Amartya Sen:

Labour Allocation in a Cooperative Enterprise,

RES 1966

Sen discusses two methods of income distribution:

1) Distribution according to need
2) Distribution according to work

SIMPLE MODEL

- Cooperative consisting of N identical families.

- In the production process the cooperative uses
  - own homegenous labor (L)
  - own land (A)
  - outside factors $F^k$ hired in perfectly competetive markets for price $P^k$.

- Consider only efficiency problems within coop (and not across coops)

- No taxation, no saving decisions
Family utility depends on income ($y^i$) and work ($l^i$):

$$U^i = U(y^i, l^i) \quad (1)$$

Where

$$U_y > 0, U_l < 0, U_{yy} < 0, U_{ll} < 0, U_{yl} = U_{yl} = 0$$

Families may also care about other members of the cooperative.

Extended preferences including social care:

$$W^j = \sum_{i=1}^{N} a_{ij} \cdot U^i \quad (2)$$

Where $a_{jj}=1$ and $0 \leq a_{ij} \leq 1$ for $i \neq j$.

Consequently:

$$W^j = U^j + \sum_{i=1}^{N} a_{ij} \cdot U^i \quad (2.1)$$
A measure of social care / “social consciousness”:

\[ S^j = \frac{1}{N} \sum_{i=1}^{N} a_{ij} \quad (4) \]

\( S^j \) range between \( 1/N \) and 1

- Cares only about his own family (\( 1/N \))
- Cares equally much for all families (1)

Assume that all families have the same “social consciousness”:

\[ S = \frac{1}{N} \sum_{i=1}^{N} a_{ij} \]
WELFARE MAXIMIZING MANAGEMENT (SOCIAL OPTIMUM)

Production given by:

\[ Q = Q(L, A, F^1, F^2, ..., F^m) \]  (6)

Welfare maximization requires equal division of total income (V) and total work (L)

Why? Everybody identical and increasing marginal disutility from work, and declining utility from income.

\[ y^i = \frac{V}{N} \equiv y \ \forall \ i \]  (7)

\[ l^i = \frac{L}{N} \equiv l \ \forall \ i \]  (8)
Social optimum is given by maximization of:

\[ W = N \cdot U(y, l) \]  (3.1)

Total income is given by:

\[ V = Q - \sum_{k=1}^{m} F^k \cdot P^k \]  (9)

Where factor prices are expressed in units of output.

FOC:

\[ Q_k = P^k, \text{ for } k = 1, \ldots, m \]  (10)

Marginal productivity of factor k equals input price

\[ Q_L = - \frac{U_l(y,l)}{U_y(y,l)} \equiv R \]  (11)

Marginal productivity of labor equals MRS between income and leisure.
INDIVIDUAL DETERMINATION OF EFFORT

Let $\alpha$ be the share of income distributed according to **needs**. ($0 < \alpha < 1$)

$(1 - \alpha)$ the share of income distributed according to **work**.

Needs are equal in this model $\rightarrow$ proportion $\alpha$ is equally distributed.

The rest is distributed such that family $i$ gets $\frac{l_i}{L}$.

Then:

$$y^i = V \left( \frac{\alpha}{N} + (1 - \alpha) \frac{l_i}{L} \right) \quad (7.1)$$

Needs only: $y^i = \frac{V}{N}$

Work only: $y^i = \frac{l_i}{L}$
Family j maximizes $W^j$ wrt to $l^j$, taking other families labor effort, as well as other input factors as given.

$$W^j = U(y^j, l^j) + \sum_{\substack{i=1 \atop i \neq j}}^{N} a_{ij} \cdot U(y^i, l^i)$$

Inserting from (7.1):

$$W^j = U \left( V \left( \frac{\alpha}{N} + (1 - \alpha) \frac{l^j}{L} \right), l^j \right) + \sum_{\substack{i=1 \atop i \neq j}}^{N} a_{ij} \cdot U \left( V \left( \frac{\alpha}{N} + (1 - \alpha) \frac{l^i}{L} \right), l^i \right)$$
Inserting from (9):

\[
W^j = U \left( (Q - \sum_{k=1}^{m} F^k \cdot P^k) \left( \frac{\alpha}{N} + (1 - \alpha) \frac{l^j}{L} \right), l^j \right) \\
+ \sum_{i=1}^{N} a_{ij} \cdot U \left( (Q - \sum_{k=1}^{m} F^k \cdot P^k) \left( \frac{\alpha}{N} + (1 - \alpha) \frac{l^i}{L} \right), l^i \right)
\]

FOC:

\[
U^j_y \left( Q_L \left( \frac{\alpha}{N} + (1 - \alpha) \frac{l^j}{L} \right) + V \left( 1 - \alpha \right) \frac{L - l^j}{L^2} \right) + U^j_l \\
+ \sum_{i=1}^{N} a_{ij} \cdot U^i_y \left( Q_L \left( \frac{\alpha}{N} + (1 - \alpha) \frac{l^i}{L} \right) + V \left( 1 - \alpha \right) \frac{-l^i}{L^2} \right) = 0
\]

\[
-U^j_l = \sum_{i=1}^{N} a_{ij} \cdot U^i_y \left( Q_L \left( \frac{\alpha}{N} + (1 - \alpha) \frac{l^i}{L} \right) - \frac{V}{L} \left( 1 - \alpha \right) \frac{l^i}{L} \right) + U^j_y \left( 1 - \alpha \right) \frac{V}{L}, \quad (12)
\]
When each family has the same utility fn. And the same social consciousness, they provide the same amount of labor in equilibrium.

