Environmental Economics 4910

Bård Harstad

UiO

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Bård Harstad (UiO)

Repeated Games and SPE

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Outline

- a. Concepts
- b. Repeated games and Folk theorem
- c. Repeated games with emission and pollution
- d. Continuous emission levels and policies
- e. Renegotiation proofness
- f. Uncertainty and imperfect public monitoring
- g. Technological spillovers
- h. Stocks
- i. Lessons

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 - Importance of technology transfer/develop recognized..
 - "technology needs must be nationally determined, based on national circumstances and priorities" (§114 in the Cancun Agreement, confirmed in Durban)

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Motivation - Paris 2015

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 - Compliance is the main problem

a. Important Concepts and Equilibria Refinements

Normal form game	Nash equilibrium
Extensive form game	Subgame-perfect equilibrium
Repeated game and stage game	Renegotiation proofness
Stochastic game	Markov-perfect equilibrium

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$$B\left(\overline{g}\right) - B\left(\underline{g}\right) > \left(\overline{g} - \underline{g}\right)c.$$

• The emission game is a prisonner dilemma game if both holds:

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Repeated Games and SPF

 $1 < \frac{B\left(\overline{g}\right) - B\left(\underline{g}\right)}{c\left(\overline{g} - g\right)} < n.$ February 2019 6 / 44

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- With (grim) trigger strategies, cooperation (g = g) is an SPE if

$$\frac{B(\underline{g}) - cn\underline{g}}{1 - \delta} \geq B(\overline{g}) - c\overline{g} - c(n - 1)\underline{g} + \delta \frac{B(\overline{g}) - cn\overline{g}}{1 - \delta} \Leftrightarrow B(\overline{g}) - B(\underline{g}) \leq c(\overline{g} - \underline{g})[\delta n + (1 - \delta)]$$

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• So, as long as the first best requires $g = \underline{g}$, cooperation is possible for sufficiently high discount factors:

$$\delta \geq \widehat{\delta} \equiv \frac{1}{n-1} \left[\frac{B\left(\overline{g}\right) - B\left(\underline{g}\right)}{c\left(\overline{g} - \underline{g}\right)} - 1 \right] < 1.$$

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• If $\delta < \hat{\delta}$, the unique SPE is $g = \overline{g}$.

c. Emissions and Technology

 Consider next a stage game with both emissions and technology investments (r_{i,t}):

$$u_{i,t} = B(g_{i,t}, r_{i,t}) - c(r_{i,t}) \sum_{i=1}^{n} g_{i,t} - kr_{i,t}.$$

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- Will be added below: Heterogeneity, continuous g, uncertainty, and stocks

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- Given g, every country will voluntarily invest optimally in r.
- Once g has been committed to, there is no need to negotiate r.
- With such commitments, the first-best agreement is simply $g = \underline{g}$.

• The maximization problem is:

$$\max_{r,g \in \left\{\underline{g}, \overline{g}\right\}} \frac{B(g, r) - ngc(r) - kr}{1 - \delta}$$

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• The maximization problem is:

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• subject to the two "compliance constraints" (CC^r) and (CC^g) :

$$\frac{B(\underline{g}, r) - n\underline{g}c(r) - kr}{1 - \delta} \geq B(\underline{g}^{b}(\widetilde{r}), \widetilde{r}) - [\underline{g}^{b}(\widetilde{r}) + (n - 1)\underline{g}^{b}(r)]c(\widetilde{r}) - k\widetilde{r} + \frac{\delta u^{b}}{1 - \delta} \forall \widetilde{r},$$

$$B(\underline{g}, r) - n\underline{g}c(r) - \delta kr \qquad (n - 1)\sum_{i=1}^{k} \delta u^{i} - \delta u^{i}$$

$$\frac{\overline{\sigma}(\underline{g},r) - \underline{n}\underline{g}c(r) - \delta n}{1 - \delta} \ge B(\overline{g},r) - \left[\overline{g} + (n-1)\underline{g}\right]c(r) + \frac{\delta u}{1 - \delta}$$

