Problem Set - ECON4910 (for March 27, 2019)

1. Supply-side policies.

Consider the model of Hoel '94 discussed in class, and also the extension by Golombek, Hagem and Hoel (1995), also briefly mentioned, where emission from country i was given by some function $E_i(x_i)$.

In class, we discussed the optimal extraction tax in a coalition/country M.

(i) Derive instead the optimal emission tax in M if the supply industry in M must pay an emission tax.

Solution key: *From the lecture slides, when we discussed this extension:*

$$\tau_x = \left(E'_M(x_M) - \frac{\sum_N E'_i(x_i) S'_i(p)}{S'(p) - D'(p)} \right) H' - \frac{y_M - x_M}{S'(p) - D'(p)}.$$

Thus, when τ_x is an extraction tax, then the suppliers face the marginal cost τ_x when increasing x_M. With an emission tax is proportional to $E_{X_M}(x_M)$, say $t_x E_{X_M}(x_M)$, then, when x_M is increased, suppliers will face the marginal cost $t_x E'_M(x_M)$. The two are equal if $t_x = \tau_x / E'_{X_M}(x_M)$, or:

$$t_x = \left(1 - \frac{\sum_N S'_i(p) E'_i(x_i) / E'_M(x_M)}{S'(p) - D'(p)} \right) H' - \frac{y_M - x_M}{S'(p) - D'(p)} \frac{1}{E'_M(x_M)}.$$

(ii) How does this tax depend on $E'_M(x_M)$?

Solution key: *The first term increases in the marginal damage, but the second term depends on whether M exports or imports. In any case, the effect of the last term, i.e., the one on terms-of-trade, is smaller if M's supply is having a large environmental damage.*

(iii) How does it depend on $E'_i(x_i)$, $i \neq M$? Explain the intuition.

Solution key: *If i's emission damage is large, then M's emission tax on consumers should be smaller since M's supply replaced i's supply.*

2. Deforestation (based on online lecture notes to be discussed March 25, 2019)

Use the model presented in class of illegal logging (set $b = 0$), but suppose there is a single country ($n = 1$) and no donor. Also, suppose the government cannot have a different expected penalty in one part of the forest than in another: The expected penalty must be θ everywhere.

(i) How does the uniform (!) level of θ influence x?

Solution key: *One cuts IFF $p > \theta$*, so:

$$\begin{aligned} x &= 0 \text{ if } \theta \geq \overline{p}, \\ x &= X \text{ if } \theta < \overline{p} - aX, \\ x &= \frac{\overline{p} - \theta}{a} \text{ when } \theta = \overline{p} - ax \text{ for } x \in (0, X). \end{aligned}$$

(ii) What is the optimal θ? What utility will the country C then get?

Solution key: *The country chooses x to maximize:*

$$\begin{aligned} -cpX - vx &= -c(\overline{p} - ax)X - vx, \text{ so} \\ x &= 0 \text{ if } caX < v, \text{ and} \\ x &= X \text{ if } caX > v. \end{aligned}$$

which is linear in x, and thus C decides between $x = 0$ and $x = X$. If $x = 0$, $C's$ payoff is $-c\overline{p}X$.If nothing is protected, C's payoff is: $-vX$. By comparison, conservation is strictly better iff $v > c\overline{p}$, which is intuitive.

(iii) Suppose the country has two distinct forests of equal size, A and B, and that θ_A and θ_B can be different, but the expected penalty must be the same within each of these two forests.

When it is optimal that they differ, and what should they be?

Solution key: *Let $X/2$ be the size of each forest. If one one forst is protected, then C's payoff is:*

$$\begin{aligned} -cpX/2 - vX/2 &= -c(\overline{p} - aX/2)X/2 - vX/2 \\ &= ac(X/2)^2 - (c\overline{p} + v)X/2, \end{aligned}$$

which gives highest payoff to C iff:

$$\begin{aligned} ac(X/2)^2 - (c\overline{p} + v)X/2 &> -c\overline{p}X \text{ and} \\ ac(X/2)^2 - (c\overline{p} + v)X/2 &> -vX, \text{ or} \end{aligned}$$

$$\begin{aligned} ac(X/2) &> (v - c\overline{p}) \text{ and} \\ ac(X/2) &> -(v - c\overline{p}), \end{aligned}$$

and, then, the protected part must have $\theta = \overline{p} - aX/2$.

(vi) Suppose the country's forest can be divided up in any way you want, and that in unit j of the forest, θ_j can be set different than in any other unit.

Derive the optimal θ_j, explain why (if) they may differ in j, and derive the country's utility. Compare that utility to the utility in (ii) and (iii) and explain why it is higher:

Solution key: *This is exactly what we did in the lecture. Then, C's payoff is (assuming $x \in (0, X)$):*

$$-c(\overline{p} - ax)(X - x) - vx.$$

Since C's payoff is hump-shaped and concave in x, the highest payoff (i.e., the peak of the utility function) will be at some x, which depends on the parameters. That peak is larger than C's payoff at any other x, including $x = 0$ and $x = X$, analyzing in (ii), and $x = X/2$, analyzed in (iii).

F.ex., C's payoff is maximized at the FOC:

$$\begin{aligned} ca\left(X - x\right) + c\left(\overline{p} - ax\right) - v &= 0, \text{ or} \\ x &= \frac{caX + c\overline{p} - v}{2ca}. \end{aligned}$$

Inserted into C's payoff, we get:

$$\begin{aligned} &-c\left(\overline{p} - a\frac{caX + c\overline{p} - v}{2ca}\right)\left(X - \frac{caX + c\overline{p} - v}{2ca}\right) - v\frac{caX + c\overline{p} - v}{2ca} \\ = &\left(-c\overline{p} + \frac{caX + c\overline{p} - v}{2}\right)\left(\frac{X}{2} - \frac{c\overline{p} - v}{2ca}\right) - v\frac{caX + c\overline{p} - v}{2ca}. \end{aligned}$$

with some algebra, it is easy to compare them. See figure discussed in the seminar!