ECON4910 Environmental Economics
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Corrections and comments to the seminar exercises

Some students have very kindly informed me about small mistakes and misunderstandings in the seminar exercises. I post a summery here with the remarks

Seminar 1

In the Uniformly mixing flow pollutant exercise:

8. If the firm sets the price P, the consumer will purchase abatement until his marginal willingness to pay, in units of x_i, for reduced emissions equals the price.

9. The marginal willingness to pay for abatement in the initial situation in must be greater than P. Since we don't know what the utility function looks like (or the price P), we cannot determine whether this inequality holds.

10. Note that if consumer 1 has contributed everything he prefers (given that he thinks others will contribute nothing), and everyone has the same preferences, then the first consumer has already secured that level of E that any other individual would be willing to secure. Thus, all other consumers will be free-riders.

Seminar 2.

Yes, it can be hard to grasp why the license price in district 1 goes to zero. Recall that the same emission unit from firm i has different harm on the different locations. Each locations refers to a license market. The regulator has distributed an equal amount of licenses in each market j. Since the harm in district 2 is twice as high as in district 1, the firm never needs to buy licenses in market 1, such that the license market in district 1 never binds and the price goes to zero $p_1 = 0$

Seminar 4.

The formulas for the deadweight losses in the seminar exercises are wrong... The correct are:

$$DWL_p = \frac{\big(B'(q(\bar{p})) - C'(q(\bar{p}))\big)\big(q(p^{**}) - q(\bar{p})\big)}{2}$$
$$DWL_q = \frac{\big(B'(\bar{q}) - C'(\bar{q})\big)\big(q^{**} - \bar{q}\big)}{2}$$

And since the two death weight losses are of equal size, we can also write them as:

$$DWL_p = \frac{\big(C'(q(\bar{p})) - B'(q(\bar{p}))\big)\big(q(\bar{p}) - q(p^{**})\big)}{2}$$
$$DWL_q = \frac{\big(C'(\bar{q}) - B'(\bar{q})\big)\big(\bar{q} - q^{**}\big)}{2}$$

Part I: Full info

- To implement q^* using a quantity instrument: Quota sets $q = q^*$

$$\text{SW}_{quota} = B(q^*) - C(q^*)$$
$$B(0) + \beta q^* - \frac{1}{2}bq^{*2} - C(0) - aq^* - \frac{1}{2}cq^{*2}$$
$$\big(B(0) - C(0)\big) + (\beta - a)q^* - \frac{1}{2}(b+c)q^{*2}$$
$$NB(0) + \frac{(\beta - a)^2}{2(b+c)} = \text{SW}_{quota}$$

- To implement q^* using a price instrument: Firm max profit $\pi = pq - C(q)$, which gives FOC: $p = MC(q^*)$

$$p = a + cq^*$$
$$p = a + c\Big(\frac{\beta - a}{b+c}\Big) = \frac{c\beta + ab}{b+c}$$

The Planner sets the price $p = MB(q^*)$

$$p = \beta - bq^* = \beta - b\Big(\frac{\beta - a}{b+c}\Big) = \frac{c\beta + ab}{b+c}$$
$$\Big[B(q(p^*)) + p^*q(p^*)\Big] - \Big[C(q(p^*)) + p^*q(p^*)\Big] = B(q^*) - C(q^*) = \text{SW}_{quota}$$

Which is exactly the same as SW_{quota}

- Which instrument is better in terms of maximizing the social welfare? Given complete information, prices and quotas are equally good instruments for regulation, giving the same welfare