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$$B(\underline{g}, r) - n\underline{g}c(r) - \delta kr \qquad (n - 1)g^{b}(r) - \delta u^{b}$$

$$\frac{B\left(\underline{g},r\right)-n\underline{g}c\left(r\right)-\delta\kappa r}{1-\delta} \geq B\left(\overline{g},r\right)-\left[\overline{g}+(n-1)\underline{g}\right]c\left(r\right)+\frac{\delta u^{b}}{1-\delta}$$

• Folk theorem: There exists $\hat{\delta}' < 1$ and $\hat{\delta}^{g} < 1$ such that the first-best can be sustained as an SPE iff $\delta \geq \max\left\{\hat{\delta}', \hat{\delta}^{g}\right\}$.

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• Literature says little when $\delta < \max\left\{\widehat{\delta}^{r}, \widehat{\delta}^{g}\right\}$.

Proposition

• CC^r never binds if an agreement is beneficial (i.e., $\hat{\delta}^r = 0$).

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• CC^g can be written as $(\hat{\delta}(r)$ can be defined such CC^g binds):

$$B\left(\underline{g},r\right) - n\underline{g}c - kr - (1/\delta - 1)\left[B\left(\overline{g},r\right) - B\left(\underline{g},r\right) - \left(\overline{g} - \underline{g}\right)c\right] \ge u^{b}$$

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- CC^{g} is more likely to hold for large δ , n, or c(r).
- Maximizing lhs of CC^g wrt r gives the 'best' compliance technology \hat{r} :

$$\frac{B_r(\underline{g}, \widehat{r}) - n\underline{g}c_r(\widehat{r}) - k}{1/\delta - 1} = B_r(\overline{g}, \widehat{r}) - B_r(\underline{g}, \widehat{r}) - (\overline{g} - \underline{g})c_r(\widehat{r})$$
$$\approx (\overline{g} - \underline{g})[B_{gr} - c_r] \Leftrightarrow$$
$$\widehat{r} > r^* \text{ IFF } B_{gr} - c_r < 0.$$



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Proposition

• Let $c(r) \equiv hf(r)$. For every r, we have $\hat{\delta}_h(r) < 0$ and $\hat{\delta}_n(r) < 0$.

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 - $r > r^* \uparrow$ for "green" technologies (where $B_{gr} < 0$ and $c_r = 0$)

Proposition

• $r < r^* \downarrow$ for "brown" technologies (where $B_{gr} > 0$ and $c_r = 0$)

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• CC^g only depends on individual parameters.

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Proposition

- CC^g only depends on individual parameters.
- Suppose $\delta_i \leq \hat{\delta}_i (r_i^*)$. If h_i , δ_i , n or i's size decreases, then

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- Suppose $\delta_i \leq \hat{\delta}_i(r_i^*)$. If h_i , δ_i , n or i's size decreases, then

• $r_i < r_i^* \downarrow$ for "adaptation" technologies (where $B_{gr}=0$ and $c_r < 0$)

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Proposition

- CC^g only depends on individual parameters.
- Suppose $\delta_i \leq \hat{\delta}_i (r_i^*)$. If h_i , δ_i , n or i's size decreases, then
 - $r_i < r_i^* \downarrow$ for "adaptation" technologies (where $B_{gr}=0$ and $c_r < 0$)
 - $r_i < r_i^* \downarrow$ for "brown" technologies (where $B_{gr} > 0$ and $c_r = 0$)

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- Reluctant countries should contribute **more!** (i.e., invest more in green technologies and less in brown.)
- *True:* One problem is to persuade a reluctant country to *participate*.
- However, the harder problem is to ensure that they are willing to *comply* once they expect others to comply.
- Reluctant countries should be helped to make such self-commitment, and this can be done with technology!

