Environmental Economics 4910

Problem Set 3 (Ex. 3 in PS2)

Ex. 3: Self-enforcing agreements (based on lecture notes)
This exercise builds on the lecture notes available online (semestersiden) which we will discuss 25/2. It is a good preparation for that lecture to work through this exercise (but we may postpone discussing it in the seminar until a later seminar; that will depend on time).

Consider the repeated prisoner's dilemma where the benefit from consuming energy y_i is given by (Note that the benefit of energy is negative, such t hat it becomes increasing in y_i)

$$-\frac{b}{2}\left(Y_i - y_i\right)^2$$

where Y_i is some exogenous bliss point, and energy can come from fossil fuel g_i or renewable energy sources r_i

$$y_i = g_i + r_i.$$

You can threat r_i as exogenously fixed for the time being. The variable g_i does NOT need to be binary, as in the lecture. In this case it is a continuous variable.

There are n countries. Each unit of g_i gives the environmental cost c_i:

$$u_i = -\frac{b}{2}\left(Y_i - g_i - r_i\right)^2 - c_i \sum_{i=1}^{n} g_i.$$

Let δ be the common discount factor.

(1) What is the noncooperative equilibrium, or the "business as usual" equilibrium (the equilibrium if there were only one period) in this game?

Solution key: consumption:

$$\begin{aligned} b\left(Y_i - g_i^b - r_i\right) &= c_i \text{ or} \\ g_i^b &= Y_i - r_i - c_i/b. \end{aligned}$$

(2) What is the first-best level for g_i?
Solution key:

$$g_i^* = Y_i - r_i - \sum_j c_j/b.$$

It is useful to define C as $C = \sum_j c_j$.

(3) Suppose the equilibrium under (1) is the threat point which all countries revert to after a country has free-rided, or defected, in the repeated prisoner's

dilemma game. Under which condition can the first best level for g_i be sustained? How does this possibility depend on r_i?

Solution key: The benefit of cooperation forever must be higher than polluting as in BAU (while freeriding) once before everyone goes back to BAU:

$$\frac{-\frac{b}{2}\left(Y_i - g_i^* - r_i\right)^2 - c_i \sum g_j^*}{1-\delta} \geq -\frac{b}{2}\left(Y_i - g_i^b - r_i\right)^2 - c_i g_i^b - c_i \sum_{j \neq i} g_j^* + \delta \frac{-\frac{b}{2}\left(Y_i - g_i^b - r_i\right)^2 - c_i \sum g_j^b}{1-\delta}$$

$$\begin{aligned}
\frac{-C^2/2b}{1-\delta} &\geq -\frac{c_i^2}{2b} - c_i\left(g_i^b - g_i^*\right) + \delta \frac{-c_i^2/2b - c_i \sum\left(g_j^b - g_j^*\right)}{1-\delta} \\
\frac{-C^2/2b}{1-\delta} &\geq -\frac{c_i^2}{2b} - c_i\left(C - c_i\right)/b + \delta \frac{-c_i^2/2b - c_i\left(n-1\right)C/b}{1-\delta} \\
\frac{-C^2/2b}{1-\delta} &\geq \frac{c_i^2}{2b} - \frac{c_i C}{b} + \delta \frac{-c_i^2/2b - c_i nC/b + c_i C/b}{1-\delta} \\
-C^2 &\geq \left(1-2\delta\right)c_i^2 - 2\left(1-2\delta\right)c_i C - 2\delta n c_i C \\
\left(1-2\delta\right)\left(\frac{c_i}{C}\right)^2 &- 2\left(1 - 2\delta + \delta n\right)\frac{c_i}{C} + 1 \leq 0
\end{aligned}$$

which is more likely to hold when the discount factor is large. The role of r_i plays no role here, since a larger r_i is both reducing g_i under BAU and in the first best by the same amount.

(4) Now, suppose the countries try to sustain a self-enforcing agreement with no pollution at all, $g_i = 0$. What is the condition for when this agreement is possible? How does this possibility depend on r_i? What is the smallest r_i which makes this agreement possible to sustain?

Solution key: Above, we assumed $g_i^* = Y_i - r_i - \sum_j c_j/b \geq 0$, but it is possible that the RHS is negative, and if emissions cannot be negative, this would actually mean that the first best agreement is to emit zero.

With no pollution in the agreement, the compliance constraint is:

$$\frac{-\frac{b}{2}\left(Y_i - 0 - r_i\right)^2}{1-\delta} \geq -\frac{b}{2}\left(Y_i - g_i^b - r_i\right)^2 - c_i g_i^b - c_i \sum_{j \neq i} 0 + \delta \frac{-\frac{b}{2}\left(Y_i - g_i^b - r_i\right)^2 - c_i \sum g_j^b}{1-\delta}$$

