Supplement to lecture 13

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From the lecture and Hoel (1994), eq. (9) we know that the optimal choice of $(x, y,)$ is given by

$$u'(y) = p + (y - x)p' + \frac{S'}{S' - D'}n' \tag{1}$$

$$c'(x) = p + (y - x)p' + \frac{D'}{S' - D'}n' \tag{2}$$

In addition, we have

$$p = p(y - x)$$
$$X = S(p)$$
$$Y = D(p)$$

These 5 equations determine (x, y, X, Y, p).

In the lecture we also showed that maximizing global social welfare gave the following 3 equations:

$$c'(x) = C'(X) \tag{3}$$

$$u'(y) = U'(Y) \tag{4}$$

$$u'(y) = c'(x) + n'(y + Y) \tag{5}$$

Together with $y + Y = x + X$ these three equations determine (x, y, X, Y).

Generally, the equations for the single country optimum do not coincide with the equations for the global optimum. However, consider the special case of $y = x$ and $S' = 0$ (which is relevant due to the results of

Harstad). In this case (1) and (2) may be rewritten as

$$u'(y) = p \tag{6}$$

$$c'(x) = p - n' \tag{7}$$

which implies that (5) holds. Moreover, $U'(Y) = p$, implying that (4) also holds. Eq. (3) is not part of the optimum condition if $S' = 0$, since $C'(X)$ is not well-defined for X-values where the marginal cost curve is vertical: When X is given because $S' = 0$, the remaining variables (x, y, Y) are determined by (4) and (5) together with $y + Y = x + X$. Hence, the optimum for the single country (or group of countries) coincides with the global optimum when $y = x$ and $S' = 0$.