Part II: Incomplete info: Quota instrument

Suppose the regulator sets a quota of $\bar{q}$. Deadweight loss above the optimum:

$$1/2(\bar{q} - q^{**})\big[MC(\bar{q}) - MB(\bar{q})\big]$$

$$1/2(\bar{q} - q^{**})\big[\theta + \gamma + c\bar{q} - \beta + b\bar{q}\big]$$

$$1/2(\bar{q} - q^{**})\big[-(\beta - \theta - \gamma) + \bar{q}(b + c)\big]$$

insert $q^{**} = \frac{\beta-\theta-\gamma}{b+c} \rightarrow q^{**}(b + c) = (\beta - \theta - \gamma)$

$$1/2(\bar{q} - q^{**})\big[-q^{**}(b + c) + \bar{q}(b + c)\big]$$

$$1/2(\bar{q} - q^{**})(\bar{q} - q^{**})(b + c)$$

$$\frac{b + c}{2}(\bar{q} - q^{**})^2$$

Next, show that the expected deadweight loss if the quantity $\bar{q}$ is imposed is:

$$E[\text{DWL}] = \frac{1}{2}\frac{(b + c)}{2}\Big[\frac{\beta - \gamma - \delta}{(b + c)} - \bar{q}\Big]^2 + \frac{1}{2}\frac{(b + c)}{2}\Big[\frac{\beta - \gamma + \delta}{(b + c)} - \bar{q}\Big]^2$$

Show that the best quantity to impose *ex-ante* is to minimize the *expected* DWL:

$$\hat{q} \quad \arg\min_{\bar{q}}\big\{E[DWL]\big\}$$

$$\hat{q} \quad \arg\min_{\bar{q}}\Big\{\frac{(b + c)}{4}\Big[\Big(\frac{\beta - \gamma - \delta}{(b + c)} - \bar{q}\Big)^2 + \Big(\frac{\beta - \gamma + \delta}{(b + c)} - \bar{q}\Big)^2\Big]\Big\}$$

$$\hat{q}(b + c) - \frac{\beta - \gamma - \delta}{2} - \frac{\beta - \gamma + \delta}{2} = 0$$

$$\hat{q} = \frac{\beta - \gamma}{b + c}$$

$\hat{q}$ is the best quantity to impose ex ante. Next, show that the expected deadweight loss when $\hat{q}$ is imposed is:

$$E[\text{DWL}] = \frac{(b + c)}{4}\Big[\Big(\frac{\beta - \gamma - \delta}{(b + c)} - \hat{q}\Big)^2 + \Big(\frac{\beta - \gamma + \delta}{(b + c)} - \hat{q}\Big)^2\Big]$$

insert $\hat{q}$:

$$= \frac{(b+c)}{4}\Big[\Big(\frac{\beta-\gamma-\delta}{b+c} - \frac{\beta-\gamma}{b+c}\Big)^2 + \Big(\frac{\beta-\gamma+\delta}{b+c} - \frac{\beta-\gamma}{b+c}\Big)^2\Big]$$

$$= \frac{(b+c)}{4}\Big[\Big(\frac{-\delta}{b+c}\Big)^2 + \Big(\frac{\delta}{b+c}\Big)^2\Big]$$

$$\hat{L}_q = \frac{\delta^2}{2(b+c)}$$

Interpret this expression.

Part III: Incomplete info: Price instrument

The firm maximize profit $\pi = pq - C(q)$, and this gives $p = C'(q)$:

$$\bar{p} = MC = \gamma + \theta + cq$$

$$q(\bar{p}, \theta) = \frac{\bar{p} - \gamma - \theta}{c}$$

The optimal price (p) for a given θ

$$MB(q(p^{**}, \theta)) = MC(q(p^{**}, \theta))$$

$$\beta - b\Big(\frac{p^{**} - \gamma - \theta}{c}\Big) = \gamma + \theta + c\Big(\frac{p^{**} - \gamma - \theta}{c}\Big)$$

$$p^{**} = \frac{\beta c + b(\gamma + \theta)}{b+c}$$

which is the optimal emission price

$$q(p^{**}, \theta) = \frac{\beta - \gamma - \theta}{b+c}$$

Next, show that the *expected* deadweight loss if price p is imposed before the value of θ is realized is:

$$E[\text{DWL}] = \frac{1}{2}\Big[\frac{b+c}{2}\big(q(\bar{p}) - q(p^{**})\big)^2\Big] + \frac{1}{2}\Big[\frac{b+c}{2}\big(q(\bar{p}) - q(p^{**})\big)^2\Big]$$

$$E[\text{DWL}] = \frac{(b+c)}{4}\Big[\Big(\frac{\bar{p}-\gamma+\delta}{c} - \frac{\beta-\gamma+\delta}{(b+c)}\Big)^2 + \Big(\frac{\bar{p}-\gamma-\delta}{c} - \frac{\beta-\gamma-\delta}{(b+c)}\Big)^2\Big]$$

