Dynamic Climate Change Games

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Outline

a. Games with stocks: Dynamic common-pool problems

b. Markov-perfect equilibria as "business as usual":

c. Short-term agreements (that are 'legally binding') and hold-up problems

d. Long-term contracts

e. Optimal duration

(Lecture notes permit technological spillovers (IPR/tariffs), more general functional forms, heterogeneity, and renegotiation)

Motivation

- Most environmental problems are dynamic in nature
  - Pollution accumulates over *time*
- Technological solutions are also relevant
  - Takes *time* to develop
- Countries/district often act independently
  - Must study the *game* between them
- Agreements may be 'legally binding'
- But agreements might be made on *some* aspects (like quotas)
  - ...but not everything of interest (like investments)
- Countries that have invested (e.g., Denmark) may then be required to contribute more.
- How should such an *incomplete contract* look like?
- What will it be in *equilibrium*?

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- In resource/environmental economics, the typical state is the **stock(s)** of resource or pollution.
- Note that the stock may or may not be "payoff relevant"

a. ... Stocks..

- We can reformulate last week's model to allow for stocks
- Consider a pollution stock $G_t = q_G G_{t-1} + \sum_j g_{j,t}$ with marginal cost C, and a technology stock $R_{i,t} = q_R R_{i,t-1} + r_{i,t}$, where the investment $r_{i,t}$ has the marginal cost K:

$$\widetilde{u}_{i,t} = B\left(g_{i,t}, R_{i,t}\right) - CG_t - Kr_{i,t},$$

- If we define $c \equiv C / \left(1 - \delta q_G\right)$ and $k \equiv K\left(1 - \delta q_R\right)$, maximizing $\widetilde{u}_{i,t}$ is equivalent to maximizing $u_{i,t}$, defined as: .

$$u_{i,t} = B\left(g_{i,t}, r_{i,t}\right) - c \sum g_{j,t} - kr_{i,t}.$$

- In this way, the game with stocks can be reformulated to a repeated game.
- This transformation is not possible if the stocks are "payoff relevant."

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  - So, states/stocks that are not payoff-relevant should not matter.

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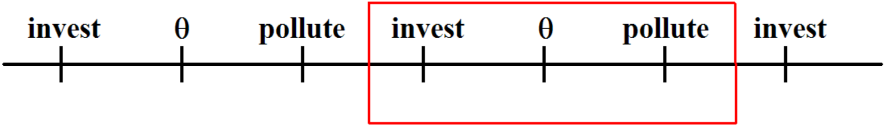
- Robust to, for example, finite time.
- We will search for a 'symmetric' MPE.

a. Model: Timing



a. A Model

- A model with $n+1$ stocks:

$$\begin{aligned}
U_{i,t} &\equiv \sum_{\tau=t}^{\infty} u_{i,\tau}\delta^{\tau-t} \\
u_{i,t} &\equiv B\left(g_{i,t}+R_{i,t}\right)-C\left(G_t\right)-kr_{i,t} \\
R_{i,t} &= q_R R_{i,t-1}+r_{i,t}, \\
G_t &= q_G G_{t-1}+\sum g_{i,t}+\theta_t,\ \ i\in\{1,2,...,n\} \\
\theta_t &\sim F\left(0,\sigma^2\right)
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- Continuation values,

$$
U_{i,t}\left(G_{t-1},R_{1,t-1},...,R_{n,t-1}\right),W_{i,t}\left(q_G G_{t-1}+\theta_t,R_{1,t},...,R_{n,t}\right)
$$

a. Simplifications

- Simplifying the model. If:

$$
\begin{aligned}
y_{i,t} &\equiv g_{i,t} + R_{i,t} \text{ and } R_t \equiv \sum R_{i,t}\text{, then:} \\
u_{i,t} &= B\left(y_{i,t}\right) - C\left(G_t\right) - kr_{i,t} \\
G_t &= q_G\, G_{t-1} + \sum y_{i,t} - R_t + \theta_t \\
R_t &= q_R\, R_{t-1} + \sum r_{i,t} \\
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$$\begin{aligned} y_{i,t} &\equiv g_{i,t} + R_{i,t} \text{ and } R_t \equiv \sum R_{i,t}, \text{ then:} \\ u_{i,t} &= B\left(y_{i,t}\right) - C\left(G_t\right) - kr_{i,t} \\ G_t &= q_G G_{t-1} + \sum y_{i,t} - R_t + \theta_t \\ R_t &= q_R R_{t-1} + \sum r_{i,t} \\ \theta_t &\sim F\left(0, \sigma^2\right) \end{aligned}$$

