Question 1[1]
"Discuss strategic aspects of transboundary pollution within a model"

a) The problem: The tragedy of the common:

The essential insight can be modelled by the following one shot (one period) normal game payoff matrix of the strategies (cooperate = pollute taking the damages in other countries into account, do not cooperate = maximize private benefits) of two players (countries):

„Tradgedy of the common" (aka „The prisoners' dilemma")		**country A**	
		cooperate	pollute individually
country B	cooperate	(3; 3)	(4; -1)
	pollute individually	(-1; 4)	(0; 0)

The point is: Comparing the relevant alternatives (e.g. for B: „If A cooperates, what should I do?") it shows that it is always **rational** for any of the countries to pollute individually – compared to the relevant alternative, not to cooperate yields the highest payoff (is a dominant strategy). But that is **socially not optimal**: if one is concerend about, for example, maximizing the sum of welfare it would be best to cooperate! But here every country has an **incentive to deviate**. This problem arises as the states are **souvereign** and no „super"regulator can impose any quotas or fees to control the **negative externality**.

b) The model

How could such payoffs result in first place? Here is a complete model (based on Hoel, 1999 with some references to the handout for lecture 7 – I decided to keep it "as general as possible" to create an "added value" to the handout): We assume *n* countries with:

$e_1, \ldots e_i .., e_n$ emissions $\qquad$ $z_1, \ldots z_j \ldots, z_n$ pollution levels

where pollution in one country is a **physical** function of **all** the emissions in all countries::

$z_j = f(e_1, \ldots e_i .., e_n)$ or assuming a linear relationship: $z_j = \sum_i a_{ij} e_i$

where a_{ij} is the transfer coefficients. These are parameters determining how a change in emissions at some source results in a change in pollution at some receptor.Each country is a source as well as a receptor. This is a flow pollutant case. If the model was extended to stock pollutants a link between the pollution level and the stock had to be modelled. We assume that the countries' welfare is represented by:

$$U_j = U_j(\overset{+}{e_j}, \overset{-}{z_j}, \overset{+}{I_j})$$ where I is the **net transfer** among countries.

Assume damages from pollution and benefits from emission are measured in monetary terms:

$$U_j = I_j + R_j(e_j) + D_j(z_j)$$ the function becomes linear

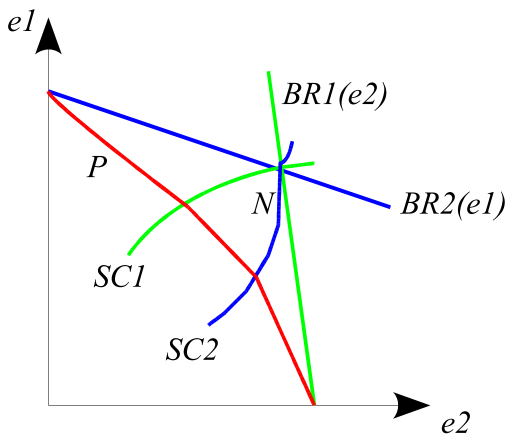
1 This topic is covered by lecture 7, slides 9 – 23 plus the additional handout.

c) The non-cooperative equilibrium

Suppose states do not interact on a diplomatic level with each other but every country maximises its own benefit with respect to its emissions taking the others' emissions as given (as in (c), just for a continuous choice). *Algebra*:

$$\max_{e_j} U_j(e_j, \textstyle\sum_i a_{ij} e_i, I_j)$$
$$FOC: dU_j / de_j = \partial U_j / \partial e_j + \partial U_j / \partial z_j \, a_{ij} = 0$$
$$\Rightarrow BR_j^* (e_1, \dots e_i \dots, e_n) \text{ for } j \neq i \qquad Equilibrium: BR_j^* = BR_k^* \text{ for all } j \text{ and } i$$

What comes out is a so called „**best response function**“, the own emissions as a function of all the other states' emissions. The non-cooperative equilibrium can be found where those functions „intersect“. In a 2-country case that can be shown *graphically*:



The crossing of the welfare indifference curves shows that the outcome is not Pareto-optimal.