Minimize the *expected* deadweight loss:

$$\hat{p} \quad \arg\min_{\bar{p}}\big\{E[DWL]\big\}$$

$$\hat{p} \quad \arg\min_{\bar{p}} \Big\{ \frac{(b+c)}{4} \Big[\Big(\frac{\bar{p}-\gamma+\delta}{c} - \frac{\beta-\gamma+\delta}{(b+c)} \Big)^2 + \Big(\frac{\bar{p}-\gamma-\delta}{c} - \frac{\beta-\gamma-\delta}{(b+c)} \Big)^2 \Big] \Big\}$$

Show that, at the optimal *ex ante* price $\hat{p}$, the expected dead weight loss is:

$$E[\text{DWL}] = \frac{(b+c)}{4} \Big[\Big(\frac{\hat{p}-\gamma+\delta}{c} - \frac{\beta-\gamma+\delta}{(b+c)} \Big)^2 + \Big(\frac{\hat{p}-\gamma-\delta}{c} - \frac{\beta-\gamma-\delta}{(b+c)} \Big)^2 \Big]$$

insert for $\hat{p} = \frac{b\gamma+c\beta}{b+c}$ and find $\hat{L}_p$:

$$= \frac{(b+c)}{4} \Big[\Big(\frac{\frac{b\gamma+c\beta}{b+c}-\gamma+\delta}{c} - \frac{\beta-\gamma+\delta}{(b+c)} \Big)^2 + \Big(\frac{\frac{b\gamma+c\beta}{b+c}-\gamma-\delta}{c} - \frac{\beta-\gamma-\delta}{(b+c)} \Big)^2 \Big]$$

$$= \frac{(b+c)}{4} \Big[\Big(\frac{b\gamma+\beta c+b\delta-b\gamma-\beta c}{c(b+c)} \Big)^2 + \Big(\frac{b\gamma+\beta c-b\delta-b\gamma-\beta c}{c(b+c)} \Big)^2 \Big]$$

$$\hat{L}_p = \frac{\delta^2}{2(b+c)} \Big(\frac{b}{c} \Big)^2$$

Interpret this expression.

Seminar 5.

In the Deforestation exercise:

It is the same whether we write v_i as the disutility of deforestation x_i, or as the positive utility value of conservation $(X - x_i)$

Discounting exercise:

question 2. How much should one discount future consumption, if the growth rate of consumption is 2% a year?

- Start out with the Euler equation:

$$e^{-rt}u'(c_0) = e^{-\rho t}u'(c_t) \tag{1}$$

$$\ln e^{-rt} + \ln u'(c_0) = \ln e^{-\rho t} + \ln u'(c_t)$$
$$-rt + \ln u'(c_0) = -\rho t + \ln u'(c_t)$$

- Take the derivative wrt time

$$-r = -\rho + \frac{u''(c_t)}{u'(c_t)}\dot{c}$$

$$-r = -\rho + \frac{u''(c_t)c_t}{u'(c_t)}\frac{\dot{c}}{c_t}$$

- Where the elasticity of substitution is defined as $\frac{u''(c_t)c_t}{u'(c_t)} \equiv -\eta_t$, and the consumption growth rate is defined as $\frac{\dot{c}}{c_t} \equiv \mu_t$:

$$r = \rho + \eta_t \mu_t$$

- Find the value of η when our utility function is given by $u_t(c_t) = 2\sqrt{c_t}$:

$$u_t(c_t) = 2\sqrt{c_t}, \quad u_t'(c_t) = \frac{1}{\sqrt{c_t}}, \quad u_t''(c_t) = -\frac{1}{2c_t^{3/2}}$$

$$-\eta = \frac{u''(c_t)c_t}{u'(c_t)} = \frac{-\frac{1}{2c_t^{3/2}}c_t}{\frac{1}{\sqrt{c_t}}} = -\frac{\frac{1}{2\sqrt{c_t}}}{\frac{1}{\sqrt{c_t}}} = -\frac{1}{2}$$

- Then $\eta = 0.5$, $\mu = 0.02$ and $\rho = 0.11$, insert:

$$r = 0.11 + 0.5 \times 0.02 = 0.12 = 12\%$$

- This rate of return on investments is higher when the discount factor is low, because the agents are less patient and demand a higher return to be willing to invest in the future.

question 3. Suppose there are two groups in the society. Half of the population are patient and have $\delta = 0.99$ (lecture), while the other half is applying discount factor $\delta = 0.90$ (exercise). Suppose you want to maximize the sum of todays' welfare (present discounted value). What is the max amount you, as the planner, would be willing to invest/pay today if the value is worth 100 consumption units in 50 years? Which annual discount rate does this correspond to?

  - \$1 at time t has the same amount as e^{-rt} dollar today
  - \$100 at time $t = 50$ has the same amount as $(100 \times e^{-rt})$ dollar today
  - **Group B:** with $\delta = 0.9$, gives $r = 12\%$,

    $$r^B = 0.11 + 0.5 \times 0.02 = 0.12$$

    \$100 dollar at time $t = 50$ is

    $$100 \times e^{-0.12 \times 50} = 0.25\$ \quad \text{today}$$

  - **Group A:** with $\delta = 0.99$, gives $r = 2\%$,

    $$r^A = 0.01 + 0.5 \times 0.02 = 0.02$$

    \$100 dollar at time $t = 50$ is

    $$100 \times e^{-0.02 \times 50} \approx 37\$ \quad \text{today}$$

- **SP** Include both groups

$$100 \times \Big(\frac{1}{2}e^{-0.02*50} + \frac{1}{2}e^{-0.12*50}\Big) = 100\big(1/2e^{-1} + 1/2e^{-6}\big) = 18.518 \quad (2)$$

  18.518 \$ is the max willingness to pay for the social planner. This corresponds to a discount rate: $r^{SP} = R$

$$100e^{-R=\times 50} = 18.518$$
$$-R \times 50 = \ln\big(\frac{18.518}{100}\big)$$
$$-R \times 50 = -1.6864$$
$$R = 0.0337 = 3.37\%$$

- Group A is the most patient, expect more from the future, and has a lower discount **rate**
- The lower discount rate, the less we discount the future, we care about future return and take them into account, then the higher present value has future cash flows
- Group B: The higher discount rate the lower present value of future cash flows
- Social planners discount rate is closest to the most patient group

question 4. What is the answers to (question 3.) if instead the 100 consumption units are materialized in 100 years, not 50 years?

- **Group B:** with $\delta = 0.9$, gives $r = 12\%$,
- with $r = 12\%$, \$100 dollar at time $t = 100$ is

$$100 \times e^{-0.12\times 100} = 0.0006\$ \approx 0 \quad \text{today} \quad (3)$$

- **Group A:** with $\delta = 0.99$, gives $r = 2\%$,
- with $r = 2\%$, \$100 dollar at time $t = 100$ is

$$100 \times e^{-0.02\times 100} \approx 14\$ \quad \text{today} \quad (4)$$

- **Social Planner**

$$100 \times \Big(\frac{1}{2}e^{-0.02*100} + \frac{1}{2}e^{-0.12*100}\Big) = 100\big(1/2e^{-2} + 1/2e^{-12}\big) \approx 7\$ \quad (5)$$

– With a longer time horizon the discounted values almost diminishes, especially for high discount rate, very impatient consumers

In the Time inconsistency exercise

If one **can commit** in period 0 to all future consumption levels, what is the optimal c_t for each period?

- With commitment the decision maker will maximize and decide the emission level/consumption for all periods.

$$\max_{c_0,c_1,c_2} c_0 - \frac{1}{2}c_0^2 + \beta\delta c_1 - \beta\delta\frac{1}{2}(c_0+c_1)^2 + \beta\delta^2 c_2 - \beta\delta^2\frac{1}{2}(c_0+c_1+c_2)^2$$

$$\mathrm{FOC}_{c_0} \quad 1 - c_0 - \beta\delta(c_0+c_1) - \beta\delta^2(c_0+c_1+c_2) = 0$$

$$c_0 = \frac{1-\beta\delta(1+\delta)c_1 - \beta\delta^2 c_2}{1+\beta\delta(1+\delta)} \tag{6}$$

$$\mathrm{FOC}_{c_1} \quad \beta\delta - \beta\delta(c_0+c_1) - \beta\delta^2(c_0+c_1+c_2) = 0$$

$$1-(c_0+c_1) - \delta(c_0+c_1+c_2) = 0$$

$$c_1 = \frac{1}{1+\delta} - c_0 - \frac{\delta}{1+\delta}c_2 \tag{7}$$

$$\mathrm{FOC}_{c_2} \quad \beta\delta^2 - \beta\delta^2(c_0+c_1+c_2) = 0$$

$$1-(c_0+c_1+c_2) = 0$$

$$c_2 = 1 - c_0 - c_1 \tag{8}$$

Insert (6) into (7)

$$c_1 = \frac{1}{1+\delta} - \Big(\frac{1-\beta\delta(1+\delta)c_1 - \beta\delta^2 c_2}{1+\beta\delta(1+\delta)}\Big) - \frac{\delta}{1+\delta}c_2$$

$$c_1(c_2) = \beta\delta - \frac{\delta}{(1+\delta)} - \frac{\delta}{(1+\delta)}c_2 \tag{9}$$

Insert (6) into (8)

$$c_2 = 1 - \Big(\frac{1-\beta\delta(1+\delta)c_1 - \beta\delta^2 c_2}{1+\beta\delta(1+\delta)}\Big) - c_1$$

$$c_2(c_1) = \frac{\beta\delta(1+\delta)}{1+\beta\delta} - \frac{1}{1+\beta\delta}c_1 \tag{10}$$

Combine (9) and (10)

$$c_2 = \frac{\beta\delta(1+\delta)}{1+\beta\delta} - \frac{1}{1+\beta\delta}\Big(\beta\delta - \frac{\delta}{(1+\delta)} - \frac{\delta}{(1+\delta)}c_2\Big)$$

Solve the system:

$$c_2^{commit} = \delta \tag{11}$$

$$c_1^{commit} = \delta(\beta - 1) \tag{12}$$

$$c_0^{commit} = 1 - \beta\delta \tag{13}$$

- If one **cannot commit** in period 0, what is the equilibrium values of c_t for each period?

$$\max_{c_2} \quad u_2 = c_2 - \frac{1}{2}(c_0 + c_1 + c_2)^2$$

Solve by backward induction, Consumption c_t becomes a function of previous consumption, and that needs to be taken into account

$$\text{FOC}_{c_2} \quad 1 - (c_0 + c_1 + c_2) = 0$$

Make a function $c_2(c_0, c_1)$

$$c_2(c_0, c_1) = 1 - c_0 - c_1$$

and insert into Utility

$$\max_{c_1} \quad u_1 = c_1 - \frac{1}{2}(c_0 + c_1)^2 + \beta\delta^2 c_2 - \beta\delta^2\frac{1}{2}(c_0 + c_1 + c_2)^2$$

$$\max_{c_1} \quad u_1 = c_1 - \frac{1}{2}(c_0 + c_1)^2 + \beta\delta(1 - c_0 - c_1) - \beta\delta\frac{1}{2}(c_0 + c_1 + 1 - c_0 - c_1)^2$$