- Write continuation values as $U(G_{t-1}, R_{t-1})$ and $W(q_G G_{t-1} + \theta_t, R_t)$.

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- **Lemma 0:** *Markov strategies depend only on* G_{t-1} *and* $R_{t-1} \equiv \sum_i R_{i,t-1}$
- So, same $y_{i,t}$ for all, even if $R_{i,t}$ differ.
- $R_{i,t}$ and R_t is a "public good" – even with no technological spillovers.

b. Business as Usual - Lemma 1

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$$\begin{aligned}
\max_{r_{i,t}} \mathrm{E}W(q_G G_{t-1} + \theta_t, q_R R_{t-1} + \sum r_{i,t}) - k r_{i,t} \quad &\Rightarrow \\
\mathrm{E}W_R\left(G_{t-1}, R_t\right) = k \Rightarrow R_t\left(G_{t-1}\right), \text{ so} \\
U^b\left(G_{t-1}, R\left(G_{t-1}\right)\right) = W\left(G_{t-1}, R\left(G_{t-1}\right)\right) \\
-\frac{k}{n}\left[R_t\left(G_{t-1}\right) - q_R R_{t-1}\right] \quad &\Rightarrow \\
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$$\max_{r_{i,t}} \mathrm{E}W(q_G G_{t-1} + \theta_t, q_R R_{t-1} + \sum r_{i,t}) - kr_{i,t} \Rightarrow$$

$$\mathrm{E}W_R(G_{t-1}, R_t) = k \Rightarrow R_t(G_{t-1}), \text{ so}$$

$$U^b(G_{t-1}, R(G_{t-1})) = W(G_{t-1}, R(G_{t-1})) - \frac{k}{n}\left[R_t(G_{t-1}) - q_R R_{t-1}\right] \Rightarrow$$

$$U_R^b = q_R k / n.$$

- *Note:* Since U_R is a constant, $U_{GR} = 0$, and U_G does not depend on R.$\|$

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- So $y_{i,t} = y_t$ is a function of $\xi_t + \theta_t$ where $\xi_t \equiv q_G G_{t-1} - R_t$, and so is G_t. Inserted, the foc for R_t comes from:

$$\max_{r_{i,t}} \mathrm{E}\left[B\left(y\left(\xi\right)\right) - C\left(G\left(\xi\right)\right) + \delta U\left(G\left(\xi\right), R_t\right)\right] - kr_{i,t} \quad (2)$$

which gives the foc, determining $\xi_t = \xi^b$ as a constant:

$$-\mathrm{E}\left[B'\left(y\left(\xi\right)\right)y'\left(\xi\right) - C'\left(G\left(\xi\right)\right)G'\left(\xi\right) + \delta U_G G'\left(\xi\right)\right] + \delta U_R = k. \quad (3)$$

b. Business as Usual - L2 - Proof Continued

- In the symmetric equilibrium:

$$
\begin{aligned}
U(G,R) &= \mathrm{E}B(y(\xi)) - \mathrm{E}C(G(\xi)) \\
&\quad -\frac{k}{n}\left[q_G G_{t-1} - \xi - q_R R_{t-1}\right] \\
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- Simplify further:
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  - **Example Q** (which we will stick to today):

$$B(y_{i,t}) = -\frac{b}{2}\left(\bar{y} - y_{i,t}\right)^2, \; C(G_t) = \frac{c}{2}G_t^2 \tag{Q}$$

b. Business as Usual - Emissions

Proposition

- *The FOC at the g-stage can be written as:*

$$g_i^b(\mathbf{R}) = \overline{y} - R_i - \frac{c\left(n\overline{y} + q_G G_- + \theta - R\right) + \delta q_G \left(1 - \delta q_R\right) k/n}{b + cn}$$

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- *The FOC at the r-stage can be written as:*

$$\begin{aligned} r_i^b &= \overline{y} - \frac{q_R}{n} R_- + \frac{q_G}{n} G_- \\ &\quad - k\left[\frac{\left(b + cn\right)^2}{cb\left(b + c\right)n}\left(1 - \frac{\delta q_R}{n}\right) - \left(1 - \delta q_R\right)\frac{\delta q_G}{cn^2}\right] \end{aligned}$$

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  - *If i pollutes a lot, every country is subsequently investing more.*

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- *Anticipating this:*
    - Investments decrease
    - Emissions increase
- A dynamic common pool problem that is **worse** than its static counterpart

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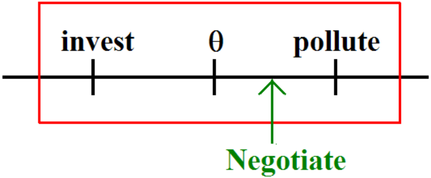
- *By comparison,*

$$r_i^b < r_i^* \text{ and } g_i^b\left(\mathbf{R}^b\right) > g_i^*\left(\mathbf{R}^b\right) > g_i^*\left(\mathbf{R}^*\right).$$

c. Short-Term Agreements



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- But what are the equilibrium (noncooperatively set) investment levels?

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- *Short-term agreements are always worse when the duration is sufficiently short (i.e., when $\delta q_R \to 1$ and $\sigma \to 0$).*

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- Agreements can be harmful ex ante - because they reduce incentives to invest.
  - But when investments are sunk, the agreement is always better than BAU.

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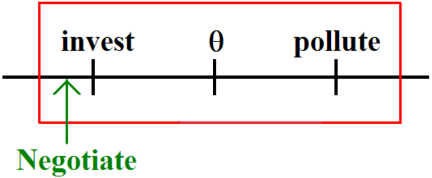

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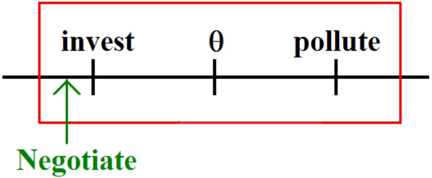

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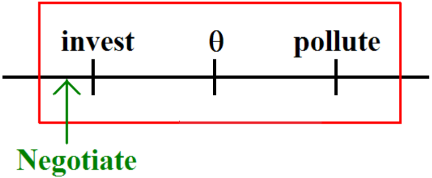

- The timing is reversed: g_i is negotiated first, then i invests.
- With this timing, there is no hold-up problem in this period.
- But **still underinvestments** - especially when the *next* bargaining round is near.

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Proposition

- *When the level g_i^{lt} is already committed to, the first-order condition for i's investment is:*

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- *Investments are suboptimally small, especially if n and δ are larger.*

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- *But since $r_i^{lt}(g_i) < r_i^{*}(g_i)$, the emission levels are suboptimally small "ex post", given the equilibrium technology level:*

$$g_i^{lt} < Eg_i^{*}\left(\mathbf{R}^{lt}\right).$$

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  - The shorter is the duration, the smaller are investments, and the smaller are the optimal quotas relative to the ex post optimal emission levels.

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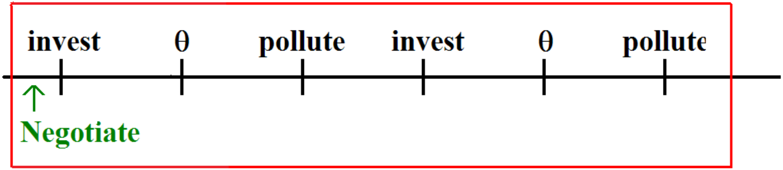

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