Environmental Problems as Repeated Games

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Lecture Note 1

Abstract

This note presents and discusses:

(1) Prisoner's dilemmas and repeated games

(2) While folk theorems point out that the first best can be sustained as a subgame-perfect equilibrium when the players are sufficiently patient, we must study the second best equilibrium when they are not

(3) Role of (green) technology

(4) Role of policies

1 Introduction

> *By lowering the relative cost of more environmentally sound technologies, technology policy can increase incentives for countries to comply with international climate obligations.*
>
> IPCC (2014:1035)

An international environmental treaty must address two major challenges to succeed. First, in the absence of international enforcement bodies, it must be *self-enforcing*. That is, countries will comply with the treaty in order to motivate other countries to do so in the future.[1] This motivation, however, may not always be sufficiently strong. For example, for many years it was clear that Canada would not meet its commitments under the Kyoto Protocol and in 2011, it simply withdrew.

[1] The need for self-enforcement is recognized by the IPCC (2014:1015): "*From a rationalist perspective, compliance will occur if the discounted net benefits from cooperation (including direct climate benefits, co-benefits, reputation, transfers, and other elements) exceed the discounted net benefits of defection.*"

The second challenge is to develop new and environmentally friendly technology. The importance of new green technology is recognized in climate treaties, but traditionally they have not quantified the extent to which countries are required to invest in these technologies.[2] Instead, negotiators focus on quantifying emissions or abatements and leave the investment decision to individual countries. Nevertheless, some countries do invest heavily in green technologies. The European Union has set itself the goal that 20 percent of its energy will come from renewable sources by 2020 and 27 percent by 2030. China is an even larger investor in renewable energy and has invested heavily in wind energy and solar technology.[3] Other countries have instead invested in so-called "brown" technology: Canada, for example, has developed its capacity to extract oil from unconventional sources, such as tar sands, and it "*risks being left behind as green energy takes off*" (*The Globe and Mail*, September 21, 2009).

The interaction between the two challenges is poorly understood by both economists and policy makers. To understand how treaties can address these two challenges and how they are related, a model is needed that allows technology investment decisions and emission decisions to be made repeatedly. Since the treaty must be self-enforcing, strategies must constitute a subgame-perfect equilibrium (SPE).

There is no such theory in the literature and therefore many important questions are left unaddressed. First, what characterizes the "best" SPE, i.e., the best self-enforcing treaty? While folk theorems have emphasized that even the first best can be sustained if the players are sufficiently patient, what distortions occur if they are not? How can technologies be used strategically to ensure that the treaty is self-enforcing? Which types of countries ought to invest the most and in what kinds of technologies?

To address these questions, we present a repeated extensive-form game, in which countries can in each period invest in technology before deciding on emission levels. In the simplest version of the model, all decisions are observable and investments are self-investments, i.e., there are no technological spillovers. Consequently, equilibrium investments would have been first best if the countries had committed to the emission levels. The first best can also be achieved if the discount factor is sufficiently high, in line with standard folk theorems. For smaller discount factors, however, the best SPE requires countries to strategically distort their investment decisions in order to reduce the

[2] Chapter 16 of the Stern Review (2007) identified technology-based schemes as an indispensable strategy for tackling climate change. However, article 114 of the 2010 Cancun Agreement, confirmed in Durban in 2011, states that "*technology needs must be nationally determined, based on national circumstance and priorities.*" In contrast and as discussed in Section 9, some of the pledges following the 2015 Paris Agreement relate to technology.

[3] For more details on the European Union's climate and energy policy strategy, see ec.europa.eu/clima/policies/strategies/2030, and for that of China, see thediplomat.com/2014/11/in-new-plan-china-eyes-2020-energy-cap/.

temptation to pollute more rather than less. We show that the distortions take the form of overinvestment in the case of "green" technologies, i.e., renewable energy or abatement technologies that can substitute for pollution. In the case of "brown" technologies, such as drilling technologies and other infrastructure investments that are strategic complements to fossil fuel consumption, investments must instead be less than the first-best amount in order to satisfy the compliance constraint. Our most controversial result states that countries should also be required to invest less than the first-best amount in the case of adaptation technologies, i.e., technology that reduces environmental harm in a country.

The comparative statics offer important policy implications. Of course, it is harder to motivate compliance if the discount factor is low or the environmental harm is on a small scale. This is also true when a small number of countries participate in the agreement, or when investment costs are high in the case of green technology or low in the case of brown or adaptation technologies. In these circumstances, the best SPE requires countries to invest more when the technology is green, and less when it is brown or when it is adaptation technology. If countries are heterogeneous, the countries that are most reluctant to cooperate because, for example, they face less environmental harm, are the most tempted to free ride. Thus, for compliance to be credible, such countries must invest the most in green technologies or the least in adaptation and brown technologies. This advice contrasts with the typical presumption that reluctant countries should be allowed to contribute less in order to satisfy their participation constraint. While incentives to *participate* require that a country's net gain from cooperating be positive, incentives to *comply* with emissions also require that this net gain outweigh the positive benefit of free riding for one period, before the defection is observed. The compliance constraint at the emission stage is therefore harder to satisfy than the participation constraint is.

Simplicity and tractability are two advantages of our baseline model. Our main results are derived in a pedagogical way with binary emission levels, while ignoring investment in technology portfolios, technological spillovers, imperfect monitoring, and policy instruments such as emission taxes or investment subsidies. However, when the model is extended to take into account these complicating factors, we obtain a deeper understanding of the interplay between agreements and technology. We show that our insight extends to the situation in which a country can invest in a portfolio of different types of technologies. Technological spillovers make it harder to design self-enforcing treaties if countries are similar; however, spillovers are necessary to facilitate technology transfers if countries are heterogeneous. When emissions are difficult to monitor, strategic investments in technologies can reduce the punishment or the risk that punishments are triggered by mistake, while still ensuring that countries are motivated to comply. The results hold with continuous emission levels and if national governments regulate firms' emissions and technology investments through taxes and subsidies. In this case, we show that optimal environmental regulation includes both emission taxes and investment subsidies if and only if the discount factor is small. Since these extensions are motivated by challenges faced by climate change agreements, the results are highly policy relevant.

The note is organized as follows. The baseline model is presented in Section 2 and analyzed in Section 3. To shed further light on optimal climate change policy, private sector decision making in investment and continuous emissions levels (Section 4). We then allow for technological spillovers and transfers (Section 5), imperfect monitoring (Section 6), and finally we show how the model can be reformulated to account for the accumulation of pollution and technology (Section 7). Section 8 discusses further readings. The Appendix contains all proofs.

2 A Model of Compliance Technology

The model we construct is motivated by global environmental problems such as climate change. Since no world government can force countries to cooperate in solving such problems, the temptation to free ride must be mitigated. The possibility of free riding is a result of the fact that if a country increases its emissions, other countries will not retaliate immediately because, for example, emissions are observed with a lag. To capture this lag, we let time $t \in \{1, ..., \infty\}$ be discrete and $\delta \in (0, 1)$ be the common discount factor between periods.

Analogously, there is also a lag between the decision to invest in a technology and the point at which it begins to contribute to consumption. This lag leads us to use an extensive-form stage game, in which each country invests in technology before deciding on how much to consume or pollute. Furthermore, the infinite time horizon relevant for climate change implies that it is unrealistic to assume that a country can invest in the capacity to produce renewable energy once and for all, without later having to invest in maintenance. To capture this effect, we start out by assuming that technology fully depreciates, so that countries must invest in every single period. We also at first abstract from technological spillovers since, in contrast to environmental externalities, technological spillovers may be relatively small when the technology is a country's capacity to produce renewable energy.

There are $n \geq 2$ players in the game, indexed by i or $j \in N \equiv \{1, ..., n\}$. In each stage game, there is an emission stage in which countries simultaneously make a binary decision $g_i \in \{\underline{g}, \overline{g}\}$ between emitting less, i.e., $g_i = \underline{g}$, or more, i.e., $g_i = \overline{g} > \underline{g}$. Whenever it is not confusing, we omit the subscripts denoting time.

Let the benefit $b_i(g_i, r_i)$ be an increasing function of country i's emissions g_i. The environmental cost from global emissions is $h_i c(r_i) \sum_{j \in N} g_j$, where parameter h_i measures country-specific environmental harm. The assumption that this cost is linear in emissions is simplifying, common, and relatively reasonable.[4] The variable $r_i \in \mathbb{R}_+$ is meant to capture the fact that a country's benefit and its environmental cost depend on the country's technology, although r_i can in fact be any variable that influences the benefit and cost of emissions. For simplicity, we assume that $b_i(g_i, r_i)$ is increasing and concave in r_i and $c(r_i)$ is decreasing and convex in r_i. We also assume that the game at the emission stage is a prisoner's dilemma, irrespective of the level of r_i, as follows:

Assumption 1 *For each $i \in N$ and $r_i \in \mathbb{R}_+$,*

(i) $b_i(\underline{g}, r_i) - h_i c(r_i)\, \underline{g} < b_i(\overline{g}, r_i) - h_i c(r_i)\, \overline{g}$;

(ii) $b_i(\underline{g}, r_i) - h_i c(r_i)\, n\underline{g} > b_i(\overline{g}, r_i) - h_i c(r_i)\, n\overline{g}$.

In words, country i benefits from emitting more for any fixed emission from other countries, but every country would be better off if everyone emitted less. Hereafter, and unless otherwise specified, we use subscripts to denote derivatives. Moreover, we abuse the notation by defining $b''_{i,gr}(r_i) \equiv (b'_{i,r}(\overline{g}, r_i) - b'_{i,r}(\underline{g}, r_i))/(\overline{g} - \underline{g})$, which captures how the benefit of emitting more rather than less varies with the level of technology.

To illustrate the relevance of technology, we will occasionally refer to the following special types:

Definition 1 *For each $r_i \in \mathbb{R}_+$,*

(A) *Adaptation technology is characterized by* $b''_{i,gr}(r_i) = 0$ *and* $c'_r(r_i) < 0$;

(B) *Brown technology is characterized by* $b''_{i,gr}(r_i) > 0$ *and* $c'_r(r_i) = 0$;

(C) *Clean technology is characterized by* $b''_{i,gr}(r_i) < 0$ *and* $c'_r(r_i) = 0$.

An adaptation technology is one that enables a country to adapt to a warmer or more volatile climate. Such technologies include agricultural reforms or more robust infrastructure and may in addition capture the effects of some geo-engineering practices that have strictly local effects. Adaptation technology is therefore complementary to polluting, since it reduces the environmental cost of emissions, i.e., $c'_r(r_i) < 0$. Brown technology can be interpreted as drilling technology, infrastructure that is beneficial in the

[4] As explained by Golosov et al. (2014:78): "*Linearity is arguably not too extreme a simplification, since the composition of a concave S-to-temperature mapping with a convex temperature-to-damage function may be close to linear.*" They also write (p. 67): "*The composition implied by Nordhaus's formulation is first concave, then convex; our function is approximately linear over this range. Overall, the two curves are quite close.*"

extraction or consumption of fossil fuel, or some other technology that is complementary to fossil fuel consumption. The complementarity is captured by $b''_{i,gr}(r_i) > 0$. In fact, most investments made in polluting industries are brown, according to our definition. Clean technology, in contrast, is a strategic substitute for fossil fuel and reduces the marginal value of emitting another unit of pollution. This is the case for abatement technology or renewable energy sources, for example. Thus, $b''_{i,gr}(r_i) < 0$ for clean technologies. Of course, both brown and clean technologies may be beneficial in that $b'_{i,r}(g_i, r_i) > 0$.