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c. Multiple Technologies

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- Suppose $\delta < \hat{\delta}^{g}$.
- Green technologies and brown technologies are strategic complements: The more countries invest in drilling technologies, the more they must invest in green technologies.
- Green technologies and adaptation technologies are strategic complements: The more countries adapt, the more they must invest in green technologies.
- Brown technologies and adaptation technologies are strategic substitutes: The more countries invest in brown technologies, the less they should adapt.

d. Continuous Emission Levels

Proposition

• (i) The Pareto optimal SPE is first best when
$$\delta \ge \max\left\{\overline{\delta}^{g}, \overline{\delta}^{r}\right\}$$
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- (i) The Pareto optimal SPE is first best when $\delta \ge \max\left\{\overline{\delta}^{g}, \overline{\delta}^{r}\right\}$;
- (ii) If k/b > 1/2, then $\overline{\delta}^r < \overline{\delta}^g$ and, when $\delta \in \left[\widehat{\delta}^r(g, r), \overline{\delta}^g\right)$, we have:

$$r = r^{*}\left(g^{*}
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- Either effect may be strongest.

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- So, green technology.
- Let the investment-cost be $\frac{k}{2}r_i^2$.
- We can define $d_i \equiv Y y_i$, so that $g_i = Y d_i r_i$, and $B = -\frac{b}{2}d_i^2$.

d. Continuous emission levels - First Best

• The socially optimal decisions are:

$$bd = b(Y - r - g) = cn \Rightarrow g^*(r) = Y - r - \frac{cn}{b}$$
$$kr = cn = bd = b(Y - r - g) \Rightarrow r^*(g) = \frac{b(Y - g)}{k + b}.$$

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• Combined, the first-best is

$$g^* = Y - rac{cn}{b} - rac{cn}{k}$$
 and $r^* = rac{cn}{k}$.

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Repeated Games and SPE

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d. Continuous emission levels - BAU

• The Nash equilibrium/BAU of the stage game is:

$$bd = c$$
 and $kr = c = bd$, so
 $g^b = Y - \frac{c}{b} - \frac{c}{k}$ and $r^b = \frac{c}{k}$.

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• This gives the BAU payoff:

$$V^b=rac{rac{c^2}{b}\left(n-rac{1}{2}
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ight)-cnY}{1-\delta}$$

• An equilibrium gives:

$$V^{e} = \frac{-\frac{b}{2}d^{2} - \frac{k}{2}r^{2} - cn(Y - d - r)}{1 - \delta}.$$

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- Then, if $\delta \in \left(\overline{\delta}^{r}, \overline{\delta}^{g}\right)$, $g^{e} > g^{*}$ while $r^{e} = r^{*}$.
- Thus, $r^e > r^*(g^e)$, and countries over-invest conditional on g.

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• To ensure that
$$kr = cn$$
, $\zeta = cn - \tau > 0$ decreases in $\delta \in \left(\overline{\delta}^r, \overline{\delta}^g\right)$.



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d. Carbon Taxes and Investment Subsidies

- Investment subsidy ς_i set before the investment stage and emission tax τ_i set before the emission stage by each country.
- International agreement: Defines taxes/subsidies to implement the best SPE.
- τ_i does not affect (CC^g_i), while ζ_i relaxes (CC^r_i).

Corollary

When δ declines from one, (CC_i^g) is always the first compliance constraint to bind;

- ii. If $\delta \geq \overline{\delta}^{g}$, the outcome is first best and implemented by $\tau_{i} = cn$ and $\varsigma_{i} = 0$;
- iii. If $\delta < \overline{\delta}^{g}$, the best SPE is implemented by $\tau_{i} = cn \phi(\delta)$ and $\zeta_{i} = \phi(\delta)$ with $\phi'(\delta) < 0$.

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d. Carbon Taxes and Investment Subsidies


e. Renegotiation-Proofness

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- Grim-trigger strategy is not renegotiation-proof;
- Allowing for renegotiation reduces the effective penalty if a country defects by emitting more;
- To satisfy the compliance constraint, the benefit of emitting more must be reduced as well.

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- In other words, there is no time at which both players would strictly benefit from following the strategies specified for a different time (where the identity of the next mover is preserved). Let S^w denote the set of weakly renegotiation-proof equilibria. Note that S^w must be independent of time.