d) The cooperative equilibrium

Suppose first there is **no transfer payments.** The problem is then to maximise the (weighted) sum of the countries' welfares with respect to all emission levels.*Algebra:*

$$\max_{\text{all } e_j} \textstyle\sum_k \alpha_k U_j(e_j, \sum_i a_{ij} e_i) \qquad \text{with } \alpha_k \text{ as weights.}$$
$$FOCs: dU_j / de_j \Rightarrow \alpha_j \partial U_j / \partial e_j = -\textstyle\sum_k \alpha_k a_{kj} \partial U_j / \partial z_j \qquad \text{for all j.}$$

This is *n* equations in *n* unknowns – problem solved. The FOCs show that the marginal benefits in country j should equal the sum of marginal environmental damages its emissions cause in all countries (including itself). **With transfer payments**, the problem becomes to maximise the sum of the countries' welfares with respect to all emission levels and all net transfers. *Algebra:*

$$\max_{\text{all } e_j} \textstyle\sum_k \alpha_k U_j(e_j, \sum_i a_{ij} e_i, I_j) \text{ s.t. } \sum_k I_j = 0 \qquad \text{by Lagrange}$$
$$FOCs: \partial L_j / \partial e_j \Rightarrow \quad \alpha_j \partial U_j / \partial e_j = -\textstyle\sum_k \alpha_k a_{kj} \partial U_j / \partial z_j \qquad \text{for all j}$$
$$\partial L_j / \partial I_j \Rightarrow \quad \alpha_j \partial U_j / \partial I_j = \lambda \qquad \text{for all j.}$$
$$\textstyle\sum_k I_j = 0$$

which can be transformed to a condition equivalent to the FOC without transfers:

$$(\partial U_j / \partial e_j) / (U_j / \partial I_j) = -\textstyle\sum_k \{a_{kj} (\partial U_k / \partial z_k) / (U_k / \partial I_k)\}$$

Graphically: **Any** socially optimal outcome would be located on the Pareto-frontier (P), as this is the locus of all optimal allocations. But with some strategic considerations, one can see that with the non-cooperative outcome as the status quo and no transfer payments, the possible outcome is restricted to the „eye" of the indifference curves as here both players are better off. Allowing for transfer payments, the whole Pareto-frontier is theoretically available as the country loosing from the new equilibrium could be compensated by the one gaining. Any distribution can be achieved.
However, there will be one **unique** Pareto-optimal outcome to above's maximisation problem. Whether transfer payments are required to make it attainable depends on where it is located.

e) Versus the cooperative equilibrium – the free riding alternative

If it is possible given our assumptions to increase the welfare or all states by reducing emissions, why is the social optimum then not chosen? Because in the social optimum each state has an **incentive to deviate.** If all the other states decide to cooperate and emit less, the best response function calls for each state individually to in that case emit more! Hence a state will try to take a „**free ride**". Even if the country might receive transfer payment and the country is better off with the agreement than without, the **relevant alternative** is „staying out" of the agreement and might yield even higher benefits.

Algebra: Maximise the private benefits taking the **cooperative** emission levels (from f) of all other countries as given constants (same method as in e). One ends up in a certain point on the above established best response function. According to game theory, this consideration is made by every country and the whole agreement breaks down.

From the numerical example in the handout for lecture 7 it can be seen that the larger the number of countries participating in the international agreement, (1) the larger the incentive to deviate and (2) the higher the amount of pollution by the deviating country.

f) Further complications

There are many limitations to the applicability of this model.
(1) Who has ever seen a Pareto-frontier?
(2) How to choose a point on it (it is too simple to assume that one can really compare welfare by just assuming a representative consumer and monetary measures and then maximising the weighted sum sum...)?
(3) How to detect whether one really keeps to an agreement (which country can control what another country emits? What if the legislation in a country is in place but it doesn't care to enforce it?)?
(4) How to prevent countries from emitting too much (Trade sanctions are only credible threats if the country does't harm itself by that)?
(5) And finally, how to prevent countries outside the agreement from emitting more as a best response to the new agreement (Kolstad calls this the **five problems of maintaining an effective environmental agreement**)?

