ECON4910, Spring 2010

Lecture 13: Discounting

Reading list:
Perman et al.: Sec. 3.1-3.4
Dasgupta

Outline of lecture:

1. The concept of intertemporal efficiency with an application to environmental issues
2. Intertemporal optimization in a one person two period economy: maximize subject to $C_1 = X - I$ and $C_2 = X + F(I)$
3. The role of the interest rate in a market economy
4. The special case of $U(C_1, C_2) = u(C_1) + \beta u(C_2)$
5. Extension of 4 to include environmental goods.
6. Optimal growth with a stock pollutant

Optimal growth with a stock pollutant

Time references are omitted where this cannot cause any misunderstanding. Maximize

$$\int_0^\infty e^{-\rho t} \left[u(C) - hS \right]$$

s.t.

$$\begin{aligned} \dot{K} &= F(K, e, t) - C \\ \dot{S} &= e - \delta S \end{aligned}$$

Notice that the marginal cost (WTP) of S is hC^α under the assumption that $u'(C) = C^{-\alpha}$.

The current value Hamiltonian is

$$H = u(C) - hS + \lambda \left[F(K, e, t) - C \right] + \mu \left[e - \delta S \right]$$

and the optimum conditions are

$$\dot{\lambda} = \rho\lambda - \frac{\partial H}{\partial K} = (\rho - F_K)\lambda \tag{1}$$

$$\dot{\mu} = \rho\mu - \frac{\partial H}{\partial S} = (\rho + \delta)\mu + h \tag{2}$$

$$H_C = u'(C) - \lambda = 0 \tag{3}$$

$$H_e = \lambda F_e + \mu = 0 \tag{4}$$

Using $u' = C^{-\alpha}$ and $g = \frac{\dot{C}}{C}$, equations (1) and (3) give

$$F_K = \rho + \alpha g \tag{5}$$

Rewriting (4) gives

$$F_e = \frac{-\mu}{\lambda} \equiv q \tag{6}$$

which together with (2) implies

$$\frac{\dot{q}}{q} = \frac{\dot{\mu}}{\mu} - \frac{\dot{\lambda}}{\lambda} = \left[\rho + \delta + \frac{h}{\mu}\right] - [\rho - F_K]$$

Using (6) this may be rewritten as

$$\frac{\dot{q}}{q} = F_K + \delta - \frac{h}{qu'}$$

or

$$\dot{q} = (F_K + \delta)q - hC^{\alpha}$$

If we as a simplification regard F_K as constant, this gives (using the transversality condition of the optimization problem)

$$q(t) = \int_t^{\infty} e^{-(F_K+\delta)(\tau-t)} hC(t)^{\alpha} e^{\alpha g((\tau-t))} d\tau$$

or, using (5)

$$q(t) = hC(t)^{\alpha} \int_t^{\infty} e^{-(\rho+\delta)(\tau-t)} d\tau = \frac{h}{\rho+\delta} C(t)^{\alpha}$$