This implies that

\[ y^i = \frac{V}{N} \equiv y \ \forall \ i \quad \text{and} \quad l^i = \frac{L}{N} \equiv l \ \forall \ i \]

Then (12) simplifies into:

\[ -\frac{u_i(y,l)}{u(y,l)} = \sum_{i=1}^{N} a_{ij} \left( Q_L \left( \frac{\alpha}{N} + (1 - \alpha) \frac{1}{N} \right) - \frac{V}{L} \left( 1 - \alpha \right) \frac{1}{N} \right) + (1 - \alpha) \frac{V}{L} \]

\[ R = \sum_{i=1}^{N} a_{ij} \left( Q_L \left( \frac{1}{N} \right) - \frac{V}{L} \left( 1 - \alpha \right) \frac{1}{N} \right) + (1 - \alpha) \frac{V}{L} \]

\[ R = Q_L \sum_{i=1}^{N} a_{ij} \frac{1}{N} - \sum_{i=1}^{N} a_{ij} (1 - \alpha) \frac{1}{N} \frac{V}{L} + (1 - \alpha) \frac{V}{L} \]

\[ R = Q_L \sum_{i=1}^{N} a_{ij} \frac{1}{N} + (1 - \alpha) \frac{V}{L} \left( 1 - \sum_{i=1}^{N} a_{ij} \frac{1}{N} \right) \]
Using $S = \frac{1}{N} \sum_{i=1}^{N} a_{ij}$:

$$R = Q_L S + (1 - \alpha)(1 - S) \frac{V}{L}$$

$$R = Q_L (S + (1 - \alpha)(1 - S) \frac{V}{Q_L L})$$

$$R = Q_L (S + (1 - \alpha)(1 - S) \frac{\frac{V}{Q_L}}{Q})$$

$$R = Q_L (S + (1 - \alpha)(1 - S) \frac{\beta}{\eta}) \quad (13)$$

Where $\beta = \frac{V}{Q}$, the ratio of income to total output
And $\eta = \frac{Q_L}{Q}$, elasticity of output wrt labor.
With income distribution purely according to needs (α=1):

\[ R = Q_L S \]

Corresponds to the optimal rule only if \( S=1 \), i.e. complete sympathy.

Since generally \( S < 1 \), then labor effort will be too low, \( R < Q_L \).

**Too little work is done**
With income distribution purely according to work ($\alpha=0$):

$$R = Q_L \left( S + (1 - S) \frac{\beta}{\eta} \right)$$

Corresponds to the optimal rule only if

1) $S=1$, i.e. complete sympathy. (does not bother the worker that other people are getting a share of his marginal product).

2) Or if $\beta=\eta$.

Note that with CRS technology, and the cooperative do not own any factor other than labor, then $\beta=\eta$.

- Average income per unit of labor ($V/L$) will be equal to marginal productivity ($Q_L$).

But when the cooperative own another factor of production (land), then $\beta>\eta$ and $R>Q_L$ (assuming CRS tech.).

Labor will be applied beyond the optimal point.

**Too much work is done**
Why overprovision?

An additional unit of labor yields an increase in income bc:

1) The income of the coop increases and the worker gets a share of this marginal increase
2) The worker gets a larger share of the total income

The first effect is unsufficient to induce the worker to provide sufficient effort. But the second effect **overcompensates** him for the effort when the average income per unit of labor is greater than the marginal product of labor.

Try to exploit the system of distribution according to work, to get a larger share of the pie.

When average income per unit of labor \((V/L)\) is exactly equal to marginal productivity \((Q_L)\) (i.e. CRS tech. with no other input than labor), then the two effects exactly balance out.
The optimal rule

A mixture of the two compensation systems is able to obtain the social optimum irrespective of social preferences, S.

This holds only iff:

\[(S + (1 - \alpha)(1 - S)^\frac{\rho}{\eta}) = 1 \quad (14)\]

\[(1 - \alpha) = \frac{\eta}{\beta} \quad (14b)\]

\(\frac{\eta}{\beta}\) should be distributed according to work and the remainder according to needs.

With CRS technology then labor’s share in the cooperative income will be \(\frac{\eta}{\beta}\).

The remainder is non-labor productive factor’s share in the cooperative income owned by the cooperative. This should be distributed according to needs.
Summary

Distribution according to needs tend to result in under-allocation of labor in a cooperative enterprise.

Distribution according to work tend to result in over-allocation of labor in a cooperative enterprise.

Optimization requires a mixed system of distribution according to needs and work.

Generally: worker effort depends on social preferences.