Maximize wrt the previous period, c_1

$$\text{FOC}_{c_1} \quad 1 - (c_0 + c_1) - \beta\delta = 0$$

$$1 - (c_0 + c_1) - \beta\delta = 0$$

Make a function $c_1(c_0)$

$$c_1(c_0) = 1 - c_0 - \beta\delta$$

and insert into U

$$\begin{aligned}\max_{c_0} \quad u_0 = c_0 - \frac{1}{2}c_0^2 + \beta\delta(1 - c_0 - \beta\delta) - \beta\delta\frac{1}{2}\big(c_0 + (1 - c_0 - \beta\delta)\big)^2 \\ +\beta\delta^2\big(1 - c_0 - (1 - c_0 - \beta\delta)\big) - \beta\delta^2\frac{1}{2}\end{aligned}$$

Maximize wrt c_0

$$\text{FOC}_{c_0} \quad 1 - c_0 - \beta\delta = 0$$

Solve the system:

$$c_0^{non-c} = 1 - \beta\delta \tag{14}$$

$$c_1^{non-c} = 0 \tag{15}$$

$$c_2^{non-c} = \beta\delta \tag{16}$$

Compare

$$c_1^{commit} < c_1^{non-c}$$

$$\delta(\beta - 1) < 0 \quad \text{since} \quad 0 < \beta < 1$$

you should compare:

c_1^{non-c} to c_1^{commit}
c_2^{non-c} to c_2^{commit}

to get some interesting interpretation. We see that the social planner would prefer a smaller c_1 when he plan and commit ahead.

Seminar 6.

Ex. 1, Supply-side policies Total emissions: $e = E_M(x_M) + E_N(x_N) = E_M\big(x_M\big) + E_N\big(S(p(x_M, y_M)\big)$ (Instead of writing $S_M(p)$ as I did on the board in the seminar, we just keep x_M)

- The utility of the coalition:

$$U_M = B(y_M) - C(x_M) - p(x_M, y_M)(y_M - x_M) - H\Big(E_M\big(x_M\big) + E_N\big(S_N(p(x_M, y_M))\big)\Big)$$

- Derive the coalitions optimal tax on consumption y_M:

$$\frac{\partial U_M}{\partial y_M} = B' - p - \frac{dp}{dy_M}(y_M - x_M) - H'\Big(E_N'(x_N)S_N'(p)\frac{dp}{dy_M}\Big) = 0$$

Rearrange:

$$B' - p = \frac{dp}{dy_M}(y_M - x_M) + H'E_N'(x_N)S_N'(p)\frac{dp}{dy_M} \equiv t^y$$

where $\frac{dp}{dy_M} = \frac{1}{S_N'(p) - D_N'(p)}$, which gives:

$$t^y = \frac{(y_M - x_M)}{S_N'(p) - D_N'(p)} + \frac{S_N'(p)}{S_N'(p) - D_N'(p)}H'E_N'(x_N)$$

- Derive the coalitions optimal tax on production x_M:

$$\frac{\partial U_M}{\partial x_M} = -C' + p - \frac{dp}{dx_M}(y_M - x_M) - H'\Big(E'_M(x_M) + E'_N(x_N)S'_N(p)\frac{dp}{dx_M}\Big) = 0$$

Rearrange:

$$p - C' = \frac{dp}{dx_M}(y_M - x_M) + H'\Big(E'_M(x_M) + E'_N(x_N)S'_N(p)\frac{dp}{dx_M}\Big) \equiv t^x$$

where $\frac{dp}{dx_M} = -\frac{1}{S'_N(p) - D'_N(p)}$, which gives:

$$t^x = -\frac{(y_M - x_M)}{S'_N(p) - D'_N(p)} + H'\Big(E'_M(x_M) - \frac{S'_N(p)}{S'_N(p) - D'_N(p)}E'_N(x_N)\Big)$$

Assumption 1. *Assume the coalition M is net exporter, then$y_M < x_M$*

The we see that t^x should be higher thus higher is the emission content in the coalition's supply, (t^x is increasing in E'_M)

> Key interpretation: If a net exporter is supplying dirty fuels to the market, supply side policies should be stricter than the demand side policies. We want to prevent the dirty fuels from being produced and supplied to the market.

