ECON4910 Environmental Economics
Spring 2017

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Problem set 1

Ex. 1. Uniformly mixing flow pollutant

Consider an economy with N consumers, who enjoy consumption of a private good x and a pure public good E.

1. Come up with some pure public good that are related to nature or the environment.

   The preferences of consumer $i \in N$ are:

   $$U_i = u_i(x_i, E) \tag{1}$$

   where u_i is quasiconcave and increasing in both arguments, and x_i is i's private consumption of x and E is i's consumption of the public good.

2. How much are consumers willing to pay for a marginal improvement in the environment?

   The private good is produced by K producers where the production function y_k is firm k's production of x, for each firm $k \in K$:

   $$y_k = f_k(m_k) \tag{2}$$

   with $f_k' > 0$ and $f_k'' < 0$. Where m_k is firm k's emissions of a uniformly mixing pollutant. We assume that there is no waste of the private good, so that $\sum_{i=1}^{N} x_i = \sum_{k=1}^{K} f_k(m_k)$ (total consumption of the private good equals total production).

3. What is the society's marginal benefit from emission $B'(M)$?

   The public good/the environment E is negatively affected by the pollution function $z(M)$, decreasing environmental quality from the baseline level E_0:

   $$E = E_0 - z(M) \tag{3}$$

   where the damage increases in emissions $z' > 0$ and E_0 is exogenously given

4. What are the society's marginal damage from emissions $D'(M)$?

5. Derive the first order conditions for Pareto optimality (efficiency in consumption) in this economy, and explain their economic interpretation.

   Using the same economy described above, we now assume:

   - the production function is $y_k = f_k(m_k) = 30 \ln m_k - m_k$,
   - the price of the private good x is 1
   - every consumer i has the same exogenous income, with a budget given by $Y \leq x_i$, and the same preferences
   - the number of firms $K = 100$,
   - $E_0 = 1000$
   - and $z(M) = M/500$.

6. Is there any bargaining opportunities in this economy?

   Assume that production costs are fixed, so the profit for firm k is:

   $$\pi_k = f_k(m_k) - b_k \tag{4}$$

7. If consumers cannot bargain with producers about emission levels, and there is no environmental regulation, how large will total emissions M be? What will the level of E be?

8. Assume now that consumers can bargain with producers about emission levels. One consumer, call him i, discovers this first, and let him assume that no-one else will bargain on emissions. Assume, for simplicity, that he is approached by a firm offering him abatement at a fixed price $P > 0$ per unit of abatement. Will he purchase a strictly positive level of abatement? Explain and characterize the solution.

9. Can we know for sure that there exists a strictly positive price P that will make him purchase a strictly positive level of abatement? (Hint: Start by finding an expression for M from consumer i's point of view.)

10. Lastly, assume that the other $N-1$ consumers hear about the first consumer's contribution and now understand that bargaining is possible. How much will each of them offer to pay firms in order to reduce emissions, if they take other consumers' contribution as exogenously given? Will the resulting situation be Pareto optimal? (A brief, intuitive answer is sufficient.)

Ex. 2 Efficient provision of a public good

Consider a simple economy with two agents, A and B, and two goods. One is a private good x_i ($i = \{A, B\}$) can be thought as money to be spent for private consumption, and the other is a public good G that can be provided in a continuous amount. We assume the two agents are initially endowed with w_i units of consumption and they can contribute to the public good (g_i) by giving up some of their private consumption, such that: $x_i = w_i - g_i$. Their utility $u_i(G, x_i)$ depend on the consumption of public good $G = g_A + g_B$ (or some function of the sum of their contributions) and on their private consumption.

1. Define a Pareto optimal resource allocation in words.

   Now, suppose utility is

   $$u_i(G, x_i) = a_i \ln G + \ln x_i \tag{5}$$

   where $a_i > 0$ is some parameter.

2. Find the efficient (Pareto optimal) provision G^* of the public good.

3. How does the optimal amount of the public good (G^*) change when using a quasilinear utility function:

   $$u_i(G, x_i) = b_i \ln G + x_i \tag{6}$$

4. Using the utility function (5), find the best response functions for g_A and g_B.

5. Using the utility function (6), find the private provision of the public good G^{NE}.

   - Explain when the agents decide to cooperate, and when it is optimal to free ride.

- Compare the total level of the public good in the Nash equilibrium with the one in the Pareto allocation (compare G^{NE} in (d) and G^* in (b))

6. *Optional*: Compare the two levels of the public good in (c) and (a) and show that $G^* > G^{\mathrm{NE}}$. (A description of the method is also fine.)