We endogenize the technology level by permitting an investment stage, in each period, during which countries simultaneously and non-cooperatively decide on investment, before they decide on whether to emit less or more. As already noted, the sequential timing follows directly from the fact that there is a minimum length of time $l \in (0,1)$ between the investment decision and the time at which the technology is operational. The lag implies that if the actual marginal investment cost is, say, $\widehat{k}_i > 0$, then its present discounted value, evaluated at the time of the emission, is $k_i \equiv \delta^l \widehat{k}_i$. With this reformulation, we do not need to explicitly discount between the two stages within the same period. Note that assuming a linear investment cost is without loss of generality, since r_i can enter a country's benefit function in arbitrary ways.[5] Country i's per-period utility can then be written as:

$$u_i = b_i(g_i, r_i) - h_i c(r_i) \sum_{j \in N} g_j - k_i r_i.$$

Benchmarks. Before analyzing self-enforcing agreements, we examine two polar cases in which emissions and investments are chosen at every decision stage either non-cooperatively by each individual country or by a planner with full enforcement power.

Consider first non-cooperative investments. Suppose that each country is expected to pollute at the same level, that is, $g_i = g$ for each i. For every g, country i's optimal investment level $r_i(g)$ is obtained by solving the following first-order condition:

$$b'_{i,r}(g, r_i) - h_i c'_r(r_i) ng - k_i = 0, \tag{1}$$

while the second-order condition holds trivially.

At the emission stage, Assumption 1 implies that $g_i = \overline{g}$ is a dominant strategy for every country. Thus, there is a unique subgame perfect equilibrium (SPE) of the stage game, that is, $(g_i, r_i) = (\overline{g}, r_i(\overline{g}))$. Using terminology from the literature on environmental agreements, we refer to this equilibrium as the business-as-usual (BAU) equilibrium and label it with the superscript *bau*. Note that BAU also coincides with the worst SPE, that

[5] If the investment cost were a different function $\kappa_i(r_i)$, we could simply define $\widetilde{b}_i(g_i, \kappa_i(r_i)) \equiv b_i(g_i, r_i)$ and $\widetilde{c}(\kappa_i(r_i)) \equiv c(r_i)$, treat $\kappa_i(r_i)$ as the decision variable, and then proceed as we do in the paper.

is, the min-max payoff of the stage game, since every country is always guaranteed at least that utility level, i.e., $u_i^{bau} \equiv b_i\left(\overline{g}, r_i^{bau}\right) - h_i c\left(r_i^{bau}\right) n\overline{g} - k_i r_i^{bau}$ with $r_i^{bau} \equiv r_i\left(\overline{g}\right)$.

The first-best outcome is characterized by $(g_i, r_i) = (\underline{g}, r_i(\underline{g}))$ for each i and coincides with the case in which a benevolent planner makes all its decisions in order to maximize the sum of countries' utilities. It follows that the first-best level of utility is $u_i^* \equiv b_i\left(\underline{g}, r_i^*\right) - h_i c\left(r_i^*\right) n\underline{g} - k_i r_i^* > u_i^{bau}$ with $r_i^* \equiv r_i\left(\underline{g}\right)$. Since the first-best investment level also follows from condition (1), we can state the following preliminary result:

Proposition 0 *If all countries commit to the emission level $g_i = \underline{g}$, every non-cooperative investment is first best, i.e., r_i^*.*

Proposition 0 provides support for the presumption that it is not necessary to negotiate investments in addition to negotiating emissions. Under a commitment to $g_i = \underline{g}$, each country's investment would be socially optimal and the first best would be sustainable as an SPE. In what follows, we consider the more realistic scenario in which countries cannot commit to low emission levels.

3 Self-enforcing Agreements

When actions are observable, an international environmental agreement can specify every country's levels of emission and investment at every point in time. For such an agreement to be self-enforcing, the decisions must constitute an SPE. As in many dynamic games with an infinite time horizon, there are multiple SPEs. When countries can communicate and negotiate at the outset, it may be reasonable to assume that they will coordinate on a Pareto-optimal SPE. Since the game is a prisoner's dilemma at the emission stage, we are especially interested in SPEs in which n countries emit less on the equilibrium path, i.e., in which $g_{i,t} = \underline{g}$ for each $i \in N$ and any $t \geq 1$.

Note that we do not require that all countries "in the world" emit less. Rather, we can let N refer to the set of countries emitting less under the agreement. If there exist other countries that always emit more, they will be irrelevant to the game and the equilibrium subsequently analyzed, since the emissions of these other countries are not payoff relevant when the environmental harm is linear in the sum of emissions. When there is a unique Pareto-optimal SPE outcome among the n countries emitting less, we refer to an equilibrium that supports it as a best equilibrium.

Definition 2 *An equilibrium is referred to as "best" if and only if it supports the unique Pareto-optimal SPE outcome involving $g_{i,t} = \underline{g}$ $\forall i \in N$ and $t \geq 1$ on the equilibrium path.*

The best equilibrium must also specify the consequences if a country fails to emit less. Since this never occurs on the equilibrium path, there is no loss in assuming that the countries would respond by playing the worst SPE, i.e., BAU, forever. The observation that punishments are never observed in equilibrium also implies that, in a setting with a common discount factor, the best equilibrium outcome must be stationary, i.e., it supports $r_{i,t} = r_i$ for every $t \geq 1$ (Abreu, 1988). Therefore, we can omit the t subscripts for brevity. The normalized (to one period) continuation value when complying with the best SPE is $u_i(r_i) \equiv b_i\left(\underline{g}, r_i\right) - h_i c(r_i)\, n\underline{g} - k_i r_i$.

Deviations can occur during either the investment stage or the emission stage. At the investment stage, a country will compare the continuation value it receives from complying with the SPE by investing in the r_i with the maximal continuation value it can obtain by deviating. Since deviating at the investment stage implies that every country will emit more starting from that period, the compliance constraint at the investment stage is as follows:

$$\frac{u_i(r_i)}{1-\delta} \geq \max_{r_i} b_i(\overline{g}, r_i) - h_i c(r_i)\, n\overline{g} - k_i r_i + \frac{\delta u_i^{bau}}{1-\delta}. \tag{CC$_i^r$}$$

The right-hand side of constraint (CC$_i^r$) is maximized when $r_i = r_i^{bau}$, implying that the compliance constraint at the investment stage simplifies to $u_i(r_i) \geq u_i^{bau}$, which actually coincides with the *participation* constraint. If a country deviates at the investment stage, the penalty is imposed before the country can benefit from free riding on emissions. Thus, the temptation to free ride at the investment stage is weak since a country does not care about other countries' investment levels per se, but only about its own emission levels.

At the emission stage, the investment cost in the current period is sunk and the compliance constraint becomes:

$$\frac{u_i(r_i)}{1-\delta} \geq b_i(\overline{g}, r_i) - h_i c(r_i)\left(\overline{g} + (n-1)\,\underline{g}\right) - k_i r_i + \frac{\delta u_i^{bau}}{1-\delta}. \tag{CC$_i^g$}$$

As δ tends to one, (CC$_i^g$) approaches (CC$_i^r$). For any $\delta < 1$, however, (CC$_i^g$) is harder to satisfy than (CC$_i^r$) because of the free-riding incentive at the emission stage. It is not sufficient that the best equilibrium be better than BAU. In addition, the discount factor must be large or the temptation to free ride on emissions must be small. For notational convenience, we rewrite constraint (CC$_i^g$) as follows:

$$\Delta_i(r_i, \delta) \equiv u_i(r_i) - u_i^{bau} - \frac{1-\delta}{\delta}(\overline{g} - \underline{g})\psi_i(r_i) \geq 0, \text{ where}$$
$$\psi_i(r_i) \equiv \frac{b_i(\overline{g}, r_i) - b_i(\underline{g}, r_i)}{\overline{g} - \underline{g}} - h_i c(r_i)$$

relates to the one-period benefit from free riding on emissions, which is positive according to Assumption 1. For every i, the equation $\Delta_i(r_i,\delta)=0$ identifies a threshold discount factor $\delta_i(r_i)$ that depends on the level r_i. Let $\overline{\delta}_i$ be defined as the level of δ that solves $\Delta_i(r_i^*,\delta)=0$. It follows that, if $\delta\geq\max_i\overline{\delta}_i$, every ($CC_i^g$) holds (even) for $r_i=r_i^*$ and the best equilibrium is simply the first best. There is also a lower bound on the discount factor, denoted by $\underline{\delta}_i$, such that if $\delta<\underline{\delta}_i$, there is no r_i that satisfies both (CC_i^g) and (CC_i^r). In this case, there does not exist any r_i such that country i will emit less. When $\delta\in[\underline{\delta},\overline{\delta}_i)$, with $\underline{\delta}\equiv\max_i\underline{\delta}_i$, country i is willing to participate in the climate agreement, but compliance with less emissions is not satisfied if $r_i=r_i^*$. To ensure that the compliance constraint at the emission stage is satisfied, the temptation to free ride must be reduced by ensuring that r_i is such that $\delta_i(r_i)\leq\delta$. This requires that $r_i>r_i^*$ if $\delta'_{i,r}(r_i^*)<0$, and $r_i<r_i^*$ if $\delta'_{i,r}(r_i^*)>0$. It is straightforward to verify that:

$$\delta'_{i,r}(r_i^*)<0 \quad\text{if } b''_{i,gr}(r_i^*)<h_ic'_r(r_i^*)\,; \tag{G_i}$$

$$\delta'_{i,r}(r_i^*)>0 \quad\text{if } b''_{i,gr}(r_i^*)>h_ic'_r(r_i^*)\,. \tag{NG_i}$$

Condition (G_i) stands for "green" technology and implies that making more investments relaxes the compliance constraint at the emission stage by reducing the threshold $\delta_i(r_i)$. Clearly, this condition is satisfied in, for example, the case of clean technology as defined in Definition 1, since additional investment reduces the gain from emitting more rather than less. Condition (NG_i) stands for "non-green" technologies and implies that making less investments relaxes the compliance constraint. Adaptation and brown technologies are special cases in which this condition holds. For these types of technologies, the benefit of emitting more is reduced if there is less investment in technology. When the benefit of emitting more is reduced, the compliance constraint (CC_i^g) is relaxed and is satisfied for a larger set of discount factors. Since the results will depend on these two conditions, we henceforth will relate to green and non-green technologies, while occasionally discussing the relevant implications of the results for the specific types of technologies described in Definition 1.

Let $r_i(\delta)$ be the level of r_i that maximizes $u_i(r_i)$ subject to $\Delta_i(r_i,\delta)\geq 0$. The following proposition specifies the conditions under which the best equilibrium exists and characterizes the optimal distortion of the investment in technology from the first-best level.

Proposition 1 *There exists a best equilibrium if and only if $\delta\geq\underline{\delta}$. For each $i\in N$, it supports $r_i=r_i^*$ when $\delta\geq\overline{\delta}_i$. Otherwise,*

(i) $r_i=r_i(\delta)>r_i^*$ if technology is green;

(ii) $r_i = r_i(\delta) < r_i^*$ *if technology is non-green. Furthermore,* $|r_i(\delta) - r_i^*|$ *is decreasing in* δ*.*

The result that the first best is achievable when the discount factor is sufficiently large is standard in the literature on repeated games.[6] Thus, the contribution of Proposition 1 is to characterize the distortions that must occur if the discount factor is small. When the discount factor is so small that the first best cannot be achieved, countries are motivated to comply with an agreement and emit less only if they have previously invested more if technology is green or less if technology is non-green. Investment levels are required to increasingly differ from the first-best level when δ declines from the level $\overline{\delta}_i$ in order to reduce the temptation to deviate from the equilibrium.[7]

For the special types of technologies described in Definition 1, the following result holds:

Corollary 1 *In the best equilibrium and relative to the first best, countries will:*

(A) Underinvest in the case of adaptation technology;

(B) Underinvest in the case of brown technology;

(C) Overinvest in the case of clean technology.