$$\begin{aligned}
\frac{-\frac{b}{2}\left(Y_i - r_i\right)^2}{1-\delta} &\geq -\frac{c_i^2}{2b} - c_i\left(Y_i - r_i - c_i/b\right) + \delta \frac{-c_i^2/2b - c_i \sum\left(Y_j - r_j - c_j/b\right)}{1-\delta} \\
\frac{-\frac{b}{2}\left(Y_i - r_i\right)^2}{1-\delta} &\geq \frac{c_i^2}{2b} - c_i\left(Y_i - r_i\right) + \delta \frac{c_i^2/2b - c_i\left(Y_i - r_i\right) - c_i \sum_{j \neq i}\left(Y_j - r_j - c_j/b\right)}{1-\delta} \\
-\frac{b}{2}\left(Y_i - r_i\right)^2 &\geq \frac{c_i^2}{2b} - c_i\left(Y_i - r_i\right) - \delta c_i \sum_{j \neq i}\left(Y_j - r_j - c_j/b\right) \\
-\frac{b}{2}\left(Y_i - r_i\right)^2 + c_i\left(Y_i - r_i\right) &\geq \frac{c_i^2}{2b} - \delta c_i \sum_{j \neq i}\left(Y_j - r_j - c_j/b\right) \equiv L_i.
\end{aligned}$$

The equation holds with equality if:

$$\begin{aligned} Y_i - r_i &= \frac{c_i}{b} \pm \frac{1}{b}\sqrt{c_i^2 - 4\left(-\frac{b}{2}\right)(-L_i)} \\ Y_i - r_i &= \frac{c_i}{b} \pm \sqrt{\left(\frac{c_i}{b}\right)^2 - \frac{2L_i}{b}} \end{aligned}$$

So, more generally, the inequality above holds when

$$Y_i - r_i \in \left[\frac{c_i}{b} - \sqrt{\left(\frac{c_i}{b}\right)^2 - 2L_i}, \frac{c_i}{b} + \sqrt{\left(\frac{c_i}{b}\right)^2 - \frac{2L_i}{b}}\right],$$

and the smallest r_i means that

$$\begin{aligned} Y_i - r_i &= \frac{c_i}{b} + \sqrt{\left(\frac{c_i}{b}\right)^2 - \frac{2L_i}{b}}, \text{ or} \\ r_i &= Y_i - \frac{c_i}{b} - \sqrt{2\delta\frac{c_i}{b}\sum_{j\neq i}(Y_j - r_j - c_j/b)}. \end{aligned}$$

So, the larger is the discount factor, the smaller is the required r_i!

(5) Compare the r_i which you just derived to the first-best level of r_i for the agreement where every country emits zero ($g_i = 0$) when the investment cost is kr_i, and when the investment is made once and for all (before the repeated game starts, and never thereafter). For which countries are the condition you derived in (4) binding (i.e., requiring country i to invest a different amount in r_i than what is first best)?

Solution key: The "second best" r_i given that emissions are zero is the level that maximizes:

$$\frac{-\frac{b}{2}(Y_i - 0 - r_i)^2}{1-\delta} - kr_i,$$

which gives:

$$\begin{aligned} b(Y_i - r_i) &= (1-\delta)k, \text{ or} \\ r_i &= Y_i - (1-\delta)k/b. \end{aligned}$$

The investment level in (4) is larger iff:

$$Y_i - \frac{c_i}{b} - \sqrt{2\delta\frac{c_i}{b}\sum_{j\neq i}(Y_j - r_j - c_j/b)} > Y_i - (1-\delta)k/b$$

$$(1-\delta)k/b - \frac{c_i}{b} > \sqrt{2\delta\frac{c_i}{b}\sum_{j\neq i}(Y_j - r_j - c_j/b)}$$

which is more likely to hold when c_i is small, and when the other countries' bliss points for consumption is small, and when the other countries technologies are large (since, in these cases, the other countries will not pollute much anyway).

(6) Return to the setting where r_i is exogenous. In fact, simplify by setting $r_i = r$, $c_i = c$, and $Y_i = Y$. Suppose the countries try to have a self-enforcing agreement where they emit only g. What is the smallest g which is possible to sustain as a self-enforcing agreement, under the threat that if one country deviates, then every country will play as in the business-as-usual equilibrium forever after? On what does this smallest level of g depend?

Solution key:

$$\frac{-\frac{b}{2}(Y-g-r)^2 - ncg}{1-\delta} \geq -\frac{b}{2}\left(Y-g_i^b-r\right)^2 - cg_i^b - (n-1)\,cg + \delta\frac{-\frac{b}{2}\left(Y-g_i^b-r\right)^2 - cng_i^b}{1-\delta}$$

$$\frac{-\frac{b}{2}(Y-g-r)^2 - ncg}{1-\delta} \geq -\frac{c^2}{2b} - c\,(Y-r-c/b) - (n-1)\,cg + \delta\frac{-c^2/2b - cn\,(Y-r-c/b)}{1-\delta}$$

$$\frac{-\frac{b}{2}(Y-g-r)^2 - cg\,(n-(n-1)(1-\delta))}{1-\delta} \geq -\frac{c^2}{2b} - c\,(Y-r-c/b) + \delta\frac{-c^2/2b - cn\,(Y-r-c/b)}{1-\delta}$$

As before, this is a second-order condition where the LHS is hump-shaped in g, so g must be within an interval. The smallest g in this interval will be an equation which depends on all the other variables, and, in particular:

- If the discount factor is larger, the smallest possible g is smaller.
- If the technology level is larger, the smallest possible g is smaller.
- If n is larger, the smallest possible g is smaller.