Problem Set 4 with Solution Key - ECON2910
Legally Binding Environmental Agreements

Take the model in the lecture notes, but suppose that there is only one period, and no uncertainty/shock, and no technological spillover, so $e = 0$.

1. What is the difference between assuming $\delta = 0$ and assuming that there is only one period? **Solution key**: *assuming $\delta = 0$ means that the players do not care about the next period, so they are essentially playing a one-period game. However, the stocks may depend on the history.*

2. Given technology stocks, the R_i's, what is the socially optimal emission levels for each country i? (I.e., the g_i which maximizes the sum of surplus?) **Solution key**:

$$\max_{g_i} \sum_{i=1}^{n} \Big\{ \sum_t \delta^t u_{i,t} \Big\} \tag{1}$$

$$\max_{g_i} \sum_{i=1}^{n} \Big\{ \sum_t \delta^t \Big(-\frac{b}{2}\big(\overline{y}_i - g_{i,t} - (q_R R_{i,t-1} + r_{i,t})\big)^2 - \frac{c}{2}\big(q_G G_{t-1} + \sum_{i=1}^{n} g_{i,t}\big)^2 - k r_{i,t}\Big) \Big\} \tag{2}$$

assume $t = 0$, with FOC:

$$g_i^* (\mathbf{R}) = \overline{y}_i - R_i - \frac{cn\,(n\overline{y} + q_G G_- - R)}{b + cn^2}$$

So, a country should emit less if R_i is large or if R (given R_i) is small.

3. Take now instead g_i as given (because they have been negotiated, for example). What is then the *equilibrium* level for R_i? **Solution key**: *Take an individual's FOC wrt r_i to get:*

$$b\,(\overline{y}_i - g_{i,t} - R_i) - k = 0.$$

4. Instead of assuming that countries negotiate the g_i's but not the r_i's, suppose instead the reverse, that countries negotiated the r_i's efficiently, but the g_i's are noncooperatively chosen. Is this situation realistic? Beneficial? What is the equilibrium? **Solution key**: *Realistic? Well, for climate agreements we do see this type of agreement when it comes to Kyoto and Paris, but there exists coalitions that focuses on technology sharing rather than emission agreements. And, for other environmental problems, it can well be that emissions are hard to monitor and regulate, while technological investments are simpler to monitor and regulate, so this situation can indeed be relevant. Beneficial? This situation seems better than BAU because countries will invest more. We know that short-term agreements can be worse than BAU, so it is possible that these types of tech agreements can be better than short-term agreements on emission levels, but exactly when that will be the case requires us to derive the payoffs for both types of agreements, and then to compare these payoss. Equilibrium? To solve each period by backwards induction, the FOC for g is as in BAU (see lecture notes but modify them by removing the continuation value, which you can do by f.ex. setting $\delta = 0$):*

$$g_i^b (\mathbf{R}) = \overline{y} - R_i - \frac{c\,(n\overline{y} + q_G G_- - R)}{b + cn}.$$

So, the effect of investing in R_i is that emissions will be reduced, and that consumption will increase (for everyone). Basically, we know that in BAU, R_i is basically a public good. In BAU, a country set marginal cost (k) of investing equal to the marginal benefit, giving (from the lecture notes):

$$\begin{aligned} r_i &= \overline{y} - \frac{q_R}{n} R_- + \frac{q_G}{n} G_- \\ &\quad - k \left(\frac{1}{cn^2} + \frac{1}{b} \right). \end{aligned}$$

If the agreement is on the r_i's, the marginal cost k should be equal to n multiplied with the private benefits, or, equivalently, the same equation as above if just k is divided by n, giving:

$$\begin{aligned} r_i^* &= \overline{y} - \frac{q_R}{n} R_- + \frac{q_G}{n} G_- \\ & -\frac{k}{n}\left(\frac{1}{cn^2} + \frac{1}{b}\right). \end{aligned}$$

5. Consider now the dynamic model where $\delta \in (0,1)$. For simplicity, suppose that the function $B(y_i)$ is so convex, or is kinked, so that the consumption level is completely fixed at some $y_i = \overline{y}$ (so, B is not quadratic). In each period, what is the first-best level of technology investments? What is the first-best pollution level? **Solution key**: *Marginal (net) costs should equal marginal benefit, meaning that in every period, R_t should satisfy*

$$k(1-\delta q_R) = n \cdot c \cdot (q_G G_{t-1} + n\overline{y} - R_t^*).$$

This gives R_t^*,

$$R_t^* = q_G G_{t-1} + n\overline{y} - \frac{k(1-\delta q_R)}{cn}$$

and $r_{i,t}$ as a function of R_{t-1},

$$r_{i,t}^* = \frac{1}{n}\left[q_G G_{t-1} + n\overline{y} - \frac{k(1-\delta q_R)}{cn} - q_R R_{t-1}\right].$$

And pollution is

$$G^* = q_G G_{t-1} + n\overline{y} - R_t = \frac{k(1-\delta q_R)}{nc}.$$

6. With the assumptions from question 5, can you show that there exists an MPE where R_t is the only (payoff-relevant) stock which strategies are conditioned on, and that in this equilibrium, every country is, at each point in time, investing the same amount as any other country? What is the equilibrium technology and pollution level in this equilibrium? **Solution key**: *Only the environmental cost of one country will be taken into account, and every country will also know that by investing now, the cost saving, which is $\delta q_R k$, which comes from the fact that every country can invest less in the next period, will be split on n countries. Thus, investments satisfy:*

$$k(1-\delta q_R/n) = c \cdot \left(q_G G_{t-1} + n\overline{y} - R_t^b\right).$$

This gives R_t^b

$$R_t^b = q_G G_{t-1} + n\overline{y} - \frac{k(1-\delta q_R/n)}{c}$$

and $r_{i,t}$ as a function of R_{t-1},

$$r_{i,t}^b = \frac{1}{n}\left[q_G G_{t-1} + n\overline{y} - \frac{k(1-\delta q_R/n)}{c} - q_R R_{t-1}\right].$$

And pollution is:

$$G^b = q_G G_{t-1} + n\overline{y} - R_t = \frac{k(1-\delta q_R/n)}{c},$$

which is naturally larger than the first best when $n > 1$.