3.1 Comparative Statics

The compliance constraints are not functions of only technology, but also depend on other parameters of the model. In this section, we consider the effect on investments in each type of technology of a change in these parameters. Compliance is particularly difficult to motivate if the cost of reverting to BAU is small, which holds true when there are few countries, i.e., when n is small, or when the environmental harm is small, i.e., when h_i is small. To satisfy the compliance constraint in these situations, it is necessary that country i invest more in clean technology, and less in brown technology or adaptation technology. The comparative statics are summarized in the following proposition:

Proposition 2 *Suppose* $\delta \in \left[\underline{\delta}, \overline{\delta}_i\right)$ *and consider the best equilibrium:*

[6] Rubinstein and Wolinsky (1995) show that Fudenberg and Maskin (1986)'s folk theorem can be generalized to repeated extensive-form games in order to account for subgame perfection within periods.

[7] Note that it is not necessary to require that investment be sufficiently small or sufficiently large that emitting less becomes a dominant strategy; it is sufficient to ensure that the benefit of emitting more be smaller (though still positive) than the present discounted value of continuing cooperation. Requiring countries to invest at a level that is inefficient, conditional on the emission levels, must be part of the self-enforcing agreement, in the same way that low emission levels are, namely any deviation leads to BAU forever.

(*i*) *If* h_i *or* n *increases,* r_i *decreases in the case of clean technology, increases in the case of brown technology, and, provided that* $c(r_i) > (c'_r(r_i))^2 / c''_{rr}(r_i)$*, increases in the case of adaptation technology;*[8]

(*ii*) *If* k_i *increases,* r_i *increases regardless of the type of technology.*

A surprising result is that investment in any type of technology will increase with the cost of investment k_i. To see this, recall that $r_i < r_i^{bau}$ for adaptation and brown technologies. For those technologies, a larger k_i reduces the value of BAU compared to the value of cooperating, i.e., $u_i(r_i) - u_i^{bau}$ increases, and makes the compliance constraint easier to satisfy at the emission stage. Thus, when k_i increases, r_i can increase toward r_i^* without violating (CC_i^g). For clean technology, we have $r_i > r_i^{bau}$, so that a larger k_i reduces the value of cooperating relative to the value of BAU. In that case, the compliance constraint becomes harder to satisfy when k_i increases and country i must invest even more to satisfy (CC_i^g).

Since countries are heterogeneous, the comparative statics are country specific. We can therefore differentiate between countries that are the most reluctant to cooperate from those that are the least. If country i has a lower level of environmental harm than country j, or has a higher investment cost in the case of clean technology or has a smaller investment cost in the case of brown or adaptation technology, then $\overline{\delta}_i > \overline{\delta}_j$, and we can say that i is more reluctant than j. Since the most reluctant countries are tempted to emit more, it is more likely that their compliance constraints bind, i.e., $\delta < \overline{\delta}_i$, and that they must invest strategically to make compliance credible.

The result that countries which benefit less from cooperation ought to make greater sacrifices is in stark contrast to the idea that countries should contribute according to their ability and their responsibility for pollution and that they must be given a better deal to motivate cooperation. It is true, of course, that a reluctant country has a participation constraint, i.e., $u_i(r) \geq u_i^{bau}$, which is more difficult to satisfy than are the constraints for other countries. However, as already shown, the compliance constraint (CC_i^g) is more difficult to satisfy than the participation constraint (CC_i^r). Although each country's benefit from cooperating, relative to BAU, must certainly be positive, it must also be larger than the benefit from free riding for one period, before the deviation is detected.

[8] If this condition is violated, investing in adaptation technology is so productive that if n or h_i increases, country i's environmental cost $h_i c(r_i) ng$ actually declines when the changes induce the country to invest more in adaptation technology, which seems unrealistic.

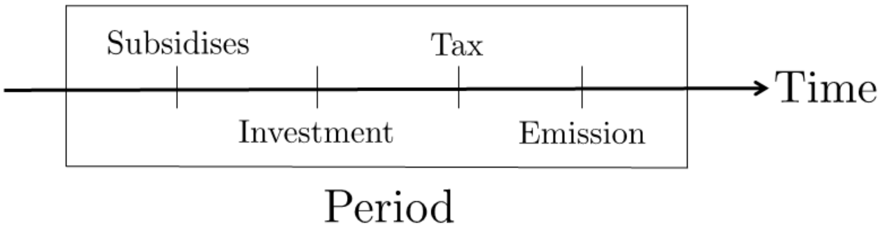

Figure 1: *The Timing*

4 Policy Instruments and Continuous Emission Levels

In this section, we study the optimal use of policy instruments, and we permit the emission level to be a continuous variable. It is natural to make these two extensions at the same time, since we cannot pin down a unique emission tax if the emission level continues to be a binary variable. (For example, any sufficiently large emission tax ensures that $\underline{g}$ is preferred to $\overline{g} > \underline{g}$.)

We assume that country i's investment subsidy, ς_i, is set by i just before the investment stage in each period, and it is observable by all countries. The actual investment is made by private investors who receive the subsidy ς_i in addition to the price paid by the consumers. The emission tax, τ_i, is set just before the emission stage, and it represents the cost of polluting paid by the consumers. If the taxes are collected and the subsidies are paid by the national governments, they do not represent actual costs or revenues—from the government's perspective—and their only effect is to influence the decisions g_i and r_i. The agreement between the countries then amounts to setting domestic taxes/subsidies such that the desired SPE is implemented.

Allowing for a continuous g_i complicates the analysis. To proceed, we restrict attention to the case in which g_i and r_i are perfect substitutes in a linear-quadratic utility function:[9]

$$u_i = -\frac{B}{2}\left(\overline{y} - (g_i + r_i)\right)^2 - \frac{K}{2} r_i^2 - c \sum_{j \in N} g_i,$$

where B and K are positive constants. Here, $\overline{y}$ is a country's bliss level for consumption, and consumption is the sum of g_i (energy from fossil fuels) and r_i (energy from renewable energy sources). Since $\partial^2 u_i / \partial g_i \partial r_i < 0$, we explicitly consider only clean technology. We can easily reformulate the utility function such that the investment cost becomes linear,[10]

[9] This utility function is also considered in Battaglini and Harstad (2014), who do not study SPEs, but instead the Markov-perfect equilibria when countries can commit to the emission levels. The first best and the BAU equilibrium are as in that paper, of course.

[10] To see this, simply define $\widetilde{r}_i = r_i^2/2$ and rewrite to $u_i = -\frac{B}{2}\left(\overline{y} - \left(g_i + \sqrt{2\widetilde{r}_i}\right)\right)^2 - K\widetilde{r}_i - c \sum_{j \in N} g_i$.

although there is no need to do so here.

Since the emission tax is the only cost of consuming fossil fuel, g_i is chosen by the consumers to satisfy the first-order condition:

$$B\left(\overline{y}-(g_i+r_i)\right)=\tau_i.$$

The left-hand side is also equal to the consumer's willingness to pay for green technology, so private investors invest according to the first-order condition:

$$Kr_i=B\left(\overline{y}-(g_i+r_i)\right)+\varsigma_i=\tau_i+\varsigma_i. \tag{2}$$

Note that the first-best outcome is

$$r^*=\frac{cn}{K}\text{ and }g^*=\overline{y}-\frac{cn}{B}-r^*,$$

which coincides with the equilibrium when the tax and the subsidy are equal to their first-best values:

$$\varsigma^*=0\text{ and }\tau^*=cn.$$

In the first best, the emission tax is set at the Pigouvian level and there is no need to additionally regulate investments, since the investors capture the entire surplus associated with their technology investments.

The BAU equilibrium is (the unique SPE in the one-period game):

$$r^b=\frac{c}{K}\text{ and }g^b=\overline{y}-\frac{c}{B}-r^b,$$

which is equivalent to

$$\varsigma^b=0\text{ and }\tau^b=c.$$

Thus, the investment subsidy is zero in the first best as well as in BAU.

To follow the same line of reasoning as in the rest of the note, we here only consider SPEs enforced by the threat of reverting to BAU, despite the fact that BAU is not the harshest penalty when $g<g^b$ is possible. Furthermore, we consider only symmetric SPEs, despite the fact that there can also be asymmetric SPEs that are Pareto optimal.

Naturally, the first best can be achieved when the discount factor is sufficiently large. When δ falls, however, each country finds it tempting to introduce a smaller emission tax than the first-best one. Once δ falls to some threshold, $\overline{\delta}$, the emission-stage compliance constraint starts to bind. For smaller discount factors, the emission tax must be allowed to fall to satisfy the compliance constraint. The associated increase in emissions can be mitigated by introducing an investment subsidy.

Note that the investment-stage compliance constraints will never bind first. As soon as one country deviates by setting a different investment subsidy, investors in all countries anticipate that cooperation will break down and demand for their technology at the emission stage will be reduced. This lowers investments everywhere, not only in the deviating country. Deviating at the investment stage immediately gives the deviator the BAU payoff, plus the benefit of the other countries' larger investments induced by their subsidies. These subsidies are zero for $\delta \geq \overline{\delta}$ and are small for discount factors close to $\overline{\delta}$. Consequently, some $\underline{\delta} < \overline{\delta}$ exists such that the compliance constraint at the investment stage is not binding when $\delta \in (\underline{\delta}, 1)$. (The proof in the Appendix derives both thresholds.)

Proposition 3 *Consider the symmetric Pareto optimal SPE sustained by the threat of reverting to BAU if a country deviates. (i) If $\delta \geq \overline{\delta}$, the equilibrium is first best: $\tau = cn$ and $\varsigma = 0$.(ii) If $\delta \in \left[\underline{\delta}, \overline{\delta}\right)$, the equilibrium is:*

$$\begin{aligned}
\tau &= cn - \phi(\delta) \ \ \textit{and} \\
\varsigma &= \phi(\delta), \ \textit{where} \\
\phi(\delta) &\equiv c(n-1)\left(1 - \delta - \sqrt{\delta^2 + \delta B/K}\right) \geq 0.
\end{aligned}$$

The function $\phi(\delta)$ decreases toward zero when δ increases to $\overline{\delta}$.[11]

Corollary 2 *The sum of the equilibrium emission tax and the investment subsidy is, for every $\delta \geq \underline{\delta}$, equal to nc, the first-best Pigouvian tax level.*

5 Technological Spillovers

Cooperation on environmental policies may be plagued by free-riding problems arising from two types of externalities. The first is the environmental harm emphasized in the baseline model, while the second is technological spillovers, especially when the protection

[11] The proposition implies that the equilibrium investment level, r_i, given by (2), stays unchanged as the discount factor falls. On the one hand, the fact that a larger g must be tolerated implies that it becomes optimal to invest less in clean technology. On the other hand, the countries can dampen the increase in g by requesting countries to invest more in green technology upfront. These two effects cancel each other out when g and r are perfect substitutes. Relative to the ex post optimal level, however, it is clear that $r - r^*(g)$ is positive and increases as δ falls, just as the equilibrium investment subsidy. The optimal investment level, conditional on the emission level g_i, is decreasing in g_i and given by:

$$r^*(g_i) = \frac{B(\overline{y} - (g_i + r_i))}{K} = \frac{B(\overline{y} - g_i)}{B + K}.$$

of intellectual property rights (IPRs) is relatively weak. Thus, one country's investment in technology and R&D benefits other countries through technological trade, diffusion, and learning by doing. The weaker the protection of IPRs, the more other countries can benefit without having to pay, and the smaller will be the fraction of the total value enjoyed by the investing country. It turns out that these spillovers alter the strategic role of technology, and that this role is different if countries are homogenous than if they are not.