Above that, **distributional consequences** are often regarded as a main deterant: It might be quite complicated to agree upon transfer payments among 50 countries to compensate for the fact that, even though marginal costs are supposedly equal, the total costs are shared very unequally (Hoel, 1999: 484).

g) Modification – infinitely repeated games

Despite all those impediments, international agreements DO actually exist. One modification that could help to explain that is to think of the game as repeated infinitely (or a finite, but uncertain number of times). Suppose there is a net benefit from joining the agreement and another one from participating but cheating. Before every period, the countries have to decide whether to cooperate or to cheat. Assume that if any country cheats the agreement breaks down at the non-cooperative outcome will prevail forever. If every country cooperates, than the socially optimal payoff is reached every period. Now, each country has to ask itself in the first period: is it worth it to benefit in the first period from cheating and then receiving the non-cooperative outcome ever after or am I better off cooperating forever? This depends on (1) the discount rate the country applies and (2) the actual payoffs.

Algebra: This is a comparison of two infinite geometric series. For cooperation, one gets the cooperative outcome forever. The present value of that is given by:

$$PV_c\text{: } c/(1-\delta)$$

If not cooperating in the first period, the country gets:

$$PV_n\text{: } a + \delta b/(1-\delta)$$

For the example in c):

$$PV_c\text{: } 3/(1-\delta) >=< PV_n\text{: } 4 + \delta(-1)/(1-\delta)$$

The comparison is the decision rule whether to cheat or not. One can resolve for δ to get the critical discount rate, for the “cheater's payoff” in the first round to establish a critical value for that, etc.

Question 2
"Emission Trading"

a) Cost efficient distribution:

We assume that emissions translate directly into pollution with no spatial or time components (for example: global warming). We assume further that the amount of pollution has been determined by the regulator in advance (given full information, the regulator should have chosen X such that the aggregated marginal benefits equal the marginal damage. Then the solution to this problem would be Pareto-optimal – see c)).
Now the firms are issued permits to pollute. **No trading** takes place so far. In order to achieve the maximum joint benefits (treating the total amount of pollution as exogenous!), the marginal benefits to the firm have to be the same.

Algebra: Setting up a **Lagrangian** (juhu!) gives us the optimal amounts as functions of the parameters b_i, A_i and X. This is the **cost-efficient** distribution. The first order conditions show that the **equi-marginal** principle holds and that the **shadow price** for the total emissions is λ, the increase in total marginal benefits when the total amount of emissions is increased by one.
If both polluters emit these amounts, the total social benefit is maximized, while in reality, they only want to maximize their own benefit by producing $x_i = b_i/2A_i$ units of pollution (NB: We are lazy. That is why we assume an **interior solution** $x_i \leq b_i/2A_i$, hence the constraint holds with equality.).

The firms are issued $\mathbf{x_1}$, $\mathbf{x_2}$ units of emission permit, which sum up to X.

The total benefit of pollution is: $TB = b_1\mathbf{x_1}\text{-}A_1\mathbf{x_1}^2+b_2\mathbf{x_2}\text{-}A_2\mathbf{x_2}^2$

So the problem to the regulator is: $\max\ b_1\mathbf{x_1}\text{-}A_1\mathbf{x_1}^2+b_2\mathbf{x_2}\text{-}A_2\mathbf{x_2}^2 \quad s.t.\ \mathbf{x_1}+\mathbf{x_2}=X$

The Lagrangian is hence: $L= b_1\mathbf{x_1}\text{-}A_1\mathbf{x_1}^2+b_2\mathbf{x_2}\text{-}A_2\mathbf{x_2}^2\text{-}\lambda(\mathbf{x_1}+\mathbf{x_2}\text{-} X)$

F.O.C: $\partial L/\partial \mathbf{x_1}=b_1\text{-}2A_1x_1\text{-}\lambda=0$

$\partial L/\partial \mathbf{x_2}=b_2\text{-}2A_2x_2\text{-}\lambda=0$

The optimal amounts are: $\mathbf{x_1}=(b_1\text{-}b_2+2A_2X)/2(A_1+A_2)$

$\mathbf{x_2}=(b_2\text{-}b_1+2A_1X)/2(A_1+A_2)$

b) Permit market outcome:

Now suppose the regulator does not know each firm's cost function[2]. **Cost-efficiency** can be obtained anyway by putting up a **market for emission permits**. The firms are issued some permits to pollute together X (lets say by "grandfathering"- on basis of historical emissions). The **initial distribution** does from a cost-efficiency point of view not matter (they need not to be the same as in a). On a perfectly functioning market the "invisible hand" will determine a market price. This market price is an **opportunity cost** for each polluter. Instead of polluting, he or she could sell the permit at the going price. Hence a rational polluter will pollute up to the point where his or her marginal benefits of pollution equal the price of the permit (the **equi-marginal principle** holds). Also, the initial distribution among the firms does not affect cost efficiency, just the **distribution of wealth** among the firms.

