ECON4910, Spring 2010

Seminar 6

Problem 1; Sketch of solution

Use notation from my paper "Global warming.".

Denote initial emissions by $(e_1^0,...,e_n^0)$, which presumably follow from each country maximizing $R_j(e_j)-D_j(\sum_i e_i)$, giving $R_j'(e_j^0)=D_j'(\sum_i e_j^0)$. (You should say a few words about the functions *R* and *D*.)

The target for total emissions is

(1) $$\sum_i e_i = E^* = \frac{1}{2}\sum_i e_i^0$$

Before considering the 4 proposed agreements, it is useful to derive the optimum conditions given this target. If side payments are considered feasible, it makes sense to maximize $\sum_i R_i(e_i)$ subject to the constraint (1) on total emissions. Setting up the Lagrangian and solving we find that this yields

(2) $$R_1'(e_1)=...=R_n'(e_n)$$

which is often called a condition for *cost-effectiveness* or, in Kolstad, the *equimarginal principle*. The equations (1) and (2) give a specific emission vector that we denote $(e_1^*,...,e_n^*)$.

It is reasonable to assume that (1) is not completely arbitrary, and that the target *E** might follow from maximization of total net benefits $\sum_j \left[R_j(e_j) - D_j(\Sigma_i e_i) \right]$. If this is the case than in addition to (2) we have

(3) $$R_1'(e_1) = ... = R_n'(e_n) = \sum_j D_j'(\Sigma_i e_i)$$

However, it is not obvious that *E** satisfies (3). It could for instance be that the value of total emissions implied from (3) is lower than *E**, but that *E** is the lowest possible emission level that it is possible to agree upon. The reason for this is that the lower are total emissions in an agreement, the stronger are the free rider incentives each country has. In other words, the stricter an agreement is, the more can a single country earn by deviating from the agreement by increasing its own emissions. In what follows, we do not assume that (3) necessarily holds. However, it is reasonable to assume that in all of the proposed agreements; the sum of net benefits is higher than in the case of no cooperation (although the sum of net benefits might not be maximized).

Proposal 1

Since each country's net benefit is increasing in its emission up to e_j^0, the country will set emissions equal to the maximal permitted level, in this case equal to $e_j^0/2$. But generally $R_j'(e_j^0/2)$ will differ between countries, so *we will not get cost-effectiveness in this case*. Also, the costs of cutting emissions in half may vary strongly between countries. This agreement may therefore be considered unfair by some of the countries. Moreover, there is no guarantee that all countries get higher net benefits under this agreement than they get without an agreement (even if the *sum* of net benefits is higher under this agreement than without an agreement).

Proposals 2 and 3

Under both of these agreements country *j* gets a quota $\overline{e}_j$ and it will maximize

$R_j(e_j) - p(e_j - \overline{e}_j)$ giving $R_j'(e_j) = p$ (see the discussion leading up to (9) in "Global warming…"). Since the quota price *p* is the same for all countries, this gives condition (2). *For agreements 2 and 3 we therefore have cost-effectiveness*. This means that the sum of net benefits are higher under 2 and 3 than under 1. In agreement 2 the quotas are the same as in 1. Since it is voluntary for a country to buy or sell quotas, each country must therefore be at least as well off under 2 as under 1. However, also for 2 the increase in the sum of net benefits (compared with no agreement) may be unevenly distributed among countries, so some countries may consider the agreement unfair. Agreement 3 has a different distribution, better (compared with 2) for countries with low initial emissions per capita and worse (compared with 2) for countries with high emissions per capita. Also for this agreement the increase in the sum of net benefits (compared with no agreement) may be unevenly distributed among countries, so some countries may consider the agreement unfair. For both 2 and 3 (as for 1) there is no guarantee that all countries get higher net benefits under this agreement than they get without an agreement (even if the *sum* of net benefits is higher under 2 and 3 than under 1).

Proposal 4

Several of you make an error here. The net benefit of country *j* is in this case $R_j(e_j) - D_j(\sum_i e_i)$: You should *not* subtract the emission tax, as the emission tax is just a domestic transfer. However, the tax affects the behaviour of producers and households, so that instead of the emission level e_j^0 we get an emission level determined by

(4) $R_j'(e_1) = t$

where *t* is the tax rate (given by the agreement, same for al countries). The higher this tax rate, the lower are emissions in each country and therefore the lower are total emissions. For a suitably chosen tax rate (which might be difficult to find in practice) we therefore can reach the goal of *E** for total emissions. Moreover, since *t* is the same for all countries, it follows from (4) and (2) that we get *cost-effectiveness also for this agreement.* Emission levels of the countries are the same as in agreements 2 and 3 (given by (1) and (2)), but unlike in 2 and 3 there are now no net transfers between countries (in 2 and 3 the net transfers are given by the payments for the traded quotas). The distribution is therefore different in 4 than in 2 and 3. Even if *marginal* abatement costs are equalized across countries, *total* abatement costs of moving from e_j^0 to e_j^* may vary significantly between countries. Just like for the other agreements, some countries may therefore consider the agreement unfair, and there is no guarantee that all countries get higher net benefits under this agreement than they get without an agreement (even if the *sum* of net benefits is higher under 4 than under no agreement).

Conclusions

- 1 is not cost-effective, while 2, 3 and 4 are cost-effective
- the four agreements differ with regard to distribution
- 2 is for certain at least as good as 1 for all countries
- in none of the agreements are we guaranteed that all countries are better off with the agreement than without.

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Problem 2

Consider the following two-stage game: In stage 1 each of N identical countries decides whether or not to join a coalition of countries. In stage 2 both the coalition members and the outsiders determine their emission levels. In the second stage, the coalition maximizes the payoff to each of its members, and each outsider maximizes its own payoff.

Emissions are either 0 or 1. The cost to a country of reducing its emissions from 1 to 0 is c. The environmental cost to each country is bE, where E is total emissions. Assume that $b < c < bN$.

Describe the outcome of this game.

Start with stage 2 of the game. assume there are k countries in the coalition. The $N-k$ outsiders will choose emissions $= 1$ since $b < c$. The coalition members (acting jointly) will choose emissions $= 1$ if $kb < c$, but emissions $= 0$ if $kb \geq c$. Let k^*be the smallest integer satisfying $kb \geq c$.

Now consider stage 1. Consider a particular country considering to join a coalition of $k-1$ other countries, when the remaining $N-k$ countries are not joining. Clearly, if $k < k^*$ it makes no difference whether the country joins or not: Emissions will in any case be 1 for a coalition of $k-1$or k for $k < k^*$.The interesting cases are $k = k^*$ and $k > k^*$.

For $k = k^*$the following holds:

Payoff to a member of a coalition of size k^*: $bk^* - c$ which is ≥ 0 by the definition of k^*.

Payoff to an outsider of a coalition of $k^* - 1$ countries: 0, since the $k^* - 1$countries choose emissions =1

So better off as a member than as an outsider.

What if $k > k^*$?

Payoff to a member of a coalition of size k: $bk - c$

Payoff to an outsider of a coalition of $k-1$ countries: bc since the $k-1$ countries choose emissions =0 since $k - 1 \geq k^*$.

So better off as outsider than than a coalition member.

Conclusion: The unique equilibrium is a coalition of size k^*.

Show how total emissions and total welfare depend on c.

Ignore the integer problem, i.e. assume $k^* = \frac{c}{b}$. Total emissions are $N - k^* = N - \frac{c}{b}$, i.e. lower c gives *higher* emissions.

Total welfare is

$$\begin{aligned} W &= \text{value of total emission reduction - total abatement costs} \\ &= Nbk^* - k^*c \ = (Nb - c)\, k^* = (Nb - c)\, \frac{c}{b} = Nc - \frac{c^2}{b} \end{aligned}$$

and

$$\frac{\partial W}{\partial c} = N - \frac{2c}{b} = 2(\frac{N}{2} - k^*)$$

so a decline in c increases W if and only if $k^* > \frac{N}{2}$.

Disutility of being an outsider

Assume that each country has some disutility of being an outsider when there exists a coalition of emission abating countries, and that this disutility is larger the larger this coalition is. Let this feature be modeled by assuming that the payoff to an outsider that does not abate is $bn - \gamma n$, where $\gamma > 0$ and n is the number of abating countries in a coalition. How does this change in the model affect the outcome of the game?

Like before, it is only coalitions of size $k \geq k^*$ that are of interest. Like before, a country is better off in a coalition of k^* countries than as an outsider of a coalition of $k^* - 1$ countries. But are the outsiders of a coalition of k^* better off as outsiders than as members of a coalition of $k^* + 1$?