Let $e \in (0,1)$ be the fraction of a country's investment that benefits the others instead of the investor. A country's per-period utility can then be written as:

$$u_i = b_i(g_i, z_i) - h_i c(z_i) \sum_{j \in N} g_j - k_i r_i, \text{ where } z_i \equiv (1-e) r_i + \frac{e}{n-1} \sum_{j \neq i} r_j. \quad (3)$$

The term $(1-e)$ is a normalization and can be removed without affecting the results.[12] The term is natural, however, when a reduction in e should be interpreted as stronger protection of IPRs, since in that case neighboring countries must pay the innovating country when using the new technology. In this context, the first-best investment level r_i^* remains unchanged as e varies, but the BAU investment level is lower when e is small, since the innovating country is then capturing more of the total gain. Thus, it is no longer true that countries invest the efficient amount conditional on emissions. Moreover, if the spillovers are sufficiently large, it may be that $r_i^* > r_i^{bau}$ regardless of the type of technology.

Instead of letting each country decide on the expenditure r_i, we find it to be more realistic (and tractable) to assume that each country decides on its technology-level target, z_i. Solving for the r_i's in (3), we get $r_i \equiv \frac{1}{n(1-e)-1}[(n-1-e) z_i - e \sum_{j \neq i} z_j]$, illustrating that j's technology reduces i's cost of achieving its target, z_i, thanks to the technological spillovers.

Unlike in the baseline model, BAU is no longer the worst SPE, since a country could, in principle, invest less than r_i^{bau} as a punishment after defection.[13] To facilitate comparison of the results to those in Section 3, we continue to focus on the Pareto-optimal SPEs that are enforced by trigger strategies in which defection leads to BAU forever.

[12] If we had $\widehat{z}_i = r_i + \frac{\widehat{e}}{n-1} \sum_{j \neq i} r_j$ instead of z_i, we could define e from $e/(1-e) \equiv \widehat{e}$ and $z_i \equiv \widehat{z}_i (1-e)$ in order to write $b_i(g_i, z_i) - h_i c(z_i) \sum_{j \in N} g_j - k r_i$.

[13] Note that it is only when $e > 0$ that a reduced r_i can be used to punish other countries.

5.1 Homogenous Countries and Intellectual Property

IPRs protection may encourage firms to innovate more than they otherwise would.

IPCC (2014:1036)

We start out with a situation in which countries are identical. Furthermore, we restrict our attention to symmetric SPEs in which every investment level is the same, so that a country's equilibrium utility can be written as $u(z) \equiv b(\underline{g}, z) - hc(z)\, n\underline{g} - kz$. The best equilibrium supports $(\underline{g}, z_i = z)_{i \in N}$, where z maximizes $u(z)$ subject to the compliance constraints. The compliance constraint at the emission stage is similar to the one in the baseline model, that is,

$$\Delta^g(z, \delta) \equiv u(z) - u^{bau} - \frac{1-\delta}{\delta}(\overline{g} - \underline{g})\psi(z) \geq 0, \tag{CC$_e^g$}$$

where $u^{bau} \equiv b\left(\overline{g}, z^{bau}\right) - hc\left(z^{bau}\right) n\overline{g} - kz^{bau}$ and $\psi(z) \equiv ((b(\overline{g}, z) - b(\underline{g}, z))/(\overline{g} - \underline{g})) - hc(z)$. The compliance constraint at the investment stage is:

$$\Delta^z(z, \delta) \equiv u(z) - u^{bau} - (1-\delta)\frac{e(n-1)}{n(1-e)-1}k\left(z - z^{bau}\right) \geq 0. \tag{CC$_e^z$}$$

Condition (CC_e^z) is trivially satisfied if $e = 0$ or if $z \leq z^{bau}$. When $e > 0$ and $z > z^{bau}$, a country that deviates at the investment stage will not only enjoy its BAU continuation value, but will also benefit from the investments made by the other countries. In that case, countries may be tempted to deviate even at the investment stage. Thus, it is no longer true that it is always harder to motivate less emissions than to motivate investment.

To show this formally, let $\delta^g(z)$ and $\delta^z(z)$ identify the thresholds of discount factors associated with the binding constraints (CC_e^g) and (CC_e^z). The upper bounds $\overline{\delta}^g$ and $\overline{\delta}^z$ are defined as the levels of δ that solve $\Delta^g(z^*, \delta) = 0$ and $\Delta^z(z^*, \delta) = 0$ at the first-best level z^*. Thus, if $\delta \geq \max\left\{\overline{\delta}^g, \overline{\delta}^z\right\}$, both compliance constraints hold for $z = z^*$ and the best equilibrium is simply the first best. When $\delta < \max\left\{\overline{\delta}^g, \overline{\delta}^z\right\}$, investment must be distorted away from its first-best level to ensure compliance with the agreement. Based on a comparison between (CC_e^g) and (CC_e^z), it is apparent that when e is sufficiently large, the compliance constraint at the investment stage is harder to satisfy than the compliance constraint at the emission stage. As we will show in the proof of the following proposition, there exists a threshold level $\widetilde{e} > 0$ such that $\overline{\delta}^z \leq \overline{\delta}^g$ for $e \leq \widetilde{e}$ and $\overline{\delta}^z > \overline{\delta}^g$ otherwise.

If spillovers are small, i.e., $e \leq \widetilde{e}$, because of, for example, the presence of strong protection of IPRs, constraint (CC_e^g) binds first as δ becomes smaller and investment

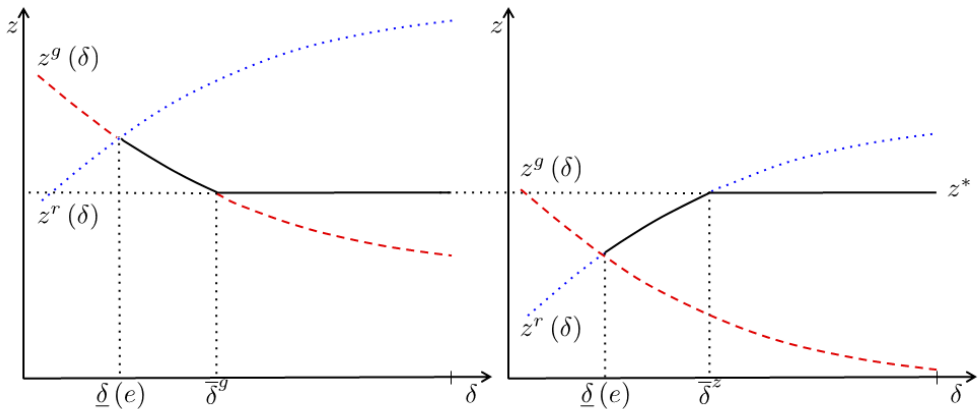

Figure 2: *With small spillovers (left panel), the emission stage compliance constraint (dashed line) will bind first and overinvestment may be necessary. With large spillovers (right panel), the investment stage compliance constraint (dotted line) becomes more difficult to satisfy and underinvestment may be necessary.*

distortions will be as described in Proposition 1: there will be overinvestment if technology is green and underinvestment if it is non-green. Formally, let $z^g(\delta)$ be defined as the z that maximizes $u(z)$ subject to $\Delta^g(z,\delta) \geq 0$. Analogously to the baseline model, the function $z^g(\delta)$ is decreasing in δ when the technology is green, but increasing when the technology is non-green.

If spillovers are large, i.e., $e > \widetilde{e}$, constraint (CC_e^z) binds first. To motivate compliance at the investment stage, the equilibrium investment levels must be lower in order to weaken the temptation to deviate. There must then be underinvestment, whatever the type of technology a country possesses. Formally, let $z^z(\delta)$ be defined as the z maximizing $u(z)$ subject to $\Delta^z(z,\delta) \geq 0$. When such a constraint binds, the function $z^z(\delta)$ increases in δ regardless of the technology type because a smaller δ increases the gain from free riding on investments when $z > z^{bau}$. Figure 2 provides an illustration of how different levels of technological spillovers affect strategic investments in the case of green technology.

As before, there exists a lower bound $\underline{\delta}(e)$, equal to the largest δ, such that if $\delta < \underline{\delta}(e)$, then there is no z that can satisfy all compliance constraints.[14]

Proposition 4 *There exists a best equilibrium if and only if $\delta \geq \underline{\delta}(e)$. For each $i \in N$, it supports $z_i = z^*$ when $\delta \geq \max\left\{\overline{\delta}^g, \overline{\delta}^z\right\}$. Otherwise,*

(i) if $e < \widetilde{e}$, then $\overline{\delta}^g > \overline{\delta}^z$ and $z_i = z^g(\delta) > z^*$ when the technology is green and $z_i = z^g(\delta) < z^*$ when technology is non-green;

[14] If $e = \widetilde{e}$, then $\overline{\delta}^g = \overline{\delta}^z = \underline{\delta}(e)$, so that the first best is possible if $\delta \geq \underline{\delta}(e)$; otherwise no equilibrium supports $g_i = \underline{g}$ for each i.

(ii) if $e > \widetilde{e}$, then $\overline{\delta}^g < \overline{\delta}^z$ and $z_i = z^z(\delta) < z^*$ regardless of the type of technology.

Compared to the basic model, the qualitative difference is that green investments decline with δ if $e > \widetilde{e}$. When countries are homogenous, large spillovers discourage investments, since they impose a constraint on the investment levels that can be sustained as SPEs. Specifically, requiring a high level of investment in green technology to motivate compliance at the emission stage may not be possible if the spillovers are large. Thus, under a policy that reduces the spillover by, for example, strengthening the protection of IPRs, compliance can be motivated by requiring more investment in green technology without concern that the compliance constraint at the investment stage will be violated.

5.2 Heterogeneous Countries and Technology Transfers

> *Protection of IPRs also works to slow the diffusion of new technologies, because it raises their cost and potentially limits their availability.*
>
> IPCC (2014:1036)

The Paris Agreement encourages technology transfers to developing countries. Article 10 states that the countries *"shall strengthen cooperative action on technology development and transfer."* In addition, "*international trade and foreign direct investment are the primary means by which new knowledge and technology are transferred between countries*" (IPCC, 2014:1035).

Thus, in terms of the model, technological transfers may require a larger e. This type of technology transfer can be rationalized in our framework. To see this, note that when the critical assumption made about homogenous countries in the previous subsection is relaxed, spillovers may be beneficial to the agreement since the possibility of technology transfers emerges. Intuitively, if the countries with the weakest compliance constraints, i.e., the least reluctant countries, are willing to invest more, then, in the presence of technological spillovers, these investments relax the compliance constraints for other countries.

To show this formally, let $\underline{\delta}_i(e)$ measure the smallest discount factor at which country i's compliance constraints at the emission and investment stages hold if every country invests at the same level, say, $\widehat{z}_i(e)$. Without spillovers, we know that a best equilibrium exists if and only if $\delta \geq \underline{\delta}(0) \equiv \max_j \underline{\delta}_j(0)$. Let $i = \arg\max_j \underline{\delta}_j(0)$ denote the most reluctant country in the absence of spillovers. We will say that country j is less reluctant than country i if, whenever i's compliance constraints hold, j's compliance constraints are non-binding. This implies that, at $\delta = \underline{\delta}_i(e)$, country j can set any $z_j \in [\widehat{z}_i(e), \widehat{z}_i(e) + \theta_{j,i}]$ for some $\theta_{j,i} > 0$, without violating its own compliance constraints, even if the other countries specify only $\widehat{z}_i(e)$. Since heterogeneity can originate from a variety of sources, $\theta_{j,i}$ is a measure of the degree of heterogeneity between i and j, for any given e. The highest level of heterogeneity is defined as $\theta \equiv \max_j \theta_{j,i}$.

Let $\underline{\delta}(e)$ be the smallest discount factor at which we can sustain a best equilibrium, i.e., an SPE which involves less emissions by all countries, for some investment levels. With these definitions, we are able to show that spillovers can improve the possibility of sustaining a best equilibrium.