2(The bad thing about that is that now it is not possible to "straightforward" determine the optimal amount of pollution, X, in advance. So Pareto-optimality most likely cannot be reached.)

Algebra: Each firm maximises its private benefits.
Let $\boldsymbol{x_1}$,$\boldsymbol{x_2}$ be the permits given to firm 1 and firm 2 by the government or…the king…
Let x_1,x_2 be the actual amount of pollution emitted by each firm.
Let p be the price of permits. Then

$$\max \quad B_1 = b_1 x_1 - A_1 x_1^2 - p(x_1 - \boldsymbol{x_1})$$
$$B_2 = b_2 x_2 - A_2 x_2^2 - p(x_2 - \boldsymbol{x_2})$$
$$\text{s.t.} \quad x_1 + x_2 = \boldsymbol{x_1} + \boldsymbol{x_2} = X$$

F.O.C: $\partial B_1/\partial x_1: b_1 - 2A_1 x_1 - p = 0 \qquad \partial B_1/\partial x_2: b_2 - 2A_2 x_2 - p = 0$:

gives: $x_1 = (b_1 - p)/2A_1 \qquad x_2 = (b_2 - p)/2A_2$ and $x_1 + x_2 = X$

In the permit market, demand and supply determine

$$p = (A_2 b_1 + A_1 b_2 - 2A_1 A_2 X)/(A_1 + A_2)$$

inserting p back into (1) and (2),get the optimal amounts as:

$$x_1 = (b_1 - b_2 + 2A_2 X)/2(A_1 + A_2) \qquad x_2 = (b_2 - b_1 + 2A_1 X)/2(A_1 + A_2)$$

Exactly the same as a)! Haha!

c) The optimal level of pollution under incomplete information:

c1) The Pareto-optimal level of pollution

To determine the Pareto-optimal amount of pollution one has to equate the **aggregated marginal benefits** with the **marginal damages** of pollution. (More precisely, the social welfare optimiser optimises social welfare by maximising the sum of benefits minus the total damage. The first order condition is above's statement. Generalized: In the "market" for pollution, supply must equal demand (anybody who thinks that sounds familiar...?)). Suppose not. If the aggregated marginal benefits from pollution are higher than the damages, than society would gain from polluting more and vice versa. Neither can be optimal.

Algebra: Either one solves the social welfare maximisers' problem. Or one takes the shortcut: Aggregating the marginal benefits **horizontally** (as they are "private") by first **inverting** them, then summing them and then inverting them again gives the total "demand" for pollution. The "supply" of pollution is giving by the society's marginal damage from pollution. The resulting optimal amount is (please refer to the blackboard for the calculation):

$$X = A_1 b_2 + A_2 b_1 / 2A_1 A_2 + \beta(A_1 + A_2)$$

c2) Marginal benefits not known

Above's calculus requires to know at least the aggregated marginal benefits – and by assumption we don't. One option is to kindly ask the firms to tell. But we know that they would highly **overstate** their benefits to get out of it as cheap as possible. Another possibility is to create some kind of incentives such that it is beneficial to truthfully reveal the marginal benefits. That would require some **incentive feasible scheme** of differentiated lump sum transfers and fees/permits such that incentive compability and participation are given.

But here is how we might end up in the optimum anyway: Use the market forces! Suppose the total amount of pollution is determined each period anew. Further, assume the marginal benefits are stable over time. In period 1, the regulator sets some amount X. She knows the marginal damage at that X (which is βX). And the regulator **observes** the outcome of the perfect market: the distribution of permits and especially the permit price p=p(X). Now, in the social optimum these must be the same. If the price is lower, this indicates that there is too much pollution, if it is higher, there is too little. Hence in the next period, the regulator can adjust the allowed pollution upwards or downwards and over time will close in on the optimal amount.

Graphically: