Discounting

Lecture slides

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2019

Pay C to later get B?

Public Investments with Long-term Consequences

- Abate, reduce emission, recycle
- Conserve exhaustible/renewable resources
- Infrastructure (windmills, roads, bridges)
- Technology
- Academic research, knowledge

The Standard Approach

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- Koopman (1960): axiomatic foundation
- "the simplicity and elegance of this formulation was irresistible" and the criterion became "dominant... largely due to its simplicity... not as a result of empirical research demonstrating its validity" (Frederick et al, '02: 355-6;352-3)

1. Continuity
2. Sensitivity
3. Non-Complementarity
4. Stationarity
5. Boundedness

- Koopmans (1960): With 1-5, $v_0 = \sum_{t=0}^{\infty} \delta^t u_t$.

The Value of a future dollar (in cents today)

interest rate \ years:	**50**	**100**	**200**
$r = 1\%$	60	37	13
$r = 4\%$	13	1,8	0,03
$r = 8\%$	1,8	0,03	0,00001

Stern-review vs Nordhaus: debate on interest rate.

Ramsey's social discount rate for consumption

- A dollar at time t has the same value as $a(t) \equiv e^{-rt}$ dollars today (time 0) if:

$$\begin{aligned} a(t)\, u'(c_0) &= e^{-\rho t} u'(c_t) \Rightarrow \\ \frac{a'(t)}{a(t)} &= -\rho + \frac{u''(c_t)}{u'(c_t)} c_t \frac{\partial c_t / \partial t}{c_t} \Rightarrow \\ r &= \rho + \eta_t \mu_t . \end{aligned}$$

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- This used to be the recommendation in Norwegian public cost-benefit analysis.
- Note that with CRRA (constant relative risk aversion); $u(c) = c^{1-\eta} / (1-\eta)$, then $u''(c_t)\, c_t / u'(c_t) = -\eta$.

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- Suppose $y_t \equiv \sum_{\tau=1}^{t} \mu_\tau$ is uncertain and distributed as $f(y_t)$.
- The expected future value of a dollar is today worth:

$$a(t) = \mathrm{E} \exp\left(-\rho t - \eta \sum_{\tau=1}^{t} \mu_\tau\right) = \int e^{-\rho t - \eta y} f(y)\, dy.$$

The discount rate under uncertainty - continued

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- May well be negative.

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- Thus, for the far-distant future, apply $\lim_{t\to\infty} R(t) = r_1 = \min_j r_j$.

Revisiting The Standard Approach

- Samuelson careful: "It is completely arbitrary to assume that the individual behaves so as to maximize an integral of [this] form". And: "any connection between utility as discussed here and any welfare concept is disavowed" (Samuelson '37: 159;161)

$$v_0 = \int_{t=0}^{\infty} e^{-\rho t} u_t dt \approx \sum_{t=0}^{\infty} e^{-\rho t} u_t = \sum_{t=0}^{\infty} \delta^t u_t .$$

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- But how?

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- **Intuitively:** The difference between t and $t+1$ vanishes as t grows

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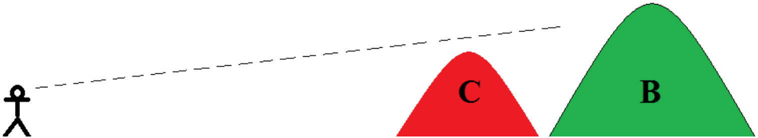

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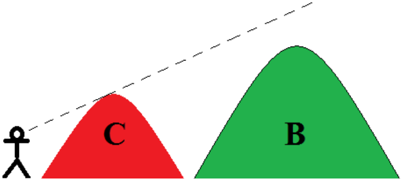

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- Phelps and Pollak (1968): "Imperfect altruism" between generations. David Laibson adopts this function to within-lifetime choices.
- Alternative names: (β, δ)-discounting, quasi-geometric discounting, quasi-exponential discounting, hyperbolic discounting,present bias.

Quasi-hyperbolic discounting and the environment

- With quasi-hyperbolic discounting (requiring discrete time):

$$w_t = u_t + \beta \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} u_\tau .$$

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- With commitment at time t, the best plan is to emit as follows at future time $\tau > t$:

$$\begin{aligned} B_\tau'\left(g_\tau^{co}\right) - c &= \frac{c\delta q}{1-\delta q} > \frac{c\beta\delta q}{1-\delta q} \Rightarrow \\ g_\tau^{co} &< g_\tau^{eq}. \end{aligned}$$

Quasi-hyperbolic discounting and time inconsistency

- It is always best to start polluting little (g_τ^{co}) tomorrow.

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  - By reducing investments in "brown technology" which would have increased the marginal benefit of emitting in the future.