ECON4910, Spring 2010

Seminar 5

Problem 1. Sketch of solution

There are two sources of energy that are perfect substitutes. Production of "brown" energy is x with total cost bx where b is fixed and positive. Production of the brown energy gives carbon emissions e equal to fx. Production of "green" energy is y with total cost $\frac{g}{2}y^2$ where g is fixed and positive. Energy demand is exogenously given equal to A, so that $x + y = A$. Assume that $gA > b$.

0.0.1 Show what the optimal values of x, y and e are in the absence of any concern for the environment.

Total costs of producing energy in the amount A is $\left(\frac{g}{2}y^2 + b(A - y)\right)$, which is minimized for $gy = b$, i.e. $y = \frac{b}{g}$, $x = A - \frac{b}{g}$, $e = f\left(A - \frac{b}{g}\right)$.

0.0.2 Show that marginal abatement costs are equal to $\frac{gA-b}{f} - \frac{g}{f^2}e$.

From $e = fx = f(A - y)$ it follows that $y = A - \frac{e}{f}$, inserting this into the cost function $\left(\frac{g}{2}y^2 + b(A - y)\right)$ gives

$$C(e) = \frac{g}{2}\left(A - \frac{e}{f}\right)^2 + b\left(A - \left(A - \frac{e}{f}\right)\right)$$

Differentiating w.r.t. e gives

$$C'(e) = g\left(A - \frac{e}{f}\right)\left(-\frac{1}{f}\right) + \frac{b}{f}$$

which after rearranging gives the marginal abatement cost $-C'(e)$ as

$$-C'(e) = \frac{gA - b}{f} - \frac{g}{f^2}e$$

0.0.3 Show how a reduction in g will affect carbon emissions (in the absence of any environmental policy) and the abatement cost function.

We showed above that $e = f\left(A - \frac{b}{g}\right)$ in the absence of any environmental policy. This is smaller the smaller is g. The steepness of the marginal abatement curve (measured positively) is given by $C''(e) = \frac{g}{f^2}$, which is smaller the smaller is g. Since the intersection with the horizontal axis is further to the left the lower is g , and the curve is flatter the smaller is g, the whole curve $-C'(e)$ must be lower the smaller is g.

0.0.4 Show how a reduction in f will affect carbon emissions (in the absence of any environmental policy) and the abatement cost function.

We showed above that $e = f\left(A - \frac{b}{g}\right)$ in the absence of any environmental policy. This is smaller the smaller is f. The steepness of the marginal abatement curve (measured positively) is given by $C''(e) = \frac{g}{f^2}$, which is bigger the smaller is g. The marginal abatement cost curve with the the smaller f must intersect the original marginal abatement cost curve at some positive value of e, since the total abatement cost $C(e)$ (= the area under the marginal abatement cost curve) is independent of f fir $e = 0$. The interpretation is obvious: To completely eliminate emissions, we must completely phase out the use of brown energy (for $f > 0$), and the cost of doing this is independent of f.

0.0.5 Assume that marginal environmental costs are constant equal to v. What is the social value of reducing f to zero? (You do not have to calculate the explicit expression for this value.)

The social cost of producing brown energy goes down from $b + vf$ to b when f drops to zero. The value W of reducing f to zero is the difference in minimized total costs for these two cases

$$W = \min_y \left[\frac{g}{2}y^2 + b\left(b + vf\right)\left(A - y\right)\right] - \min_y \left[\frac{g}{2}y^2 + b\left(A - y\right)\right]$$

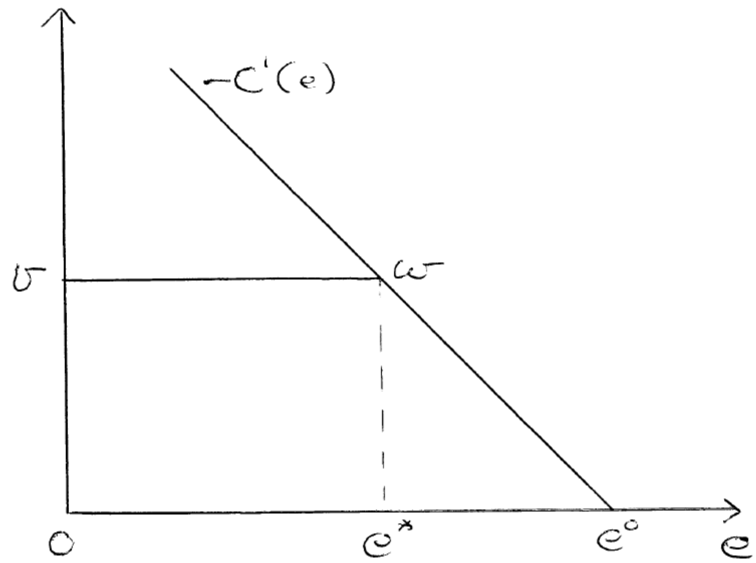
If I have calculated correctly, this gives (assuming that $gA > b + vf$, so that optimal emissions e^* are positive when $f > 0$):

$$W = vf\left(A - \frac{b}{g} - \frac{fv}{2g}\right)$$

As an alternative to this derivation, we can illustrate W using a figure: In the figure below, $e^0 = f\left(A - \frac{b}{g}\right)$, i.e. the optimal value of e with the original f in the absence of environmental concern. The value W in the figure is the area $Ovwe^0$. This is the sum of avoided environmental costs $Ovwe^*$ and avoided abatement costs e^*we^0.

0.0.6 If there is a carbon tax equal to v, what is the maximal profit an innovator can achieve from reducing f to zero?

An innovator can charge a price up to v for its technology. If it charges v, the technology will be used in the amount e^*, and the profit to the innovator will be $Ovwe^*$, which is smaller than W. The profit might be higher than $Ovwe^*$, but it will in any case be smaller than W.



0.0.7 Once a new technology reducing f to zero has become available, what is the optimal carbon tax?

The optimal carbon tax is marginally above zero. The best response from the innovator to this tax is to set a price between zero and the tax. Since the price of the technology and the carbon tax both are approximately zero, we get the optimal outcome with (approximately) zero emissions and with (almost) all "brown" energy being produced with the new technology.

Derivation of w:

$W = \min_y \left[\frac{g}{2}y^2 + (b+vf)(A-y)\right] - \min_y \left[\frac{g}{2}y^2 + b(A-y)\right]$

$y = \frac{b+vf}{g}$ in first and $y = \frac{b}{g}$ in second

$$\left[\frac{g}{2}\left(\frac{b+vf}{g}\right)^2 + (b+vf)\left(A - \left(\frac{b+vf}{g}\right)\right)\right] - \left[\frac{g}{2}\left(\frac{b}{g}\right)^2 + b\left(A - \left(\frac{b}{g}\right)\right)\right]$$

$$\left[\frac{g}{2}\left(\frac{b+vf}{g}\right)^2 + (b+vf)\left(A - \left(\frac{b+vf}{g}\right)\right)\right] = -\frac{1}{2g}(b+fv)(b-2Ag+fv)$$

$$\left[\frac{g}{2}\left(\frac{b}{g}\right)^2 + b\left(A - \left(\frac{b}{g}\right)\right)\right] = -\frac{1}{2}\frac{b}{g}(b-2Ag)$$

$$-\frac{1}{2g}(b+fv)(b-2Ag+fv) + \frac{1}{2}\frac{b}{g}(b-2Ag) = -\frac{1}{2}\frac{f}{g}v(2b-2Ag+fv)$$

Problem 2. Sketch of solution (updated May 18)

There is a large number n of polluting firms in the economy, each having an abatement cost function $c(a_i, y_i)$, where a_i and y_i are abatement and technology levels of firm i, respectively. The function c has the properties assumed in Hoel: "Environmental R&D".

The technology level of firm i is given by

$$y_i = x_i + \gamma \sum_{j \neq i} x_j$$

where x_i is the number of useful ideas firm i purchases from the R&D sector. Give an interpretation of this equation.

Each firm benefits (in the form of improved technology) from the ideas it purchases itself, and also (for $\gamma > 0$) from the ideas purchased by other firms. See Hoel (2005) where a similar modeling is used for technology spillovers across countries.

The R&D sector produces ideas at a cost $f(X)$ per idea, where $X = \sum_j x_j$, and where f' may be non-negative. Discuss what the sign of f' might be.

It could be either positive or negative, see the discussion in Hoel (2010), section 5.3.

Assume that there is one firm per produced idea in the R&D sector, and that there are zero profits in this sector in equilibrium.

The government has an environmental cost function $D(E)$ with $D' > 0$ and $D'' \geq 0$, where E denotes total emissions. Prior to the production and trade of useful ideas, the government can commit to a particular level of total emission quotas.

Give a characterization of the government's optimal policy (as in equation 5 in Hoel: "Environmental R&D").

Zero profit in the innovating sector implies that the price of an idea is f. Firm i takes f as given and chooses x_i to minimize $c(a_i, x_i + \gamma \sum_{j \neq i} x_j) + f x_i + q(e_i^0 - a_i)$, giving $c_y + f = 0$. Since all firms make the same choices, it therefore follows that

$$c_y\left(a, [1 + \gamma(n-1)]\, x\right) + f(nx) = 0 \tag{1}$$

This equation is a constraint the government must take into consideration when minimizing (where $E = E^0 - na$)

$$H(a, x) = nc\left(a, [1 + \gamma(n-1)]\, x\right) + nxf(nx) + D(E^0 - na) \tag{2}$$

Without the constraint (1), maximizing (2) would give

$$\begin{aligned} c_a &= D'(E) \\ -c_y\left[1+\gamma(n-1)\right] &= f + nxf' \end{aligned}$$

With the constraint (2) we form the Lagrangian

$$L = H(a,x) + \mu\left[c_y\left(a,\left[1+\gamma(n-1)\right]x\right) + f(nx)\right]$$

Setting the first-order derivatives of L equal to zero gives, after some rearranging:

$$c_a - D'(E) = \frac{C_{ya}\left[-\gamma(n-1)f + nxf'\right]}{c_{yy}\left[1+\gamma(n-1)\right] + nf'}$$

Assume that the denominator is positive (i.e. if $f' < 0$ this term is dominated by the positive term)[1]. Since $C_{ya} < 0$, it follows that the optimum is characterized by $c_a - D'(E) > 0$ if $\gamma > 0$ or $f' < 0$.

Does your result depend on whether the quotas are allocated for free to the polluting firms or are auctioned?

No, these two forms of quotas give the same outcome provided free quotas are given to firms in a way that firms cannot influence.

How would the outcome be changed if instead of committing to a quota the government committed to an emission tax?

No change: Also with a tax the constraint facing the government would be (2), where a as before is chosen by the government (in this case indirectly through the choice of the tax).

[1] If the denomiator was negative, the zero-profit equilibrium would not be a stable solution to $\dot{X} = h \cdot (p - f)$, where p is the short-run price of ideas for a given X (and h is positive).