Environmental Economics 4910

Problem set 2 with Solution Key + Problem Set 3 (Ex. 3)

Ex. 1: Market in Licenses (based on Montgomery 1972)

When we in class (8/2) briefly mentioned the analysis of “Markets in licenses and efficient pollution control programs” by Montgomery (1972), we simplified by assuming interior solutions and that all license prices were strictly positive.

1. Is this a reasonable assumption? Why/why not? **Solution key**: This refers to the matrix model we just briefly mentioned in class, in which there are many locations on which you can pollute. In general, given caps in the various districts, and given the constant transport coeffients (in the matrix) it may well be that one of the caps is not binding, so that the price there will be zero. However, then the price/cap is not optimally set: Since we proved that the cap should be such that the equilibrium price equals the marginal cost of pollution in the district.

2. How does Montgomery (1972) distinguish between “pollution licenses” and “emission licenses”? **Solution key**: A pollution license owned by firm i is a permit to polute in a district, say j. If i’s emission pollutes in multiple districts, then i needs pollution licenses in all these districts. An emission license is a license to emit.

   Suppose there are $n = 10$ firms and $m = n = 10$ locations but, referring to the notation in the paper, $h_{i1} = 1$, $h_{i2} = 2$, and $h_{ij} = 0$ for every firm i, and every location $j \notin \{1, 2\}$.

3. Can you draw the harm matrix from Montgomery (1972) of the unit diffusion coefficients for this exercise? **Solution key**:

$$
\begin{array}{c|cccccccccc}
 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{firms} & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

   Suppose, further, that $\sum_i l_{ij}^0 = 40\ \forall j$ and that the firms are identical in that the loss functions are the same and given by $F_i(e_i) = 10 - e_i(20 - e_i)$.

4. Can you derive the "first best" when the number of licenses are given at an arbitrary level? **Solution key**: No, this is not possible, because the "first best" requires that the number of licenses are optimally set, rather than being given at an arbitrary level.

5. How would you derive the second best when the number of licenses are given? **Solution key**: This is possible. Then, the solution is to maximize welfare (e.g., sum of utilities, or profits, or minimize the costs) given the number. Since $\sum_i l_{ij}^0 = 40$ for both locations, while location 2 receives most pollution, the constraint will only bind in location 2. Then, the problem is:

$$\min_{\{q_i\}_i} \sum_i F_i(q_i) = \min_{\{q_i\}_i} \sum_i [10 - q_i(20 - q_i)] \text{ s.t. } \sum_i 2q_i = 40.$$

Since all firms are identical, and $F_i(q_i)$ is convex, it is optimal that each firm emits the same amount, so $q_i = 2$.

6. If the firms take the permit prices as given, what is the firm's optimal purchase of licenses to pollute in the various districts, as a function of the permit prices? **Solution key**: A firm i will minimize the total costs:

$$\min_{\{l_{ij}\}_j} [10 - q_i(20 - q_i)] + \sum p_j \left(l_{ij} - l_{ij}^0\right) \text{ s.t. } q_i h_{ij} \leq l_{ij},$$

which leads to the solution:

$$q_i = 10 - \frac{p_1}{2} - p_2.$$

7. Can you characterize the equilibrium license prices and allocation of licenses? **Solution key**: Since $\sum_i l_{ij}^0 = 40$, the constraint will only bind in location 2, and then $q_i = 2$. Since the constraint does not bind in location 1, $p_1 = 0$. Then, for $q_i = 10 - \frac{p_1}{2} - p_2$ to hold when $q_i = 2$, we must have $p_2 = 8$.

8. How many license markets do we need for the market outcome to coincide with the efficient social outcome in this simple model? **Solution key**: The answer is 1, because firms' decisions are uniquely pinned down by

$$\sum p_j h_{ij} = p_1 + 2p_2.$$

Ex. 2: Capital mobility and the environment (based on Oates and Schwab 1988)

Consider the small country in Oates and Schwab (and in class 11/2) and assume it has not initially liberalized capital mobility.

1. Suppose both k and t are given. What is the consumer's optimal level of c and $\alpha = E/L$? **Solution key**: The consumer maximizes (since the wage is $f(k, \alpha) - kf_k'(k, \alpha))$):

$$u(c, E) \text{ s.t. } c = y + f(k, \alpha) - kf_k'(k, \alpha) + tk \text{ and } \alpha = E/L.$$

So, it boils down to one variable, since pinning down α determines c. The FOC for α is, if we just substitute in the equation for c and E:

$$
\begin{aligned}
u_c \frac{\partial c}{\partial \alpha} + u_E \frac{\partial E}{\partial \alpha} &= u_c \left[f'_\alpha (k, \alpha) - k f''_{k\alpha} (k, \alpha) \right] + L u_E = 0, \text{ so} \\
\frac{-u_E}{u_c} &= \frac{f'_\alpha (k, \alpha) - k f''_{k\alpha} (k, \alpha)}{L}.
\end{aligned}
$$

2. Suppose the consumer owns a fraction γ of the capital and the firm(s) and receives that share γ of the profit. What is now the optimal c and α in the closed economy? How does α depend on γ, and why? **Solution key**: In a closed economy, the per-capita profit is (assuming the product price is normalized to 1):

$$
f (k, \alpha) - \left[f (k, \alpha) - k f'_k (k, \alpha) \right] = k f'_k (k, \alpha),
$$

where the expression in the bracket is the wage, which is here the only expenditures of production. Assumption: I will assume for now that t is given. Then, the consumer solves:

$$
u (c, E) \text{ s.t. } c = y + f (k, \alpha) - k f'_k (k, \alpha) + tk + \gamma k f'_k (k, \alpha) \text{ and } \alpha = E/L.
$$

Again, this boils down to one variable, since pinning down α determines c. The FOC for α is, if we just substitute in the equation for c and E:

$$
\begin{aligned}
u_c \frac{\partial c}{\partial \alpha} + u_E \frac{\partial E}{\partial \alpha} &= u_c \left[f'_\alpha (k, \alpha) - k f''_{k\alpha} (k, \alpha) + \gamma k f''_{k\alpha} (k, \alpha) \right] + L u_E = 0, \text{ so} \\
\frac{-u_E}{u_c} &= \frac{f'_\alpha (k, \alpha) - (1 - \gamma) k f''_{k\alpha} (k, \alpha)}{L}.
\end{aligned}
$$

Since the RHS is increasing in γ, a larger γ means that $-u_E/u_c$ is larger, so consumption is larger and emission is larger when the consumer owns more of the firm/capital. We could repeat this exercise with endogenous/optimal t.

3. Now consider the open economy in which $\gamma = 0$ and the optimal tax was $t = 0$. Suppose the other countries in the world introduces a capital tax, t^*. What are the possible effects of t^* on our country's choice of environmental standard? **Solution key**: In this case, we had (in the class) the FOC:

$$
\frac{-u_E}{u_c} = \frac{F_E}{L} = \frac{f'_\alpha (k, \alpha)}{L},
$$

so emission is larger and consumption is larger in the open economy—assuming k is the same. The FOC is the same if hte other countries raise their capital taxes, but, everything else equal ,this will increase k in our home country. A larger k leads to more production, higher wages, and more consumption, for any given α and t. The positive taxes abroad does not change the conclusion that the optimal tax in the home country is zero, because before we derived this result for any fixed capital rate of return (r) abroad, and thus this result also holds if r is replaced by $r-t^*$. However, the larger k affects the FOC for α, above, since $f'_\alpha(k,\alpha)$ increases. This, in turn, increases $\frac{-u_E}{u_c}$. When $\frac{-u_E}{u_c}$ increases, the emission level in the home country or the consumption level in the home country must increase (or both). Since we have already concluded that c increases, so that u_c decreases and $1/u_c$ increases, we cannot conclude whether $-u_E$ will increase or decrease when t^* increases. Intuitively, more consumption and smaller u_c means that it is not so important to attract more capital/consumption, and this suggest that emission should decline. On the other hand, the larger k means that the benefit of allowing more pollution is higher, since emission and k are complements. In the end, the effect on emission will depend on the curvature of u w.r.t. c and E, as well as how strong complements emission and capital are in the production function.

4. Suppose a consumer's utility is $u(c,E)+\beta Z$, where $Z=tk$ is a public good financed by the capital taxes, and parameter $\beta>0$ measures the value of the public good. Can you describe the consumer's preferred allocation of c, α, t and the corresponding k and Z? **Solution key**: Assumption: I assume we do this in the open economy and that the consumer owns no capital or firm, just as in the paper and in class. The consumer's problem can be writtten as:

$$\begin{aligned} & \max u(c,L\alpha)+\beta tk \text{ s.t.} \\ c &\leq y+f(k,\alpha)-kf'_k(k,\alpha)+tk \\ f_k-t &\geq r \end{aligned}$$

so the KTL problem is:

$$\max u(c,L\alpha)+\beta tk-\lambda_1(c-(y+f(k,\alpha)-kf'_k(k,\alpha)))+\lambda_2(f_k-t-r)$$

which leads to the FOC wrt c, α, k and t:

$$\begin{aligned} u_c-\lambda_1 &= 0, \\ Lu_E+\lambda_1(f_\alpha(k,\alpha)-kf''_{k\alpha}(k,\alpha))+\lambda_2(f''_{k\alpha}) &= 0 \\ \beta t+\lambda_1(f'_k(k,\alpha)-f'_k(k,\alpha)-kf''_{kk}(k,\alpha))+\lambda_2 f''_{kk} &= 0 \\ \beta k-\lambda_2 &= 0 \end{aligned}$$

which we can solve:

$$\begin{aligned} Lu_E+u_c(f_\alpha(k,\alpha)-kf''_{k\alpha}(k,\alpha))+\beta k(f''_{k\alpha}) &= 0 \\ \beta t+(u_ck-\beta k)(-f''_{kk}) &= 0 \end{aligned}$$

$$\begin{aligned} \frac{-u_E}{u_c} &= \frac{f_\alpha(k,\alpha)+f''_{k\alpha}k(\beta-u_c)/u_c}{L} \\ t &= \frac{\beta-u_c}{-f''_{kk}}k \end{aligned}$$

so

$$\frac{-u_E}{u_c} = \frac{f_\alpha(k,\alpha) + t f''_{k\alpha}(-f''_{kk})/u_c}{L}$$

$$t = \frac{\beta - u_c}{-f''_{kk}} k$$

The second equation suggests that although $t = 0$ if $\beta = u_c$, as was the case in the lecture (since then the benefit from tk was that consumption could increase). Generally, t increases in β, which is the value of the public good. The first equation suggests that a larger t (because of the larger β) increases $\frac{-u_E}{u_c}$, so emission goes up / consumption up. Intuitively, consumption is not likely to go up since t increases above the level which is optimal without the public good (i.e, 0). So, emission goes up. The intuition is that to finance the public good, the consumer must increase the capital tax and to nevertheless attract capital to the country, one must permit more local emission (since more emission raises the return of capital).

Ex. 3: Self-enforcing agreements (based on lecture notes)

This exercise builds on the lecture notes available online (semestersiden) which we will discuss 25/2. It is a good preparation for that lecture to work through this exercise (but we may postpone discussing it in the seminar until a later seminar; that will depend on time).

Consider the repeated prisoner's dilemma where the benefit from consuming energy y_i is given by (Note that the benefit of energy is negative, such t hat it becomes increasing in y_i)

$$-\frac{b}{2}(Y_i - y_i)^2$$

where Y_i is some exogenous bliss point, and energy can come from fossil fuel g_i or renewable energy sources r_i

$$y_i = g_i + r_i.$$

You can threat r_i as exogenously fixed for the time being. The variable g_i does NOT need to be binary, as in the lecture, in this case it is a continuous variable.

There are n countries. Each unit of g_i gives the environmental cost c_i:

$$u_i = -\frac{b}{2}(Y_i - g_i - r_i)^2 - c_i \sum_{i=1}^{n} g_i.$$

Let δ be the common discount factor.

(1) What is the noncooperative equilibrium, or the "business as usual" equilibrium (the equilibrium if there were only one period) in this game?

(2) What is the first-best level for g_i?

(3) Suppose the equilibrium under (1) is the threat point which all countries revert to after a country has free-rided, or defected, in the repeated prisoner's dilemma game. Under which condition can the first best level for g_i be sustained? How does this possibility depend on r_i?

(4) Now, suppose the countries try to sustain a self-enforcing agreement with no pollution at all, $g_i = 0$. What is the condition for when this agreement is possible? How does this possibility depend on r_i? What is the smallest r_i which makes this agreement possible to sustain?

(5) Compare the r_i which you just derived to the first-best level of r_i for the agreement where every country emits zero ($g_i = 0$) when the investment cost is kr_i, and when the investment is made once and for all (before the repeated game starts, and never thereafter). For which countries are the condition you derived in (4) binding (i.e., requiring country i to invest a different amount in r_i than what is first best)?

(6) Return to the setting where r_i is exogenous. In fact, simplify by setting $r_i = r$, $c_i = c$, and $Y_i = Y$. Suppose the countries try to have a self-enforcing agreement where they emit only g. What is the smallest g which is possible to sustain as a self-enforcing agreement, under the threat that if one country deviates, then every country will play as in the business-as-usual equilibrium forever after? On what does this smallest level of g depend?