Proposition 5 *For every $e > 0$, we have:*

(*i*) $\underline{\delta}(e) < \underline{\delta}(0)$ *if the heterogeneity, θ, is sufficiently large;*

(*ii*) *When $\delta \in (\underline{\delta}(e), \underline{\delta}(0))$, some countries will invest more in order to motivate the most reluctant countries to comply.*

In other words, if countries are sufficiently heterogeneous, then the set of discount factors that support a best equilibrium can be expanded if the spillover is positive rather than zero. This is because spillovers allow countries taking advantage of the heterogeneity, so that the compliance constraints of the most reluctant country can be weakened by the investments of the less reluctant countries. In this way, technological transfers facilitate compliance.

6 Technology and Imperfect Transparency

> *Measurement, reporting, and verification may be beneficially complemented by enforcement strategies.*
>
> IPCC (2014:1015)

The baseline model assumes that domestic emissions can be perfectly observed. In reality, emissions at a country level are difficult to monitor, while global emissions as well as installation of technologies are easier to track (Sterner, 2003). This imperfection leads to new roles for strategic investment in technologies.

In this section, we assume that domestic emissions cannot be observed and that even aggregate emissions are imperfectly monitored.[15] This imperfection leads to two types of errors: First, with probability $q > 0$ there is a *type I error* when it appears as if there has

[15] Note that assuming unobserved domestic emissions would not affect the above results if we continued to assume that aggregate emissions were perfectly monitored, since countries revert to BAU as soon as some country has defected (even if there is no public information regarding its identity).

been a defection, even if there has not been. Second, with probability $1-p \in [0,1]$ there is a *type II error* when a country has emitted more but the defection goes undetected. In previous sections, it was assumed that $p = 1$ and $q = 0$; in the next subsection, the best equilibrium as a function of the errors p and q is derived; while in the following subsection both errors are endogenous.

In this context, we derive a unique Pareto-optimal public perfect equilibrium (PPE) outcome in which every country emits $g_i = \underline{g}$ in the cooperation phase.[16] A PPE that supports this outcome is referred to as a best equilibrium. Since investments are assumed to be perfectly observable, deviations at the investment stage can be detected and discouraged if they are punished by reverting to BAU forever.[17] Just as in the baseline model, the compliance constraint at the investment stage requires only that a country's utility be higher than the BAU level. To simplify and isolate the effects of imperfect monitoring, we henceforth assume that all countries are identical.

6.1 Errors and Punishments

We start out by taking the probabilities p and q as given. In order to provide a simple microfoundation for this situation, suppose that, at the end of each period, the countries observe a binary signal $\varrho \in \{0,1\}$, which conveys information on the imperfectly observed aggregate emissions. If everyone emits less, $\varrho = 1$ with probability q. If a single country emits more, $\varrho = 1$ with probability p. Let $p > q$, so that $\varrho = 1$ is more likely if a country defects. In equilibrium, the signal $\varrho = 1$ will be followed by a punishment phase.

Let $\underline{\omega}(r)$ be a country's continuation value after $\varrho = 1$, while $\overline{\omega}(r)$ is the continuation value after $\varrho = 0$, conditioned on countries having invested a level r in the current period. Following Abreu et al. (1990), the payoff associated with the best equilibrium is the largest utility that satisfies the following constraints:

$$\frac{u(r)}{1-\delta} = b(\underline{g}, r) - hc(r)n\underline{g} - kr + \frac{\delta}{1-\delta}\left[(1-q)\overline{\omega}(r) + q\underline{\omega}(r)\right], \text{ subject to} \tag{4}$$

$$\frac{u(r)}{1-\delta} \geq b(\underline{g}, r) - hc(r)((n-1)\underline{g} + \overline{g}) - kr + \frac{\delta}{1-\delta}\left[(1-p)\overline{\omega}(r) + p\underline{\omega}(r)\right], \tag{5}$$

$$u(r) \geq \overline{\omega}(r), \underline{\omega}(r) \geq u^{bau}. \tag{6}$$

[16] In any PPE: (i) each country's strategy depends only on public history, which is a sequence of global emission levels and investment levels, and (ii) no country wants to deviate following any public history. See Fudenberg and Tirole (1991) for a definition of this equilibrium concept.

[17] It is straightforward to also allow for unobservable investments. Suppose a country can invest r in an observable technology type, and $\widehat{r}$ in a different unobservable technology type at cost $\widehat{k}\widehat{r}$. If both technology types enter the benefit function $\widehat{b}(g, r, \widehat{r})$, then our analysis will remain valid if we simply define $b(g, r) \equiv \max_{\widehat{r}} \widehat{b}(g, r, \widehat{r}) - \widehat{k}\widehat{r}$ for $g \in \{\underline{g}, \overline{g}\}$, since the unobservable investment will always be set equal to the individually optimal level, conditional on the agreed-upon emission level.

Eq. (4) defines the intertemporal utility, which is decomposed into current and continuation utilities according to the realization of the signal ϱ. Inequality (5) is the compliance constraint at the emission stage, while (6) ensures that continuation utilities are feasible.

In order to determine the best equilibrium, it is clear that $\underline{\omega}(r)$ must be as large as compliance constraint (5) permits and $\overline{\omega}(r) = u(r)$, which implies that:

$$\underline{\omega}(r) = u(r) - \frac{1-\delta}{\delta(p-q)}(\overline{g} - \underline{g})\psi(r),$$

where $\psi(r)$ is as in Section 2. Replacing the optimal values of $\underline{\omega}(r)$ and $\overline{\omega}(r)$ into (4), we get:

$$u(r) = b(\underline{g}, r) - hc(r)n\underline{g} - kr - \frac{q}{p-q}(\overline{g} - \underline{g})\psi(r). \tag{7}$$

Embedded in equation (7) is the efficiency loss $(q/(p-q))(\overline{g} - \underline{g})\psi(r)$ associated with the punishment that is triggered with some probability, even on the equilibrium path. For q approaching zero, the efficiency loss term vanishes and $u(r)$ tends to the first-best value.

Let $\widetilde{r}$ be the arg max of $u(r)$ in (7). If $\underline{\omega}(\widetilde{r}) \geq u^{bau}$, the punishment $\underline{\omega}(\widetilde{r})$ is feasible and the best equilibrium sustains $\widetilde{r}$. Due to the efficiency loss term in (7), the equilibrium level $\widetilde{r}$ is larger than the first-best investment level if technology is green and smaller if it is not. A feasible punishment $\underline{\omega}(\widetilde{r})$ can be supported if countries play BAU after the signal $\varrho = 1$ for $T \leq \infty$ periods before returning to the cooperative phase.[18] This strategy implies that:

$$\underline{\omega}(r) = \left(1 - \delta^T\right) u^{bau} + \delta^T u(r) \geq u^{bau}. \tag{8}$$

Condition (8) satisfied with equality implicitly defines the optimal length of punishment $T(\delta)$. It is then apparent that an additional strategic role of investment is to increase $\underline{\omega}(r)$, or in other words to decrease T, without violating the compliance constraint.

If $\underline{\omega}(\widetilde{r}) < u^{bau}$, there is no equilibrium in which countries invest $\widetilde{r}$, even if $T = \infty$. In this case, the best equilibrium requires $\underline{\omega}(r) = u^{bau}$, i.e., $T = \infty$, and technology investment r to be distorted even more from its first-best level in order to satisfy the compliance constraint. Such a technology investment is implicitly determined from $\underline{\omega}(r(\delta)) = u^{bau}$. Let us denote by $\overline{\delta}$ the level of δ that satisfies $\underline{\omega}(\widetilde{r}) = u^{bau}$ and by $\underline{\delta}$ the maximal δ, such that if $\delta < \underline{\delta}$, no PPE supports $g_i = \underline{g}$ in the cooperation phase.

Proposition 6 *Given the errors $(q, 1-p)$, there exists a best equilibrium if and only if*

[18] The equilibrium strategy is along the lines of Green and Porter (1984), who show that under imperfect monitoring, firms can create collusive incentives by allowing price wars to break out with positive probability.

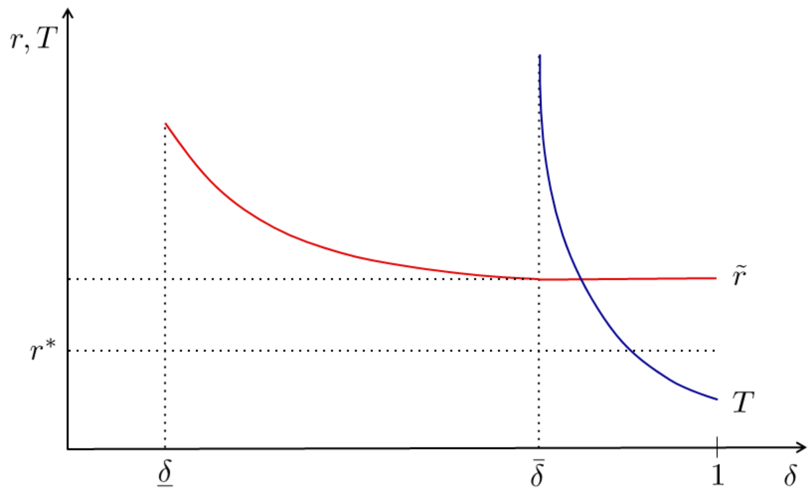

Figure 3: *Even for large discount factors, countries overinvest if technology is green. The large investments permit a shorter punishment phase without violating the compliance constraint.*

$\delta \geq \underline{\delta}$. *If* $\varrho = 1$, *it supports* $g_{i,t} = \overline{g}$ *and* $r_{i,t} = r^{bau}$ *for* T *periods. Otherwise,* $g_{i,t} = \underline{g}$ *and*

(i) *when* $\delta \geq \overline{\delta}$, $T'_\delta(\delta) < 0$ *and* $r_{i,t} = \widetilde{r} > r^*$ *if technology is green and* $r_{i,t} = \widetilde{r} < r^*$ *if technology is non-green;*

(ii) *when* $\delta \in [\underline{\delta}, \overline{\delta})$, $T = \infty$ *and* $r_{i,t} = r(\delta) > \widetilde{r} > r^*$ *if technology is green and* $r_{i,t} = r(\delta) < \widetilde{r} < r^*$ *if technology is non-green. Furthermore,* $|r(\delta) - \widetilde{r}|$ *is decreasing in* δ.

The qualitative difference between Proposition 6 and the baseline model without uncertainty is described in part (i). Since there is always a chance that the penalty will be triggered by mistake, the first best is impossible to sustain. The compliance constraint requires a penalty, but its duration should be reduced as much as the compliance constraint permits. By requiring countries to invest strategically, the temptation to raise emissions more than permitted declines and the penalty duration can be reduced without violating the compliance constraint. Figure 3 plots optimal investments and duration of punishment as functions of δ in the case of green technology.

6.2 The Optimal Punishment Probability

In order to endogenize the errors and simultaneously capture real-world uncertainty, denote global emissions by $g \equiv g_0 + \sum_{i \in N} g_i$, where g_0 is a random variable, drawn from a standard normal cdf $\Phi(\cdot)$, i.i.d. over time. The shock g_0 captures natural variations in the release of greenhouse gases.

As will be shown in the next result, the best equilibrium specifies a threshold $\widehat{g}(r)$ above which the punishment phase is initiated. Since punishment is triggered by mistake as soon as $g_0 > \widehat{g}(r) - n\underline{g}$, it is beneficial to raise the threshold $\widehat{g}(r)$. However, when doing so, T must increase for the compliance constraint to hold. By letting the punishment be as hard as possible, i.e., $T = \infty$, the threshold $\widehat{g}(r)$ can increase and the likelihood for errors can be minimized.