- Consider an emission tax that is proportional to the quantity produced: $t^x = t^e E'_M(x_M)$, then t^e is given by:

$$t^e = -\frac{y_M - x_M}{S'_N(p) - D'_N(p)}\frac{1}{E'_M(x_M)} + H' - \frac{S'_N(p)}{S'_N(p) - D'_N(p)}\frac{E'_N(x_N)}{E'_M(x_M)}H'$$

If the free riders have higher emission content, ($E'_N > E'_M$) then the tax t^e should be smaller, we want to replace the dirty fuels from N with fuels from M, no matter whether M is a net importer or exporter.

Self-enforcing contracts

The compliance constraint:

$$\underbrace{\frac{1}{(1-\delta)}\Big[\ln(b + g^{FB} + r) - cNg^{FB}\Big]}_{\text{Everyone play FB}} \geq \underbrace{\ln(b + g^{BaU} + r) - cg^{BaU} - c(N-1)g^{FB}}_{\text{one country diviates}}$$
$$+ \underbrace{\frac{\delta}{(1-\delta)}\Big[\ln(b + g^{BaU} + r) - cNg^{BaU}\Big]}_{\text{cooperation is destroyed forever after}}$$

Where $B(g^{FB}) = \ln(b + g^{FB} + r)$ and $B(g^{BaU}) = \ln(b + g^{BaU} + r)$, and we then rearrange such that it is easier to interpret the condition:

$$B(g^{FB}) - B(g^{BaU}) \geq c\big(1 + \delta(N-1)\big)(g^{FB} - g^{BaU})$$

Assumption 2. *$\delta \in (0,1)$ and $N > 1$*

We see that this condition holds if δ is high, c is high or N is high.

Consider now the scenario where where countries differ

(4) What is the non-cooperative equilibrium?

$$\max_{g_i} \quad u_i = \ln(b + g_i + r_i) - c_i \sum_{i=1}^{n} g_i$$

$$\frac{du_i}{dg_i} : \frac{1}{b + g_i + r_i} - c_i = 0$$

$$g_i^{\text{BaU}} = 1/c_i - b - r_i$$

(5) What is the first-best equilibrium?

$$\max_{g_i} \quad u_i = \ln(b + g_i + r_i) - \sum_{j=1}^{n} c_j \sum_{i=1}^{n} g_i$$

$$\frac{du_i}{dg_i} : \frac{1}{b + g_i + r_i} - \sum_{j=1}^{n} c_j = 0$$

$$g_i^{\text{FB}} = \frac{1}{\sum_{j=1}^{n} c_i} - b - r_i$$

(6) State the compliance constraint.

Solution: The compliance constraint for the countries: $1, 2, ..., i, j, s, ..., N$ becomes:

$$\frac{1}{(1-\delta)}\Big[\ln(b + g^{FB} + r) - c_i \sum_{s=1}^{N} g_s^{FB}\Big] \geq \ln(b + g^{BaU} + r) - c_i g_i^{BaU} - c_i \sum_{j \in N \setminus i}^{N-1} g_j^{FB}$$
$$+ \frac{\delta}{(1-\delta)}\Big[\ln(b + g^{BaU} + r) - c_i \sum_{s=1}^{N} g_s^{BaU}\Big]$$

separate all g_i^{FB} and g_i^{BaU} from the sums $\sum_{s=1}^{N}$

$$\frac{1}{(1-\delta)}\Big[\ln(b+g^{FB}+r)-c_i g_i^{FB}-c_i\sum_{j\in N\setminus i}^{N-1} g_j^{FB}\Big]\geq \ln(b+g^{BaU}+r)-c_i g_i^{BaU}-c_i\sum_{j\in N\setminus i}^{N-1} g_j^{FB}$$

$$+\frac{\delta}{(1-\delta)}\Big[\ln(b+g^{BaU}+r)-c_i g_i^{BaU}-c_i\sum_{j\in N\setminus i}^{N-1} g_j^{BaU}\Big]$$

rearrange, such that we can combine $\frac{1}{(1-\delta)}c_i\sum_{j\in N\setminus i}^{N-1} g_j^{FB}$ and $c_i\sum_{j\in N\setminus i}^{N-1} g_j^{FB}$:

$$\ln(1-\delta)\Big[\ln(b+g^{FB}+r)-c_i g_i^{FB}\Big]\geq \ln(b+g^{BaU}+r)-c_i g_i^{BaU}$$

$$+\frac{\delta}{(1-\delta)}\Big[\ln(b+g^{BaU}+r)-c_i g_i^{BaU}\Big]+\frac{\delta}{(1-\delta)}\Big[c_i\sum_{j\in N\setminus i}^{N-1} g_j^{FB}-c_i\sum_{j\in N\setminus i}^{N-1} g_j^{BaU}\Big]$$

$$\frac{1}{(1-\delta)}\Big[\ln(b+g^{FB}+r)-c_i g_i^{FB}\Big]\geq \frac{1}{(1-\delta)}\Big[\ln(b+g^{BaU}+r)-c_i g_i^{BaU}\Big]$$

$$+\frac{\delta}{(1-\delta)}\Big[c_i\sum_{j\in N\setminus i}^{N-1} g_j^{FB}-c_i\sum_{j\in N\setminus i}^{N-1} g_j^{BaU}\Big]$$

if $\delta\in(0,1)$

$$\ln(b+g^{FB}+r)-c_i g_i^{FB}-\delta c_i\sum_{j\in N\setminus i}^{N-1} g_j^{FB}\geq \ln(b+g^{BaU}+r)-c_i g_i^{BaU}-\delta c_i\sum_{j\in N\setminus i}^{N-1} g_j^{BaU}$$

$$\ln(b+g^{FB}+r)-\ln(b+g^{BaU}+r)\geq c_i(g_i^{FB}-g_i^{BaU})+\delta c_i(\sum_{j\in N\setminus i}^{N-1} g_j^{FB}-\sum_{j\in N\setminus i}^{N-1} g_j^{BaU})$$

- What if we force the g^{FB} to be equal to zero? Then the compliance constraint becomes:

  Insert $g^{FB}=0$:

  $$\frac{1}{(1-\delta)}\ln(b+r)\geq \ln(b+g^{BaU}+r)-cg^{BaU}+\frac{\delta}{(1-\delta)}\Big[\ln(b+g^{BaU}+r)-cNg^{BaU}\Big]$$

  Insert for $g_i^{\mathrm{BaU}}=1/c-b-r$

  $$\ln(b+r)\geq(1-\delta)\ln(1/c)-(1-\delta)c(1/c-b-r)+\delta\Big[\ln(1/c)-cN(1/c-b-r)\Big]$$

  $$\ln(b+r)\geq\ln(1/c)-(1-\delta)c(1/c-b-r)-\delta cN(1/c-b-r)$$

  $$\ln\big(c(b+r)\big)\geq\Big(c(b+r)-1\Big)\Big((1-\delta)+\delta N\Big)$$

We see that it is important to increase the investment level r when c is low, δ is low and N is low, because then it is hard to sustain cooperation.

- We now endogenize the technology levels by letting the countries simultaneously, non-cooperatively decide on their r_i's at the investment stage, which is prior to the emission stage. (We are still assuming that $g^{FB} = 0$). The CC gives us the level of g:

$$\frac{1}{(1-\delta)} \ln(b+r) \geq \ln(b+g+r) - cg + \frac{\delta}{(1-\delta)} \Big[\ln(b+g+r) - cNg \Big]$$

The level of r is given by:

$$r_i^*(g) = \arg\max_{r_i} \quad \{\ln(b+g_i+r_i) - c\sum_{i=1}^{N} g_i - kr_i\}$$

$$\text{FOC:} \quad \frac{1}{b+r_i^*+g_i} - k = 0 \tag{17}$$

Insert for $g = 1/k - b - r$ from (17) into CC:

$$\frac{1}{(1-\delta)} \ln(b+r) \geq \ln(1/k+r-r^*) - c(1/k-b-r^*) + \frac{\delta}{(1-\delta)} \Big[\ln(1/k-r^*+r) - cN(1/k-b-r^*) \Big]$$

$$\ln\big(k(b+r)\big) \geq -\big(1+\delta(N-1)\big)c(1/k-b-r)$$