ECON4910, Spring 2010

Seminar 5

Problem 1

There are two sources of energy that are perfect substitutes. Production of "brown" energy is x with total cost bx where b is fixed and positive. Production of the brown energy gives carbon emissions e equal to fx. Production of "green" energy is y with total cost $\frac{g}{2}y^2$ where g is fixed and positive. Energy demand is exogenously given equal to A, so that $x + y = A$. Assume that $gA > b$.

1. Show what the optimal values of x, y and e are in the absence of any concern for the environment.

2. Show that marginal abatement costs are equal to $\frac{gA-b}{f} - \frac{g}{f^2}e$.

3. Show how a reduction in g will affect carbon emissions (in the absence of any environmental policy) and the abatement cost function.

4. Show how a reduction in f will affect carbon emissions (in the absence of any environmental policy) and the abatement cost function.

5. Assume that marginal environmental costs are constant equal to v. What is the social value of reducing f to zero? (You do not have to calculate the explicit expression for this value.)

6. If there is a carbon tax equal to v, what is the maximal profit an innovator can achieve from reducing f to zero (for all or some of the brown energy production)? (You do not have to calculate the explicit expression for this value.)

7. Once a new technology reducing f to zero has become available, what is the optimal carbon tax?

Problem 2

There is a large number n of polluting firms in the economy, each having an abatement cost function $c(a_i, y_i)$, where a_i and y_i are abatement and technology levels of firm i, respectively. The function c has the properties assumed in Hoel: "Environmental R&D".

The technology level of firm i is given by

$$y_i = x_i + \gamma \sum_{j \neq i} x_j$$

where x_i is the number of useful ideas firm i purchases from the R&D sector. Give an interpretation of this equation.

The R&D sector produces ideas at a cost $f(X)$ per idea, where $X = \sum_j x_j$, and where f' may be non-negative. Discuss what the sign of f' might be.

Assume that there is one firm per produced idea in the R&D sector, and that there are zero profits in this sector in equilibrium.

The government has an environmental cost function $D(E)$ with $D' > 0$ and $D'' \geq 0$, where E denotes total emissions. Prior to the production and trade of useful ideas, the government can commit to a particular level of total emission quotas.

Give a characterization of the government's optimal policy (as in equation 6 in Hoel: "Environmental R&D"). Does your result depend on whether the quotas are allocated for free to the polluting firms or are auctioned? How would the outcome be changed if instead of committing to a quota the government committed to an emission tax?