Lemma 1 *There exists a unique threshold $\widehat{g}(r)$ such that, in the best equilibrium, continuation utilities are given by:*

$$\omega(g,r) = \begin{cases} \overline{\omega}(r) = u(r) & \text{if} \quad g < \widehat{g}(r), \\ \underline{\omega}(r) = u^{bau} & \text{if} \quad g \geq \widehat{g}(r). \end{cases}$$

Given this threshold, the probability of type I error is $q = 1 - \Phi(\widehat{g}(r) - n\underline{g})$, while the probability of type II error is $1 - p = \Phi(\widehat{g}(r) - (n-1)\underline{g} - \overline{g})$. To further reduce the probability of error on the equilibrium path, q, the threshold $\widehat{g}(r)$ can be increased if the temptation to defect is reduced, and it is indeed reduced if the level of green technology increases or that of non-green technology decreases. This possibility is reflected in the best equilibrium.

Proposition 7 *There exists a best equilibrium if and only if $\delta \geq \underline{\delta}$. For each $i \in N$, it supports $g_{i,t} = \overline{g}$ and $r_{i,t} = r^{bau}$ if $g_{i,\tau} > \widehat{g}(r)$ for some $\tau < t$. Otherwise, $g_{i,t} = \underline{g}$ and*

(i) $r_{i,t} = \widehat{r} > \widetilde{r} > r^*$ *if technology is green and therefore* $\widehat{g}'_r(r) > 0$*;*

(ii) $r_{i,t} = \widehat{r} < \widetilde{r} < r^*$ *if technology is non-green and therefore* $\widehat{g}'_r(r) < 0$*;*

where $\widehat{r}$ solves the following first-order condition:

$$b'_r\left(\underline{g}, r\right) - hc'_r(r)n\underline{g} - k = \left(\overline{g} - \underline{g}\right)\left(L\left(\widehat{g}(r)\right)\psi'_r(r) + \psi(r)\, L'_{\widehat{g}}\left(\widehat{g}(r)\right)\frac{d\widehat{g}(r)}{dr}\right),$$

with $L(\widehat{g}(r)) \equiv q/(p-q) \in (0, \infty)$.

This result points to a new role for technology. In the baseline model, strategic technology investments were necessary in order to motivate compliance when the discount factor is small. In the previous subsection, where p and q were given, strategic investments reduced the length of the punishment period that was necessary in order to discourage defections. When $\widehat{g}(r)$ and the errors are endogenous, the new role of technology is to reduce the probability that the punishment is triggered by mistake, i.e., type I error.

7 Technology and Pollution as Stocks

In this section, we reformulate the model to treat technology, as well as pollution, as stocks. Suppose we let $r_{i,t}$ measure i's technology stock at time t, where $q_i^r \in [0,1]$ is the fraction of past technology that survives, i.e., that has not depreciated, into the next period, and each unit of investment, $I_{i,t}$, costs $\widetilde{k}_i$. Clearly, deciding on $I_{i,t}$ is equivalent to deciding on $r_{i,t}$ once $r_{i,t-1}$ is sunk. One benefit of investing today is that investments can be reduced in the next period. Naturally, we can account for the future cost saving already today:

$$\text{With } r_{i,t} = q_i^r r_{i,t-1} + I_{i,t}, \text{ let } k_i \equiv \widetilde{k}_i \left(1 - \delta q_i^r\right)$$

be defined as the net cost of adding to the technology stock in period t, taking into account the future cost saving. If the q_i^r's were small, the above analysis would remain unchanged since countries would need to invest in every period (even off the equilibrium path) in order to maintain the technology level that is necessary to satisfy the compliance constraint, and the net cost of investing would be equal to k_i. Small q_i^r's are reasonable in the very long-run context of climate change, in which countries must expect to invest repeatedly, partly, for example, to maintain the infrastructure and the capacity to produce renewable energy. If the q_i^r's are instead large, then a country cannot easily reduce a clean technology stock to r_i^{bau} after defecting and therefore defecting would be less attractive than assumed above. In this case, an agreement is more likely to be self-enforcing because of this irreversibility.

It is also straightforward to treat pollution as a stock. Suppose G_t is the pollution stock at time t and it depreciates at the rate $q^g \in [0,1]$, and let $\widetilde{h}_i$ be environmental harm to country i's from each unit of G_t at each point in time. If $\widetilde{h}_i$ is a constant, that is, independent of the technology level, then:

$$\text{With } G_t = q^G G_{t-1} + \sum_j g_{j,t}, \text{ let } h_i \equiv \widetilde{h}_i / \left(1 - \delta q^g\right)$$

be defined as the present discounted cost of emitting another unit, evaluated at the time of the emission, while taking into account that it will depreciate only gradually. The present discounted cost h_i of every unit $g_{i,t}$ can be accounted for already at time t, allowing us to represent i's per-period payoff exactly as above.

In the case of both a technology stock and a pollution stock, the analysis continues to hold since the stocks are not payoff relevant, that is, they do not influence the marginal cost/benefit when deciding on $r_{i,t}$ or $g_{i,t}$, and thus affect neither the equilibrium nor the first-best $r_{i,t}$'s or $g_{i,t}$'s.

In fact, even with a convex investment-cost function for technology, the technology stock may be payoff irrelevant as long as it substitutes for emissions that have linear costs (which is the case in Battaglini and Harstad, 2016, for example). If the cost of pollution is nonlinear, then a larger pollution stock would reduce the temptation to pollute and would raise the incentive to invest in clean technology (as in Harstad, 2012; 2016). Since these effects have already been analyzed in the literature, the contribution of our model is best highlighted by abstracting from payoff-relevant stocks.

8 Further Readings

Lecture Notes 1 is for the most part based on a published article written jointly by Bård Harstad, Francesco Lancia, and Alessia Russo, but it is inspired by, and it builds on, a large body of literature.

Our investigation of bottom-up cooperation complements the top-down (mechanism-design) approach by, for example, Martimort and Sand-Zantman (2016). In some regard, this note fills a gap between the literature on environmental economics and that on repeated games. As mentioned, it is widely accepted that international agreements must be self-enforcing.[19] Thus, we draw heavily on the repeated games literature, although much of this literature has been concerned with folk theorems and conditions under which the first best can be sustained if only the players are sufficiently patient (see, e.g., Ivaldi et al., 2003; Mailath and Samuelson, 2006).

There are also other papers on self-enforcing environmental agreements. In previous papers, such as Barrett (1994; 2005) and Dutta and Radner (2004; 2006), technology investments are either not permitted or chosen as a corner solution at the beginning of the game.

There is an emerging literature that examines the relationship between technology investments and international environmental cooperation. Much of it focuses either on the harmful effects of technology investments on a country's bargaining position in the future, when new commitments are to be negotiated (see, e.g., Buchholz and Konrad, 1994; Beccherle and Tirole, 2011; Harstad, 2012, 2016; Helm and Schmidt, 2015) or on a country's incentive to invest in the presence of positive international externalities (see, e.g., Barrett, 2006; de Coninck et al., 2008; Golombek and Hoel, 2005; Hoel and de Zeeuw, 2010). In contrast, this note stresses how technology influences a country's incentives to comply with emission abatements.

The structure of the above model is similar to the one of Harstad (2012; 2016) and

[19] As Downs and Jones (2002:S95) observed, "*a growing number of international relations theorists and international lawyers have begun to argue that states' reputational concerns are actually the principal mechanism for maintaining a high level of treaty compliance.*"

Battaglini and Harstad (2016), where countries pollute and invest in green technologies in every period. These papers assume contractible emission levels and study Markov-perfect equilibria, while this note focuses on self-enforcing agreements and subgame-perfect equilibria. This approach leads to a new strategic effect of technology—namely that technology should be chosen so as to make future cooperation credible.

Theoretically, the note is related to the industrial organization literature, in which strategic investments can deter entry (see, e.g., Spence, 1977; Dixit, 1980; Fudenberg and Tirole, 1984) or reduce production costs and therefore improve the competitive position vis-à-vis rivals (see, e.g., Brander and Spencer, 1983; Spence, 1984; d'Aspremont and Jacquemin, 1988; Leahy and Neary, 1997).[20]

More closely related is the literature on the influence of capacity constraints on the sustainability of tacit collusion. In examining this question, Brock and Scheinkman (1985) treated the capacity constraints as exogenous, while Benoit and Krishna (1987) allowed firms to collude on capacity investments as well as on price. When capacity investments are irreversible, firms overinvest in order to make retaliation harsh and credible; but this effect vanishes when investments are reversible, since firms can always adjust the retaliation capacity later.[21] The mechanism above differs in that overinvestment in green technology or underinvestment in adaptation and brown technologies is necessary along the equilibrium path in order to undermine the short-run gain from deviation in the cooperative phase. This result holds even when investment decisions are fully reversible and is reinforced when they are not.

[20] Martin (1995) and Cabral (2000) contributed to the analysis of the role of strategic investment, by considering an infinite-period duopoly industry in which firms make R&D decisions as well as product market decisions. Both papers showed that R&D investments may encourage firms to tacitly collude on output, resulting in a welfare loss. However, the mechanism by which collusion is sustained occurs is very different from the mechanism in this note, since Martin (1995) assumes that firms commit themselves to the joint profit-maximizing level of R&D, while Cabral (2000) assumes that R&D investments are hidden and therefore cannot be part of the agreement.

[21] While Davidson and Deneckere (1990) do not allow firms to collude in capacity, they do allow them to collude on price. Like Benoit and Krishna (1987), they also show that excess capacity is present in all equilibria. The impact of asymmetry in capacity on self-enforcing collusion is instead analyzed by Lambson (1994) and Compte et al. (2002), who investigate how asymmetry in capacity influences whether collusion is self-enforcing. They conclude that, depending on parameters, asymmetric capacities may either encourage or discourage collusion.

9 Appendix

Proof of Proposition 1. Since $u_{i,t}(r_{i,t})$ is concave and single-peaked in $r_{i,t}$, the best equilibrium involving $g_{i,t} = \underline{g}$ for each $i \in N$ and every $t \geq 1$ requires $r_{i,t}$ to be the closest to r_i^*, subject to compliance constraints at both the investment and the emission stages being satisfied. Since deviations are never observed in equilibrium and the discount factor is common to all countries, the best equilibrium simply requires $r_{i,t} = r_i$ at every date t. Hence, we can remove t superscript and solve at any fixed δ the following constrained optimization problem:

$$\max_{r_i} u_i(r_i) \equiv b_i(\underline{g}, r_i) - h_i c(r_i) n\underline{g} - k_i r_i \text{ s.t.,}$$

$$u_i(r_i) \geq u_i^{bau}, \qquad (\text{CC}_i^r)$$

$$\Delta_i(r_i, \delta) \equiv u_i(r_i) - u_i^{bau} - \frac{1-\delta}{\delta}(\overline{g} - \underline{g})\psi_i(r_i) \geq 0, \qquad (\text{CC}_i^g)$$

where $\psi_i(r_i) \equiv ((b_i(\overline{g}, r_i) - b_i(\underline{g}, r_i))/(\overline{g} - \underline{g})) - h_i c(r_i)$. Since $u_i(r_i) \geq u_i^{bau}$ at r_i^*, both constraints hold if δ is close to 1. At r_i^*, $u_i(r_i)$ and condition (CC_i^r) do not change when δ falls, but (CC_i^g) will eventually bind because $\psi_i(r_i) > 0$ under Assumption 1. For each i, a threshold $\overline{\delta}_i$ is implicitly defined as the level of δ that solves $\Delta_i(r_i^*, \delta) = 0$. Thus, if $\delta \geq \max_i \overline{\delta}_i$, $r_i = r_i^*$ satisfies conditions (CC_i^r) and (CC_i^g) for all countries. If $\delta < \overline{\delta}_i$, condition (CC_i^g) is violated at r_i^*. However, compliance with low emissions can be satisfied if $r_i = r_i(\delta) > r_i^*$ for green technology, where $r_i(\delta)$ maximizes $u_i(r_i)$ subject to $\Delta_i(r_i, \delta) = 0$, since $\psi_{i,r}'(r_i) < 0$. The opposite relation holds for non-green technology, i.e., $r_i = r_i(\delta) < r_i^*$. As δ declines further, condition (CC_i^g) is satisfied only if the distortion $|r_i - r_i^*|$ increases more. For each i, there exists a lower bound $\underline{\delta}_i \in (0, \overline{\delta}_i)$, such that if $\delta < \underline{\delta}_i$, conditions (CC_i^r) and (CC_i^g) cannot be satisfied for any r_i and no SPE supporting $g_i = \underline{g}$ for each i exists. ■