Payoff to a member of a coalition of size $k^* + 1$: $b\,(k^* + 1) - c$

Payoff to an outsider of a coalition of k^* countries: $bk^* - \gamma k^*$

So better off joining the coalition if and only if $b\,(k^* + 1) - c \geq bk^* - \gamma k^*$, i.e. if and only if

$$\gamma \geq \frac{c - b}{k^*}$$

If this inequality holds, the coalition of size k^* is not an equilibrium, since the $N - k^*$ outsiders would not be happy about the choice of being an outsider. Moreover, if the inequality above holds, it holds also for all $k > k^*$. This implies that the equilibrium coalition size is N.

If the inequality above does *not* hold, the coalition of size k^* is an equilibrium. However, it still might be the case that

$$\gamma \geq \frac{c-b}{N-1}$$

If this is the case, the coalition of all N countries is also an equilibrium.

Problem 3

Emissions to a lake from a sector of the economy are given by

$$x = E - a \tag{1}$$

where E is exogenous (and positive) and a is abatement. The abatement cost function is

$$c(a) = \frac{a^2}{2} \tag{2}$$

Emissions accumulate in the lake according to the differential equation

$$\dot{A} = x - \alpha A \tag{3}$$

where A is the stock of the pollutant in the lake and the depreciation parameter is positive.

The environmental damage cost is assumed to be given by the function

$$D(A) = \frac{bA^2}{2} \tag{4}$$

where b is a positive parameter.

The environmental regulator sets an emission tax in order to regulate the amount of emissions.

Derive the qualitative properties of the time paths of the optimal emission tax and the optimal emission rate.

I first solve this using optimal control theory, then explain how you can reach the same equations using informal economic reasoning.

Optimal control theory

We want to maximize (including the possibility of cleaning, and ignoring time references where this cannot cause any misunderstanding)

$$W = \int_0^\infty e^{-rt} \left[-c(a) - ky - D(A) \right] dt$$

s.t.

$$\dot{A} = x - y - \alpha A$$

The current value Hamiltonian is

$$H = -c(a) - ky - D(A) + \lambda\left(E - a - y - \alpha A\right)$$

and the conditions for the optimum are

$$\dot{\lambda} = r\lambda - \frac{\partial H}{\partial A} = (r + \alpha) + D'(A)$$

$$\frac{\partial H}{\partial a} = -c'(a) - \lambda \leq 0 \quad [= 0 \text{ for } a > 0]$$

$$\frac{\partial H}{\partial y} = -k - \lambda \leq 0 \quad [= 0 \text{ for } y > 0]$$

The shadow price λ has the interpretation of the value of on more unit of A. Since A is a "bad", $\lambda < 0$. It is easier to interpret positive numbers, so we define $q = -\lambda$. The three equations above can then be rewritten as

$$\dot{q} = (r + \alpha)\, q - D'(A)$$

or, using (4)

$$\dot{q} = (r + \alpha)\, q - bA \tag{5}$$

$$q - c'(a) \leq 0 \quad [= 0 \text{ for } a > 0]$$

or, using (2)

$$a = q \tag{6}$$

$$q - k \leq 0 \quad [= 0 \text{ for } y > 0] \tag{7}$$

From (5) and the transversality condition that implies $Lim_{t \to \infty} e^{-rt} q(t) = 0$ it follows that

$$q(t) = \int_t^{\infty} e^{-(r+\alpha)(\tau - t)} D'\left(A\left(\tau\right)\right) d\tau$$

or, using (4)

$$q(t) = b\int_t^{\infty} e^{-(r+\alpha)(\tau-t)} A(\tau)\, d\tau \tag{8}$$

Informal economic reasoning

Let us ignore (7) for the time being.

Equation (6) is straightforward: It simply says that marginal abatement costs ($= a$) are equal to the emission tax q. Equation (8) tells us what the optimal emission tax is for a stock pollutant. Just like for a flow pollutant, it is equal to the marginal environmental damage caused by one unit of emissions:

One unit of emissions at date t gives $e^{-\alpha(\tau-t)}$ units of pollution at date τ.

$e^{-\alpha(\tau-t)}$ units of pollution at date τ give a damage equal to $e^{-\alpha(\tau-t)}D'(A(\tau)$.

The discounted value of this is $e^{-r(\tau-t)}e^{-\alpha(\tau-t)}D'(A(\tau) = e^{-(r+\alpha)(\tau-t)}D'(A(\tau)$.