7. For which discount factors are the first best, derived in 5, possible to sustain in a SPE? Assume here that the players can use trigger strategies where the punishment (after a country does not cooperate) is that the countries revert to the MPE in the game. **Solution key**: *Here you must derive utilities in the first best, utilities in the MPE from point 4, and write the "compliance constraint" and show that it is satisfied if and only if the discount factor is sufficiently large. For all these, the benefit*

$B(.)$ will be a constant, say, equal to B, in all equilibria, given the assumptions on the kink. The utility in the first best can thus simplify to the LHS in:

$$\frac{B - \frac{c}{2}(G^*)^2 - kr_i^*}{1-\delta},$$

and, with first-best also in the last period, we can easily calculate r_i^* from the above formulae:

$$\begin{aligned}
r_i^* &= \frac{1}{n}\left[q_G G_{t-1}^* + n\overline{y} - \frac{k(1-\delta q_R)}{cn} - q_R R_{t-1}^*\right] \\
&= \frac{1}{n}\left[q_G G_{t-1}^* + n\overline{y} - \frac{k(1-\delta q_R)}{cn} - q_R\left[q_G G_{t-1}^* + n\overline{y} - \frac{k(1-\delta q_R)}{cn}\right]\right] \\
&= \frac{1}{n}\left[q_G G_{t-1}^* + n\overline{y} - \frac{k(1-\delta q_R)}{cn}\right](1-q_R) \\
&= \frac{1}{n}\left[q_G \frac{k(1-\delta q_R)}{nc} + n\overline{y} - \frac{k(1-\delta q_R)}{cn}\right](1-q_R) \\
&= \frac{1}{n}\left[n\overline{y} - \frac{k(1-\delta q_R)(1-q_G)}{cn}\right](1-q_R).
\end{aligned}$$

The compliance constraint is:

$$\begin{aligned}
\frac{B - \frac{c}{2}(G^*)^2 - kr_i^*}{1-\delta} &\geq B - \frac{c}{2}\left(G^b\right)^2 - kr_i^d - \delta\left[B - \frac{c}{2}\left(G^b\right)^2 - kr_i^{dd}\right] \\
&\quad -\delta^2 \frac{B - \frac{c}{2}\left(G^b\right)^2 - kr_i^b}{1-\delta} \qquad \text{(CC)}
\end{aligned}$$

where the RHS measures the PDV of "defecting". Since investment cost is linear, defection means that the defecting country i invests zero or so little that G_t becomes G_t^b, according to the FOC's above. This means that R_t becomes equal to R_t^b:

$$R_t^b = q_G G^* + n\overline{y} - \frac{k(1-\delta q_R/n)}{c},$$

given that G_{t-1}^* was first best in the previous period. Given that the other countries cooperate and invest r_i^*, a defecting country invests such that

$$\begin{aligned}
q_R R_{t-1} + (n-1)r_i^* + r_i^d &= R_t^b, \text{ or} \\
r_i^d &= r_t^* - \left(R_t^* - R_t^b\right) \\
&= \frac{1}{n}\left[n\overline{y} - \frac{k(1-\delta q_R)(1-q_G)}{cn}\right](1-q_R) + \frac{k(1-\delta q_R)}{cn} - \frac{k(1-\delta q_R/n)}{c}.
\end{aligned}$$

Here, I will assume that even this requires i to invest a positive amount (or, we could assume that investments can be negative, as if i can use the capital for other purposes). In the next period, the pollution stock will be as in BAU, and the technology will be:

$$R_{t+1}^b = q_G G^b + n\overline{y} - \frac{k(1-\delta q_R/n)}{c}$$

and to achieve this, given R_t^b, investments must be:

$$\begin{aligned}
r_{i,t+1}^{dd} &= \frac{1}{n}\left(R_{t+1}^b - q_R R_t^b\right) \\
&= \frac{1}{n}\left(q_G G^b + n\overline{y} - \frac{k(1-\delta q_R/n)}{c} - q_R\left(q_G G^* + n\overline{y} - \frac{k(1-\delta q_R/n)}{c}\right)\right).
\end{aligned}$$

So, at t, when i defects, R_t becomes very small since G^* was small in the previous period. At $t+1$, therefore, investments must increase to $r_{i,t+1}^{dd}$, and R_{t+1} becomes constant from then on, and *investments fall to* r_t^b *from* $t+2$. *All these solutions can be inserted into (CC) and thereafter we can solve (CC) wrt* δ *to see what discount rate it takes to make the first best self-enforcing (doing this is messy and technical but it is just algebra). This might be too messy unless you make additional assumptions, such as setting . Thus, the purpose of this excercise is NOT to work on the messy algebra, but to illustrate how much more complicated the analysis of self-enforcing agreement can be if we generalize the model by not looking at the repeated-game setting (i.e., if stocks accumulate over time).*