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- Definition. A subgame-perfect equilibrium s^t ∈ S^w is strongly renegotiation proof if no continuation payoff profile is strictly Pareto-dominated by the continuation payoff profile of another s' ∈ S^w.

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e. Renegotiation-Proofness: Consequences

Proposition

Suppose that after a country deviates, the countries can renegotiate before triggering the penalty.

- i. With strong renegotiation-proofness (or with side transfers) and if a deviator has no bargaining power, the coalition of punishers will ensure that the deviator does not receive more than the BAU continuation value:
 - permitting renegotiation does not alter the set of Pareto optimal SPE;
- ii. With weak renegotiation-proofness, or if a deviator has some bargaining power, it will receive more than its BAU continuation value and the compliance constraint is harder to satisfy than without renegotiation:
 - to satisfy the compliance constraint $|r_i r^*|$ must increase more, the larger the bargaining power.

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• The probabilities will depend on the threshold \hat{g} :

$$p_I = 1 - F\left(\widehat{g} - n\underline{g}
ight)$$
 and $p_{II} = F\left(\widehat{g} - (n-1)\,\underline{g} - \overline{g}
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- Consider the following trigger strategy with **T-period punishment** phase:
 - If $r_{i,t} \neq r^*$, reversion to BAU forever
 - If $g_t > \widehat{g}$, reversion to BAU for T periods.
- When $p_l > 0$, the best SPE may require $T < \infty$.

f. Uncertainty and Imperfect Monitoring: Cooperation

Proposition

• The triplet (\underline{g}, r, T) is an SPE if $\delta \geq \hat{\delta}(r, T)$ where $\hat{\delta}_T < 0$, $\hat{\delta}_{\rho_l} > 0$, $\hat{\delta}_{\rho_{ll}} > 0$ and, as before, $\hat{\delta}_n < 0$, $\hat{\delta}_h < 0$ and

$$sign \ \widehat{\delta}_r = sign \ \left(B_{gr} - c_r\right).$$

• *Proof:* Let $V^{c}(r)$ be the continuation value in the cooperation phase:

$$V^{c}(r) = B(r,\underline{g}) - \underline{ngc}(r) - kr + \delta[p_{l}V^{p}(r) + (1 - p_{l})v^{c}(r)],$$

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• where the continuation value at the start of the punishment phase is:

$$V^{p}(r) = \sum_{\tau=0}^{T-1} \delta^{\tau} v^{b} + \delta^{T} V^{c}(r) = \frac{1-\delta^{T}}{1-\delta} v^{b} + \delta^{T} V^{c}(r), \text{ where}$$
$$v^{b} = \max_{r} B(r, \bar{g}) - n\bar{g}c(r) - kr.$$

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• As before, if the agreement is valuable, CC-r is never binding.

A country may be tempted to pollute a lot to get $V^{d}\left(r
ight)=$

$$B(r,\bar{g}) - \left[(n-1)\underline{g} + \bar{g} \right] c(r) - kr + \delta \left[(1 - p_{II}) V^{p}(r) + p_{II} V^{c}(r) \right]$$

The best equilibrium maximizes $V^{c}(r)$ subject to CC-g:

$$V^{c}(r) \geq V^{d}(r) \Rightarrow$$
 (CC-im)

$$V^{c}(r)\left[\left(1-p_{II}-p_{I}\right)\delta\left(1-\delta^{T}\right)+1-\delta\right] \geq B(r,\bar{g})-\left[\bar{g}+(n-1)\underline{g}\right]c(r)-kr+\left(1-p_{II}-p_{I}\right)\delta\left(1-\delta^{T}\right)V^{b},$$

Let $\hat{\delta}(r, T, p_{II}, p_I)$ be defined such that the inequality holds with identity. Doing comparative static w.r.t. this equation completes the proof.

