ECON4910, Spring 2010

Seminar 4

Sketch of solution

Consider a one consumer economy where consumption C is equal to $Y - G$, where Y is output and G is an exogenously given level of public expenditure. $Y = F(L, E)$, where where L is labour supply and E is aggregate emissions of some pollutant. Assume that $F_{LL} = F_{LE} = 0$ and $F_{EE} < 0$.

Unlike the function assumed in the lecture, this function cannot be homogeneous of degree one. Moreover, since $F_{LE} = 0$ and $F_{EE} < 0$, the profit maximization condition $F_E = q$, where q is the emission price, gives $E = E(q)$ or $q = q(E)$, each function declining in its argument.

1. Show that for suitable values of w, a and b (all positive and $a + b < 1$) these properties follow from output being equal to

$$Y = \max_{\ell} \left[w\ell + (L - \ell)^a E^b - pE \right]$$

Assuming an interior solution ($0 < \ell < L$) the properties follow immediately from the envelope theorem and the first-order conditions for the maximization problem.

2. Give an interpretation of this production function.

Obvious interpretation: 2-sector economy. Sector 1 only uses labor and is CRS, so that marg prod of labor is constant, denoted w. Sector 2 uses labor and fossil energy E, which has price p (could be cost of domestic production or an international price). Note that $F_E = b(L - \ell)^a E^{b-1} - p$, which is zero from profit maximization if there is no carbon tax. With a carbon tax q producers choose E so that $b(L - \ell)^a E^{b-1} = p + q$, implying $F_E = q$.

The consumer's utility function $u(C, L, E)$ is given by

$$u(C, L, E) = \log C - hL - kE$$

3. How does the willingness to pay for reduced E depend on C and L?

We have $u_C = C^{-1}$, $u_L = -h$, and $u_E = -k$. Marginal WTP for reduced E is

$$\frac{-u_E}{u_C} = kC$$

All non-wage income goes to the government (through government ownership or a hundred percent profit tax). The (gross) wage rate in this economy is w, and the person pays a tax equal to $twL - s$, where t is positive and s may be positive or negative.

4. How does the labour supply depend on t and s?

The consumer regards E, w, t and s as given and maximizes $u(C, L, E)$ subject to the budget constraint $C = (1-t)wL + s$. This gives the standard condition $\frac{-u_L}{u_C} = (1-t)w$. Using the budget constraint and the specification of u this gives

$$L = \frac{1}{h} - \frac{s}{(1-t)w}$$
$$C = \frac{(1-t)w}{h}$$

implying that L is declining in s, and also in t if $s > 0$ (which is assumed henceforth whenever s is exogenous).

5. Derive an expression for the optimal emission tax, and compare it with the Pigovian level for the following three cases:

- t and s are chosen optimally
- t is exogenously given and s is chosen so the government's budget is balanced
- s is exogenously given (and positve), and t is chosen so the government's budget is balanced

The optimal choice of E must maximize $u(F(L, E) - G, L, E)$, taking into account that L depends on E (while G is exogenous). The first-order condition of this maximization gives (remembering that $F_E = q$, $F_L = w$, $\frac{-u_L}{u_C} = (1-t)w$, and $\frac{-u_E}{u_C} = kC$)

$$q = kC - tw\frac{dL}{dE}$$

If $t = 0$, the optimal tax is equal to the Pigovian level, i.e. $q = kC$. For $t > 0$, the sign of $q - kC$ depends on the sign of $\frac{dL}{dE}$. To find this, we must use the country's (or government's) budget condition to find out how t or s must be changed as a response to a change in E.

The budget condition for the country may be written as

$$F(L(t,s),E) = C(t,s) + G$$

We could insert the specified functions for C and L, but it is simpler to use the budget equation written as above.

Assume first that s is is exogenously given. Differentiating the budget condition gives

$$[F_L L_t - C_t] \frac{dt}{dE} = -F_E$$

It is reasonable to assume that $F_L L_t - C_t$ is positive—otherwise we could reduce t and increase G without violating the budget condition. When $F_L L_t - C_t > 0$, it follows that $\frac{dt}{dE} < 0$ (for $F_E = q > 0$). Since we know from above that $L_t < 0$ in our case, an increase in E must therefore give a higher value of L. In this case we thus have $\frac{dL}{dE} > 0$, implying

$$q - kC = -tw\frac{dL}{dE} < 0$$

so the optimal emission tax is lower than the Pigovian level.

Consider next the case where t is exogenous. Proceeding as above we find

$$[F_L L_s - C_s] \frac{ds}{dE} = -F_E$$

For our case we know that $C_s = 0$ and $L_s < 0$, implying $\frac{ds}{dE} > 0$ (for $F_E = q > 0$). An increase in E must therefore give a lower value of L in this case, i.e. $\frac{dL}{dE} < 0$. Hence,

$$q - kC = -tw\frac{dL}{dE} > 0$$

so the optimal emission tax is higher than the Pigovian level.

If t and s were chosen optimally, we would set $t = 0$. In this case we would therefore have $q = kC$, i.e. the optimal emission tax would be equal to the Pigovian level.

6. Would your answers to question 5 be changed if all profits in the economy went to an owner that did not count in the economy's welfare function (e.g. a foreign owner)?

In the analysis above, the government's budget condition was automatically satisfied when the consumer's budget condition and the country's budget were in balance. However, with a third sector (a foreign owner) we must explicitly specify the government's budget condition:

$$twL(t,s) + q(E)E = G + s$$

If qE goes up, either s must go up (since $L_s < 0$) or t must decline (assuming $wL + twL_t > 0$).

Combining the budget condition of the government with the budget condition of the consumer ($C = (1-t)wL + s$) we get

$$C = wL + q(E)E - G$$

so that the optimal E follows from maximizing $u(wL(t,s) + q(E)E - G, L(t,s), E)$. This gives

$$u_C \left[w\frac{dL}{dE} + q + Eq'(E) \right] + u_L \frac{dL}{dE} + u_E = 0$$

or (remembering that $\frac{-u_L}{u_C} = (1-t)w$, and $\frac{-u_E}{u_C} = kC$)

$$q = kC - tw\frac{dL}{dE} - Eq'(E)$$

The sign of the term $\frac{dL}{dE}$ can be found as before, and will as before depend on whether t of s is held constant as E is changed. The sign of $\frac{dL}{dE}$ will also depend on in what direction $q(E)E$ changes as Echanges.

Unlike the previous case, we now have a term $-Eq'(E)$, which is positive since $q' < 0$. To illustrate the importance of this term, assume $q + Eq' = 0$. In this case qE will be unaffected by a change in E, implying that the change in E will have no effect on t or s. In this case we therefore have $\frac{dL}{dE} = 0$, implying that $q - kC = -Eq' > 0$.

It is straightforward to show the following:

If $q+Eq' > 0$ (increased E increases government revenue), an increase in E will either reduce t, and hence increase L, or increase s, and hence reduce L. In this case the term $-tw\frac{dL}{dE}$ will have the same sign as in the case discussed under point 5.

If $q + Eq' < 0$ (increased E reduces government revenue), it must be the case that $\frac{dL}{dE} > 0$ (from the equation above $q + Eq' = kC - tw\frac{dL}{dE}$). This case can only occur if the increase in E reduces s, and hence increases L.