Proof of Proposition 2. Recall that, conditional on g, r_i^* and r_i^{bau} are given by the first-order condition (1). Differentiating such a condition w.r.t. g and r_i, we get:

$$\frac{dr_i}{dg} = \frac{-b_{i,rg}''(g, r_i) + h_i c_r'(r_i) n}{b_{i,rr}''(g, r_i) - h_i c_{rr}''(r_i) gn}, \qquad (9)$$

where the denominator is the second-order condition of $u_i(r_i)$ w.r.t. r_i, which is negative. Since g is discrete, we have:

$$r_i^* - r_i^b = \int_{\underline{g}}^{\overline{g}} \frac{-b_{i,rg}''(g, r_i) + h_i c_r'(r_i) n}{b_{i,rr}''(g, r_i) - h_i c_{rr}''(r_i) gn} dg.$$

Hence, $r_i^* - r_i^b > 0$ if technology is clean, or negative otherwise. Furthermore, in the case of adaptation technology, Eq. (9) simplifies to $dr_i/dg = -c_r'(r_i)/(c_{rr}''(r_i)g)$ and in turn the term $c(r_i(g))g$ is increasing in g if and only if $c(r_i) > (c_r'(r_i))^2/c_{rr}''(r_i)$. If $\delta < \overline{\delta}_i$, the best equilibrium satisfying $g_i = \underline{g}$ for each i requires that $r_i = r_i(\delta)$, so that condition (CC$_i^g$) binds. Differentiating $\Delta_i(r_i, \delta) \equiv u_i(r_i) - u_i^{bau} - \frac{1-\delta}{\delta}(\overline{g} - \underline{g})\psi_i(r_i) = 0$ w.r.t. r_i yields $\Delta_{i,r}' = u_{i,r}'(r_i) - \frac{1-\delta}{\delta}(\overline{g} - \underline{g})\psi_{i,r}'(r_i)$. For $r_i \simeq r_i^*$, we can then state the following results: (i) since $\Delta_{i,n}' = -h_i(c(r_i)\underline{g} - c(r_i^{bau})\overline{g})$ and $\Delta_{i,h}' = -n(c(r_i)\underline{g} - c(r_i^{bau})\overline{g}) + \frac{1-\delta}{\delta}(\overline{g} - \underline{g})c(r_i)$, $dr_i/dn = -\Delta_{i,n}'/\Delta_{i,r}'$ and $dr_i/dh_i = -\Delta_{i,h}'/\Delta_{i,r}'$ are negative if technology is clean, and positive otherwise (for the case of adaptation provided that $c(r_i) > (c_r'(r_i))^2/c_{rr}''(r_i)$); and (ii) since $\Delta_{i,k}' = -(r_i - r_i^{bau})$, $dr_i/dk_i = -\Delta_{i,k}'/\Delta_{i,r}'$ is positive if technology is of any type. ■

Proof of Proposition 3. If we define $d_i \equiv \overline{y} - g_i - r_i$ to be the decrease in consumption relative to the bliss level $\overline{y}$, the first-best emission (and consumption) level is simply given by $d^* = cn/B$, while, in BAU, $d^b = c/B$. We can write the continuation value as:

$$v = d(nc - Bd/2) + r(nc - Kr/2) - cn\overline{y}.$$

■

(i) The compliance constraint at the emission stage can be written as:

$$\frac{v}{1-\delta} \geq nc(r - r^b) - \frac{K}{2}\left(r^2 - (r^b)^2\right) + c(n-1)(d - d^b) + \frac{v^b}{1-\delta}, \qquad (\text{CC}_c^g)$$

which implies that:

$$\delta \geq \widehat{\delta}(g, r) \equiv 1 - \frac{-cn\overline{y} + d(nc - Bd/2) + r(nc - Kr/2) - v^b}{nc(r - r^b) - \frac{K}{2}\left(r^2 - (r^b)^2\right) + c(n-1)(d - d^b)}, \text{ so}$$

$$\overline{\delta} \equiv \widehat{\delta}(g^*, r^*) = 1 - \frac{(n-1)^2(1/2B + 1/2K)}{(n-1)^2/2K + (n-1)^2/B} = \frac{K}{B + 2K}.$$

(ii) Note that $r_i = r^*$ is both maximizing v and weakening (CC$_c^g$). Given this r^*, the optimal d is the largest d satisfying (CC$_c^g$). Substituting for v and then solving (CC$_c^g$) for the largest d, we get:

$$\begin{aligned} dB &= nc - \phi, \text{ where} \\ \phi(\delta) &\equiv c(n-1)\left[1 - \delta - \sqrt{\delta^2 + \delta B/K}\right] \\ &= c(n-1)\left[1 - \delta - \sqrt{(1-\delta)^2 - \left(\overline{\delta}^g - \delta\right)/\overline{\delta}^g}\right], \end{aligned}$$

where $\phi(\delta)$ decreases from $c(n-1)$ to 0 as δ increases from 0 to $\overline{\delta}^g$. This d is implemented by the emission tax $nc - \phi$. To ensure $r_i = r^*$, the subsidy must be ϕ.

Note that the investment-stage compliance constraint can be written as

$$\frac{v}{1-\delta} \geq \varsigma(n-1)\frac{c}{K} + \frac{v^b}{1-\delta}, \tag{CC$_c^r$}$$

which always holds when $\varsigma = \phi(\delta) \to 0$. When δ falls, v declines and $\varsigma = \phi(\delta)$ increases. The threshold $\underline{\delta}$ is defined implicitly by requiring (CC$_c^r$) to hold with equality at $\underline{\delta}$. *QED*

Proof of Proposition 4. To determine the best equilibrium in the presence of technological spillovers and homogenous countries, we must solve at any fixed δ the following constrained optimization problem:

$$\max_z u(z) \equiv b(\underline{g}, z) - hc(z)\, n\underline{g} - kz \text{ s.t.},$$

$$\Delta^z(z,\delta) \equiv u(z) - u^{bau} - (1-\delta)\frac{e(n-1)}{n(1-e)-1}k\left(z - z^{bau}\right) \geq 0, \tag{CC$_e^z$}$$

$$\Delta^g(z,\delta) \equiv u(z) - u^{bau} - \frac{1-\delta}{\delta}(\overline{g} - \underline{g})\psi(z) \geq 0, \tag{CC$_e^g$}$$

where $\psi(z) \equiv (b(\overline{g}, z) - b(\underline{g}, z))/(\overline{g} - \underline{g}) - hc(z)$ and z^{bau} is determined from $b'_z\left(\overline{g}, z^{bau}\right) - hc'_z\left(z^{bau}\right)n\overline{g} - (n-1-e)k/(n(1-e)-1) = 0$. The thresholds $\overline{\delta}^z \equiv 1 - ((n(1-e) - 1)/(e(n-1)k(z^* - z^{bau})))(u(z^*) - u^{bau})$ and $\overline{\delta}^g \equiv (\overline{g} - \underline{g})\psi(z^*)/((u(z^*) - u^{bau}) + (\overline{g} - \underline{g})\psi(z^*))$ are equal to the levels of δ implicitly defined from $\Delta^z(z^*, \delta) = 0$ and $\Delta^g(z^*, \delta) = 0$, respectively. Hence, if $\delta \geq \max\left\{\overline{\delta}^g, \overline{\delta}^z\right\}$, conditions (CC$_e^z$) and (CC$_e^g$) are satisfied for $z_i = z^*$ and the best equilibrium is first best. Let $\delta < \max\left\{\overline{\delta}^g, \overline{\delta}^z\right\}$. Note that constraint (CC$_e^z$) can bind first only if $z^* > z^{bau}$. Under this condition, since $dz^{bau}/de = (k(n-1)^2/(n(1-e)-1)^2)/(b''_{zz}(\overline{g}, z) - hc'_{zz}(z)n\overline{g}) < 0$ and $du^{bau}/de = (((n-1)ke)/(n(1-e)-1))\left(dz^{bau}/de\right) < 0$, we have $d\overline{\delta}^g/de < 0$ and $d\overline{\delta}^z/de > d\overline{\delta}^g/de$, which implies that there exists a threshold level $\widetilde{e} > 0$ implicitly defined from $\overline{\delta}^z = \overline{\delta}^g$, such that $\overline{\delta}^g \geq (<)\, \overline{\delta}^z$ if $e \leq (>)\, \widetilde{e}$. Let $e \leq \widetilde{e}$ and $\delta \in [\overline{\delta}^z, \overline{\delta}^g)$. Then $z_i = z^g(\delta)$ where $z^g(\delta)$ is the level z that maximizes $u(z)$ subject to $\Delta^g(z,\delta) = 0$. For $z \simeq z^*$,

$dz^g/d\delta \approx \psi(z)/(\delta(1-\delta)\psi'_z(z))$, which implies that $z^g(\delta) > z^*$ if technology is green and $z^g(\delta) < z^*$ otherwise. Let now $e > \widetilde{e}$ and $\delta \in [\overline{\delta}^g, \overline{\delta}^z)$. Then $z_i = z^z(\delta)$ where $z^z(\delta)$ is the level z that maximizes $u(z)$ subject to $\Delta^z(z,\delta) = 0$. For $z \simeq z^*$, $dz^z/d\delta \approx z - z^{bau}$, which implies that $z^z(\delta) < z^*$ if technology is of any type. Inspecting constraints (CC_e^g) and (CC_e^z), it is easy to see that there exists a lower bound $\underline{\delta}$ that is the largest level of δ such that if $\delta < \underline{\delta}$, there is no level of z that can simultaneously satisfy compliance with investments and emissions, i.e., $g_i = \underline{g}$ for each i cannot be enforced for any z.