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Proposition

• Let $\hat{\delta}(r(T), T) = \delta$. If T decreases or p_I or p_{II} increases, then

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Let δ̂(r(T), T) = δ. If T decreases or p_l or p_{ll} increases, then
r(T)>r* ↑ for "green" technologies (B_{gr}<0 and c_r=0)

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- $r(T) < r^* \downarrow$ for "adaptation" technologies ($B_{gr}=0$ and $c_r < 0$)

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Repeated Games and SPE

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- increases in r for "adaptation" technologies

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- Let θ_t be drawn from a cdf $\Phi(\cdot)$ with variance σ^2 and zero mean defined over a finite support, and measures the net emission from Nature.
- Let $\phi(y|g)$ be the density function of y conditioned on countries' emissions $g = y - g_0$ and assume that the monotone likelihood ratio property holds: The ratio $\phi(y|g')/\phi(y|g)$ is strictly increasing in y when g' > g.
- Then, it is optimal to increase ĝ even though T must increase also (to ∞). (This is the "bang-bang result of Abreu, Pearce, and Stacchetti '90)
- In equilibrium, the strategic value of r is that it increases g and thus the probability p₁.

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g. Technological Spillovers

• With technological spillovers, the country *i*'s per capita utility is:

$$B(g_i, z_i(r_i, r_{-i})) - hc(z_i(r_i, r_{-i})) \sum g_j - kr_i$$

where

$$z_i(r_i, r_{-i}) \equiv (1-e)r_i + \frac{e}{n-1}\sum_{j \neq i} r_j$$

- The first-best r_i^{*} is as before, but countries will not invest optimally conditionally on g_i;
- Noncooperative investments decline in e;
- When $r_i^* > r_i^b$, countries are tempted to deviate from the first-best even at the investment stage.

g. Technological Spillovers

• At the investment stage, (CC_e^r) is:

$$\frac{v}{1-\delta} \geq \frac{e}{1-e}k\left(r-r^{b}\right) + \frac{v^{b}}{1-\delta},$$

- At the emission stage, (CC_e^g) is as before;
- Let $\hat{\delta}^{r}(r)$ and $\hat{\delta}^{g}(r)$ be the level of δ_{i} such that (CC_{e}^{r}) and (CC_{e}^{g}) holds with equality;

g. Technological Spillovers

Proposition

An SPE exists in which $g_i = \underline{g} \quad \forall i \in \mathbb{N}$ if and only if $\delta \geq \underline{\delta}$. In this case, the Pareto optimal SPE is unique and:

$$i \quad If \ \delta \ge \max\left\{\widehat{\delta}^{r}\left(r^{*}\right), \widehat{\delta}^{g}\left(r^{*}\right)\right\}, \ then \ r = r^{*};$$

$$ii. \quad If \ \delta \in \left[\underline{\delta}, \max\left\{\widehat{\delta}^{r}\left(r^{*}\right), \widehat{\delta}^{g}\left(r^{*}\right)\right\}\right), \ then:$$

$$r = \begin{cases} r^{g}\left(\delta\right) > r^{*} \ when \ e \le \overline{e} \ if \ (G); \\ r^{r}\left(\delta\right) < r^{*} \ when \ e > \overline{e} \ if \ (G); \\ \min\left\{r^{g}\left(\delta\right), r^{r}\left(\delta\right)\right\} < r^{*} \ if \ (NG). \end{cases}$$

Corollary

Stronger intellectual property right may be necessary to sustain a self-enforcing treaty.

Bård Harstad (UiO)

Repeated Games and SPE

• We can reformulate the model to allow for stocks

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- We can reformulate the model to allow for stocks
- Consider a pollution stock $G_t = q_G G_{t-1} + \sum g_{i,t}$ with marginal cost C, and a technology stock $r_{i,t} = q_R r_{i,t-1} + \Delta r_{i,t}$, where the investment $\Delta r_{i,t}$ has the marginal cost K:

$$\widetilde{u}_{i,t} = B\left(g_{i,t}, r_{i,t}\right) - CG_t - K\Delta r_{i,t},$$

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• If we define $c \equiv C/(1 - \delta q_G)$ and $k \equiv K(1 - \delta q_R)$, maximizing $\tilde{u}_{i,t}$ is equivalent to maximizing $u_{i,t}$, defined as:

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Image: Image:

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- But if tech binds future emissions, investments cannot be too high.

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