Summed over all future dates this gives the optimal emission tax:

$$q(t) = \int_t^{\infty} e^{-(r+\alpha)(\tau-t)} D'(A(\tau))\, d\tau$$

and differentiating this with respect to t gives

$$\dot{q} = (r+\alpha) - D'(A)$$

We have thus found the equations (6), (5) and (8) using informal economic reasoning.

Qualitative properties

It is useful to first consider the long-run stationary solution.

Show how the long-run levels of the emission tax and the emission rate depend on the sizes of the parameters α and b.

A stationary solution is characterized by q and A constant . Ignoring for now the possibility of cleaning (i.e. setting $y = 0$) it follows from (3), (5) and ((6)that q and A are constant for

$$\begin{aligned} x^* &= \alpha A^* \\ (r+\alpha)\, q^* &= bA^* \\ x^* &= E - q^* \end{aligned}$$

Solving gives

$$A^* = \frac{r+\alpha}{b+\alpha\,(r+\alpha)}E$$
$$q^* = \frac{b}{b+\alpha\,(r+\alpha)}E$$
$$x^* = \frac{\alpha\,(r+\alpha)}{b+\alpha\,(r+\alpha)}E$$

The development towards these long-run values is described in the phase diagram in Figure 1. There are two stable trajectories leading to the stationary outcome (A^*, q^*); one from below and one from above. In Figure 1 it is assumed that $A(0) \in (0, A^*)$, giving $q\,(0) \in (0, q^*)$. Both A and q grow over time in the equilibrium.

Possibility of cleaning

From (7) it is clear that $q(t)$ can never exceed k. The interpretation is obvious: If $q(t) > k$, we would immediately clean at an infinite rate, making A immediately drop downwards . This downward drop would from (8) give a downward drop in $q(t)$, restoring the equality $q(t) = k$.

The stationary equilibrium is now given by

$$\begin{aligned} q^* &= k \\ x^* + y^* &= \alpha A^* \\ (r+\alpha)\,q^* &= bA^* \\ x^* &= E - q^* \end{aligned}$$

giving

$$\begin{aligned} q^* &= k \\ A^* &= \frac{r+\alpha}{b}k \\ x^* &= E - k \\ y^* &= \frac{\alpha\,(r+\alpha)+b}{b}k - E \end{aligned}$$

This stationary solution as well as the development of A and q towards it is described in Figure 2.

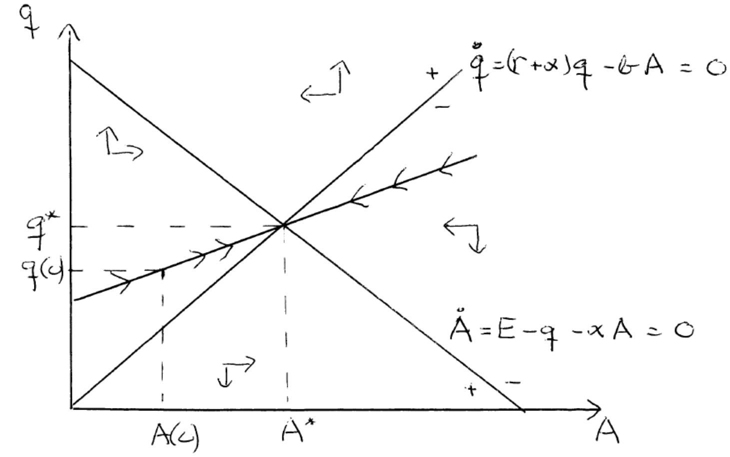

Figure 1

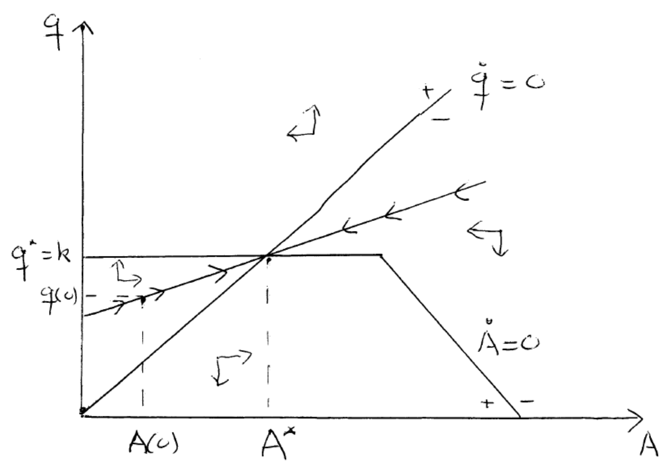

Figure 2