm's price is not observable

$$\underbrace{D_i + p_i}_{\substack{\text{observed}\\ \text{by firm } i}} = \underbrace{p_j}_{\substack{\text{controlled}\\ \text{by firm } j}} + \underbrace{a}_{\substack{\text{stochastic}\\ \text{term}}}$$

In a symmetric equilibrium, each firm sets the same price in equilibrium, α, so that: $D_i = a - \alpha + \alpha = a$

But which price?

If firm II sets the price α and believes firm I does the same, what price would firm I want to set?

Firm II's estimate of a after period 1:

$$\tilde{a} = D_{II}^1 = a - \alpha + p_I^1 \quad \rightarrow \quad \tilde{a} = \tilde{a}\left(p_I^1\right)$$

In period 2, firm II believes it is playing a game of complete information where $a = \tilde{a}\left(p_I^1\right)$.

$$\rightarrow \quad p_{II}^2 = \tilde{a}\left(p_I^1\right)$$

What are the incentives for firm I to set a price in period 1 that differs from α?

First, consider period 2: Firm I has been able to deduce the true a and solves:

$$\max_{p_I^2} \left[a - p_I^2 + \tilde{a}\left(p_I^1\right)\right] p_I^2$$

$$\rightarrow \; p_I^2 = \frac{a + \tilde{a}\left(p_I^1\right)}{2} = \frac{a + a - \alpha + p_I^1}{2} = a + \frac{p_I^1 - \alpha}{2}$$

Firm I's period-2 profit:

$$\pi_I^2 = \left(a + \frac{p_I^1 - \alpha}{2}\right)^2$$

Period 1:

What is the optimum price for firm I in period 1, given firm II's price α?
Discount factor: $\delta \in (0, 1]$

Firm I solves:

$$\max_{p_I^1} E_a \left[\left(a - p_I^1 + \alpha\right) p_I^1 + \delta \left(a + \frac{p_I^1 - \alpha}{2}\right)^2 \right]$$

FOC: $a^e - 2p_I^1 + \alpha + \delta\left(a^e + \frac{p_I^1 - \alpha}{2}\right) = 0$

In a symmetric equilibrium: $p_I^1 = \alpha$.

$$a^e - 2\alpha + \alpha + \delta a^e = 0$$

$\Rightarrow$ First-period price: $\alpha = a^e(1 + \delta)$

- Manipulation of learning fails.
- The firms set higher prices in period 1 than if manipulation of each other's learning were not possible.
- Puppy-dog strategy: A high price today in order for the other firm to believe demand is high and therefore set a high price tomorrow.

Strategic interaction in one market – incomplete information in another

A version of *predation*:
The stronger firm competes aggressively in order to reduce the weaker firm's financial resources.

Product market: Duopoly – complete information

Capital market: Competitive – incomplete information

Two periods.

The two firms differ in financial strength:
The "long purse" story.

In order to operate in the market in period 2, each firm has to incur an investment K.

Firm 1 has internal funds in excess of K.

Firm 2 has to borrow on the capital market: Its internal funds equal $E < K$.

Firm 2 borrows $D = K - E$, and has to pay back: $D(1 + r)$

Interest rate: r

Firm 2's gross profit in period 2 is stochastic: $\tilde{\pi} \in [\underline{\pi}, \overline{\pi}]$

Cumulative distribution function: $F(\tilde{\pi})$; $F'(\tilde{\pi}) = f(\tilde{\pi})$

Expected value: π^e

If $\pi < D(1 + r)$, then firm 2 goes bankrupt.

Bankruptcy:
The bank receives π and incurs bankruptcy costs B.

Competitive capital market – banks' profits 0.

Banks' cost of funds: r_0

The interest rate in equilibrium solves:

$$(1+r)D[1-F(D(1+r))]+\int_{\underline{\pi}}^{D(1+r)}[\tilde{\pi}-B]f(\tilde{\pi})d\tilde{\pi}=(1+r_0)D$$

The expected bankruptcy costs will have to be covered by the borrowers.

So firm 2's capital costs is

$$[(1 + r_0)E] + [(1 + r_0)D + BF(D(1 + r))] =$$

$$(1 + r_0)K + BF((K - E)(1 + r))$$

Firm 2's expected net profit in period 2:

$$W = \pi^e - (1 + r_0)K - BF((K - E)(1 + r))$$

The higher is firm 2's internal funds, the more likely is it that firm 2 will undertake the period-2 investment:

An increase in E

- lowers debt $K - E$
- lowers interest rate r

Thus: $\frac{dW}{dE} > 0$

Period 1:

- E is a function of firm 2's period-1 profits.
- Firm 1 can lower E by reducing prices in period 1.
- Predatory pricing.