The Linear city model
The Linear city model

- X-axis: consumers are located uniformly from 0 to 1.
- Y-axis: consumer’s utility from buying from firm 1 and firm 2 respectively
- Locations:
  - Firm 1 is located at $a$
  - Firm 2 is located at $1-b$
The Linear city model

- Consumers have a reservation utility $s$
- Utility decreases quadratically in the distance to the firm:
  - $u_1 = s - p_1 - t(x - a)^2$
  - $u_2 = s - p_2 - t(1 - b - x)^2$
- Consumers who are located at the same place as a store, utility is $s - p$
- From there, utility decreases in distance.
- Demand is determined by the indifferent consumer $\bar{x}$:
  - Firm 1: $D_1(p_1, p_2, a, b) = \bar{x}$
  - Firm 2: $D_2(p_1, p_2, a, b) = 1 - \bar{x}$
The Linear city model

- Timing of the game:
  1. Firms choose location – this moves the «umbrellas» left/right
The Linear city model

• Timing of the game:
  1. Firms choose location – this moves the «umbrellas» left/right
  2. Firms choose prices
The Linear city model

- Timing of the game:
  1. Firms choose location – this moves the «umbrellas» left/right
  2. Firms choose prices

- What location is the optimal choice?
  ➢ Solution: Backward induction
  1. Step: Solve the price game:
     - What are the optimal prices given a and b?
     - Prices become functions of a and b:
       \[ p_1 = p_1(a, b) \]
       \[ p_2 = p_2(a, b) \]
  2. Step: Solve the location game.
     - What are the optimal locations a and b?