■

Proof of Proposition 5. To determine the best equilibrium in the presence of technological spillovers and heterogenous countries, we must solve, for any fixed δ, the following constrained optimization problem:

$$\max_{z_i, z_{-i}} u_i(z_i, z_{-i}) \equiv b_i\left(\underline{g}, z_i\right) - h_i c(z_i)\, n\underline{g} - k_i \frac{(n-1-e)\, z_i - e z_{-i}}{n(1-e)-1} \text{ s.t.,}$$
$$\Delta_i^g(z_i, z_{-i}, \delta) \equiv u(z_i, z_{-i}) - u_i^{bau} - \frac{1-\delta}{\delta}(\overline{g} - \underline{g})\psi_i(z_i) \geq 0,$$
$$\Delta_i^z(z_i, z_{-i}, \delta) \equiv u_i(z_i, z_{-i}) - u_i^{bau} - (1-\delta)\frac{e}{n(1-e)-1} k_i\left(z_{-i} - z_{-i}^{bau}\right) \geq 0,$$

where $z_{-i} \equiv \sum_{j \neq i} z_j$ and $\psi_i(z_i)$ is defined in the proof of Proposition 5. Let i be the country with the largest $\underline{\delta}_i$. Suppose $e = 0$ and $\delta = \underline{\delta}_i$, and let $\widehat{z}_i(0)$ be the investment level that is satisfying with equality both compliance constraints for i. Then, there is an SPE in which every country invests $\widehat{z}_i(0)$ and all compliance constraints are satisfied. Next, consider the situation in which $e > 0$. When everyone continues to invest $\widehat{z}_i(0)$, $u_i(z_i, z_{-i})$ is invariant in e, u_i^{bau} decreases in e, and thus both $\Delta_i^g(z_i, z_{-i}, \delta)$ and $\Delta_i^z(z_i, z_{-i}, \delta)$ decrease in e. Obviously, the magnitude of these shifts is independent of θ, where θ is defined in the main text. For some $\theta > 0$, country j can choose $z_j = \widehat{z}_i(0) + \theta$ and still satisfy j's compliance constraints. Thus, consider the SPE in which $z_j = \widehat{z}_i(0) + \theta$, while everyone else invests $\widehat{z}_i(0)$. The larger z_j benefits i. This is because $u_i(z_i, z_{-i})$ and $\Delta_i^g(z_i, z_{-i}, \delta)$ increase by $\theta k_i e/[n(1-e)-1]$, while $\Delta_i^z(z_i, z_{-i}, \delta)$ increases by $\theta\delta k_i e/[n(1-e)-1]$, according to the formulas above. Consequently, for a sufficiently large θ, the positive effects on $\Delta_i^g(z_i, z_{-i}, \delta)$ and $\Delta_i^z(z_i, z_{-i}, \delta)$ are larger than the direct negative effect following an increase in e. For such a large θ, when z_j decreases by θ, both compliance constraints of the most reluctant country become nonbinding, i.e., $\underline{\delta}_i$ declines. ■

Proof of Proposition 6. To determine the best equilibrium in the presence of imperfect monitoring for given probabilities p and q, we must solve at any fixed δ the following constrained optimization problem:

$$\max_r u(r) \equiv b(\underline{g}, r) - hc(r)\, n\underline{g} - kr + \delta\left[(1-q)\,\overline{\omega}(r) + q\underline{\omega}(r)\right] \text{ s.t.,}$$
$$u(r) \geq b(\overline{g}, r) - hc(r)(n-1)\underline{g} - hc(r)\,\overline{g} - kr + \delta\left[(1-p)\,\overline{\omega}(r) + p\underline{\omega}(r)\right], \quad (\mathrm{CC}_c^g)$$

where $\overline{\omega}(r) = u(r) \geq u^{bau}$ and $\underline{\omega}(r) \geq u^{bau}$. Constraint (CC_c^g) necessarily binds at the optimum, which implies that the intertemporal utility can be written as $u(r) \equiv b(\underline{g}, r) - hc(r)\, n\underline{g} - kr - (q/(p-q))(\overline{g} - \underline{g})\psi(r)$, where $\psi(r)$ is reported in the text. Let $\widetilde{r}$ be the level of r that solves the following first-order condition:

$$b'_r(\underline{g}, r) - hc'_r(r)n\underline{g} - k - \frac{q}{p-q}(\bar{g} - \underline{g})\psi'_r(r) = 0, \tag{10}$$

where the second-order condition is satisfied for $b(\underline{g}, r) - hc(r)n\underline{g}$ sufficiently concave in r. Replacing $\overline{\omega}(\widetilde{r})$ with $u(\widetilde{r})$ into condition (CC_c^g) satisfied with equality, yields:

$$\underline{\omega}(\widetilde{r}) = u(\widetilde{r}) - \frac{(1-\delta)}{\delta(p-q)}(\overline{g} - \underline{g})\psi(\widetilde{r}), \tag{11}$$

which is feasible if it is at least equal to u^{bau}. Let $\overline{\delta}$ be the level of δ that solves $\underline{\omega}(\widetilde{r}) = u^{bau}$ and consider the following two cases.

(i) If $\delta \geq \overline{\delta}$, then $\underline{\omega}(\widetilde{r}) \geq u^{bau}$ and $r = \widetilde{r}$. Differentiating Eq. (10) with respect to r and q, we get $dr/dq \approx -(p(\overline{g} - \underline{g})/(p-q)^2)\psi'_r(r)$, which is positive and implies $\widetilde{r} > r^*$ for green technologies. The opposite relation holds for non-green technologies, i.e., $\widetilde{r} < r^*$. Combining $\underline{\omega}(\widetilde{r}) = \delta^T u(\widetilde{r}) + \left(1 - \delta^T\right) u^{bau}$ with Eq. (11), the optimal length of punishment $T(\delta)$ is determined from the following equation:

$$u(\widetilde{r}) - u^{bau} - \frac{(1-\delta)}{\delta\left(1-\delta^T\right)(p-q)}(\overline{g} - \underline{g})\psi(\widetilde{r}) = 0. \tag{12}$$

Differentiating Eq. (12) w.r.t. T and δ, we get $dT/d\delta = ((1-\delta^T) - (1-\delta)T\delta^T)/(\delta(1-\delta)\delta^T \ln\delta) < 0$.

(ii) If $\delta < \overline{\delta}$, then $\underline{\omega}(\widetilde{r}) < u^{bau}$, which implies that the optimal investment $r \neq \widetilde{r}$ is obtained from constraint (CC_c^g) when $T = \infty$ and in turn $\underline{\omega}(r) = u^{bau}$, that is,

$$b(\underline{g}, r) - hc(r)n\underline{g} - kr - \frac{1-\delta(1-q)}{\delta(p-q)}(\overline{g} - \underline{g})\psi(r) = u^{bau}. \tag{13}$$

Differentiating Eq. (13) w.r.t. r and δ, we get:

$$\frac{dr}{d\delta} = -\frac{\frac{1}{\delta^2(p-q)}(\overline{g}-\underline{g})\psi(r)}{b'_r(\underline{g},r) - hc'_r(r)n\underline{g} - k - \frac{1-\delta(1-q)}{\delta(p-q)}(\overline{g}-\underline{g})\psi'_r(r)}. \tag{14}$$

For $r \simeq \widetilde{r}$, using Eq. (10), the denominator of (14) is $-((1-\delta)\delta/(p-q))(\overline{g}-\underline{g})\psi'_r(\widetilde{r})$, which is positive and implies $r > \widetilde{r} > r^*$ for green technologies. The opposite relation holds for non-green technologies, i.e., $r < \widetilde{r} < r^*$. Inspecting condition (13), we find a lower bound $\underline{\delta}$, such that if $\delta < \underline{\delta}$, there exists no level of r that can satisfy such a condition and less emissions by all countries cannot be enforced for any level of r. ■

Proof of Lemma 1. Continuation utilities $\omega(g,r)$ must maximize:

$$\frac{u(r)}{1-\delta} = b(\underline{g},r) - hc(r)n\underline{g} - kr + \frac{\delta}{1-\delta}\int_g \omega(g,r)\,\phi(g|n\underline{g})dg, \text{ s.t.},$$

$$\frac{u(r)}{1-\delta} \geq b(\underline{g},r) - hc(r)((n-1)\,\underline{g}+\overline{g}) - kr + \frac{\delta}{1-\delta}\int_g \omega(g,r)\,\phi(g|\,(n-1)\,\underline{g}+\overline{g})dg, \tag{15}$$

$$u(r) \geq \omega(g,r) \geq u^{bau}. \tag{16}$$

Ignoring for a moment constraint (16) and letting ν be the multiplier associated with (15), the first-order condition with respect to $\omega(g,r)$ is:

$$\int_g \omega(g,r)\left[\phi(g|n\underline{g}) - \nu\phi(g|\,(n-1)\,\underline{g}+\overline{g})\right]dg.$$

By the monotone likelihood ratio property and given that g is continuous, there is a unique $\widehat{g}(r)$ for which $\frac{\phi(\widehat{g}(r)|n\underline{g})}{\phi(\widehat{g}(r)|(n-1)\underline{g}+\overline{g})} = \nu$ and such that if $g > (<)\,\widehat{g}(r)$ then $\frac{\phi(g|n\underline{g})}{\phi(g|(n-1)\underline{g}+\overline{g})} < (>)\,\nu$. We can then conclude that we must have $\omega(g,r) = u^{bau}$ for $g \geq \widehat{g}(r)$ and $\omega(g,r) = u(r)$, otherwise. ■

Proof of Proposition 7. To determine the best equilibrium in the presence of imperfect monitoring when $\widehat{g}$ is endogenously determined, we must solve the constrained optimization problem reported in the proof of Proposition 7 for the levels $\widehat{g}$ and r, where $q = 1-\Phi(\widehat{g}-n\underline{g})$ and $1-p = \Phi(\widehat{g}-(n-1)\,\underline{g}-\overline{g})$. Using constraint ($CC^g_c$) and replacing $\overline{\omega}(r) = b(\underline{g},r) - hc(r)n\underline{g} - kr - (q/(p-q))(\overline{g}-\underline{g})\psi(r)$ and $\underline{\omega}(r) = u^{bau}$, we get:

$$b(\underline{g},r) - hc(r)n\underline{g} - kr - u^{bau} - \frac{1-\delta(1-q)}{\delta(p-q)}(\overline{g}-\underline{g})\psi(r) \geq 0. \tag{17}$$

Differentiating Eq. (17) w.r.t. $\widehat{g}$ and δ, we have $d\widehat{g}/d\delta \approx \partial[(1-\delta+\delta q)/(\delta(p-q))]/\partial\widehat{g}$. Since abandoning cooperation consists in emitting more rather than less, a tail test prescribes to trigger a punishment when aggregate emissions fall in the upper tail of their distribution, i.e., in the critical region $[\widehat{g}, \infty)$. In such a region, the monotone likelihood ratio property implies that $d\widehat{g}/d\delta > 0$. Differentiating Eq. (17) w.r.t. $\widehat{g}$ and r, we have:

$$\frac{d\widehat{g}}{dr} = \frac{b_r'\left(\underline{g}, r\right) - hc_r'(r)n\underline{g} - k - \frac{1-\delta+\delta q}{\delta(p-q)}\left(\overline{g} - \underline{g}\right)\psi_r'(r)}{\frac{\partial}{\partial \widehat{g}}\left[\frac{1-\delta+\delta q}{\delta(p-q)}\right]\left(\overline{g} - \underline{g}\right)\psi(r)}. \tag{18}$$

Let $\widehat{r}$ be the level of r that solves the following first-order condition:

$$b_r'\left(\underline{g}, r\right) - hc_r'(r)n\underline{g} - k = \left(\overline{g} - \underline{g}\right)\left(L\left(\widehat{g}(r)\right)\psi_r'(r) + \psi(r)\, L_{\widehat{g}}\left(\widehat{g}(r)\right)\frac{d\widehat{g}(r)}{dr}\right), \tag{19}$$

where $L\left(\widehat{g}(r)\right) \equiv q/(p-q)$ with $L_{\widehat{g}}'\left(\widehat{g}(r)\right) < 0$. Replacing Eq. (19) into Eq. (18), yields:

$$\frac{d\widehat{g}}{dr} = -\frac{1}{p-q}\frac{\psi_r'(r)}{\psi(r)}\left(\frac{\partial}{\partial \widehat{g}}\left[\frac{1}{p-q}\right]\right)^{-1},$$

with $\partial\left[1/(p-q)\right]/\partial\widehat{g} > 0$ in the upper tail of the distribution of global emissions. If technology is green, $\psi_r'(\widehat{r}) < 0$ and $d\widehat{g}(r)/dr > 0$. Using Eq. (19), this implies that $\widehat{r} > \widetilde{r} > r^*$, where $\widetilde{r}$ solves the first-order condition (10) reported in the proof of Proposition 7. If technology is non-green, the opposite relation holds, i.e., $\widehat{r} < \widetilde{r} < r^*$. Inspecting Eq. (17), there exists a lower bound $\underline{\delta}$, such that if $\delta < \underline{\delta}$, less emissions by all countries cannot be enforced for any r. ■

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