ECON4910 Environmental Economics — Seminar 1

January 26, 2015

Problem 1: Efficient provision of a public good

Consider a simple economy with two agents, A and B, and two goods. One is a private good x_i ($i = \{A, B\}$) can be thought as money to be spent for private consumption, and the other is a public good G that can be provided in a continuous amount. We assume the two agents are initially endowed with w_i units of consumption and they can contribute to the public good (g_i) by giving up some of their private consumption, such that: $x_i = w_i - g_i$. Their utility $u_i(G, x_i)$ depend on the consumption of public good $G = g_A + g_B$ (or some function of the sum of their contributions) and on their private consumption.

(a) Define a Pareto optimal resource allocation. Now, suppose utility is $u_i(G, x_i) = a_i \ln G + \ln x_i$, where $a_i > 0$ is some parameter. Find the efficient (Pareto optimal) provision G^* of the public good.

(b) How does the optimal amount of the public good (G^*) change when using a quasi-linear utility function, i.e. $u_i(G, x_i) = b_i \ln G + x_i$? Try to explain why.

(c) Using the utility function in (a), find the best response functions for g_A and g_B.

(d) Using the utility function in (b), find the private provision of the public good (G^{NE}).

  (i) Explain when the agents decide to cooperate and when it is optimal to free ride.

  (ii) Compare the total level of the public good in the Nash equilibrium with the one in the Pareto allocation (compare G^{NE} in (d) and G^* in (b))

(e) *Optional*: Compare the two levels of the public good in (c) and (a) and show that $G^* > G^{\text{NE}}$. (A description of the method is also fine.)

Problem 2: Pareto Optimum in an economy with a uniformly mixing flow pollutant

Consider an economy with N consumers, who enjoy consumption of a private good x and a pure public good G
The preferences of consumer $i \in N$ are:

$$U_i = u_i(G, x_i) \tag{1}$$

where u_i is quasiconcave and increasing in both arguments, and x_i is i's private consumption of x.

The private good is produced by K producers, for which the production function of each firm $k \in K$ is:

$$y_k = f_k(m_k) \tag{2}$$

where

- y_k is firm k's production of x,
- m_k is firm k's emissions of a uniformly mixing pollutant,
- with $f_k' > 0$ and $f_k'' < 0$

We assume that there is no waste of the private good, so that $\sum_{i=1}^{N} x_i = \sum_{k=1}^{K} f_k(m_k)$ (total consumption of the private good equals total production).

The public good G is affected by pollution in the following way:

$$G = G_0 - z(M) \text{ with} \tag{3}$$

where $z' > 0$, G_0 is exogenously given and $M = \sum_{k=1}^{K} m_k$.

(a) Derive the first order conditions for Pareto optimality in this economy, and explain their economic interpretation.

Using the same economy described above, we now assume:

- the production function is $y_k = f_k(m_k) = 30\ln(m_k) - m_k$,
- the price of the private good x is 1
- every consumer i has the same income F (exogenous) and the same preferences,
- the number of firms $K = 100$,
- and $G = G_0 - z(M) = 1000 - M/500$.

(b) Assume here that production costs are fixed, so the profit for firm k is: $\pi_k = f_k(m_k) - b_k$. If consumers cannot bargain with producers about emission levels, and there is no environmental regulation, how large will total emissions M be? What will the level of G be?

(c) Assume now that consumers can bargain with producers about emission levels. One consumer, call him i, discovers this first, and let him assume that no-one else will bargain on emissions. Assume, for simplicity, that he is approached by a firm offering him abatement at a fixed price $P > 0$ per unit of abatement. Will he purchase a strictly positive level of abatement? Explain and characterize the solution. Can we know for sure that there exists a strictly positive price P that will make him purchase a strictly positive level of abatement? (Hint: Start by finding an expression for M from consumer i's point of view.)

(d) Lastly, assume that the other $N - 1$ consumers hear about the first consumer's contribution and now understand that bargaining is possible. How much will each of them offer to pay firms in order to reduce emissions, if they take other consumers' contribution as exogenously given? Will the resulting situation be Pareto optimal? (A brief, intuitive answer is sufficient.)