Consumption Growth in an Economy with Natural Resources and Zero Investment.

By

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Abstract:

A quite general growth model with several capital assets, some of which are natural resources, is presented. The consumption development is analyzed for the case in which the total value of net investment is zero. In this case consumption growth is explained exclusively by growth of exogenous factors of production. Whether or not such a development will be feasible and give positive consumption over an infinite horizon will depend on the exact specification of the production possibility set.

1. *Introduction.*

In two recent articles, Hartwick [1,2] has demonstrated that if investment in producible capital is equal to the rents from exhaustible resources, consumption will stay constant over time. The present paper demonstrates that this result is just a special case of a property of a general growth model with several capital goods. Within the framework of such a model, it will be shown that if the total value of net investment in all capital goods, measured by competitive prices, is zero, then any growth in consumption will be a result of growth in exogenous factors.

2. *A general growth model.*

We have m capital goods with stocks $k = (k_1,\ldots,k_m)'$ and rates of change $\dot{k} = u = (u_1,\ldots,u_m)'$, n consumption goods $c = (c_1,\ldots,c_n)'$ and q exogenous factors of production $z = (z_1,\ldots,z_q)'$. Time references for all variables are omitted to simplify notation. The production possibilities are given by the transformation function

(1) $$F(z, k, u, c) \leqq 0,$$

where

(2) $$F_{zh} = \frac{\partial F}{\partial z_h} \leqq 0,\ F_{ki} = \frac{\partial F}{\partial k_i} \leqq 0,\ F_{ui} = \frac{\partial F}{\partial u_i} > 0 \text{ and } F_{cj} = \frac{\partial F}{\partial uj} > 0,$$

$$h = 1,\ldots,q$$

$$i = 1,\ldots,m$$

$$j = 1,\ldots,n.$$

Disregarding consumption satiation, $F_{cj} > 0$ implies that (1) must hold with a strict equality in an efficient solution. The notation F_z will be used for the vector $(F_{z1},\ldots,F_{zq})$, while F_k, F_u and F_c are defined similarly.

The production structure given by (1) is very general. It covers economies with heterogeneous produced capital goods but without natural resources as well as economies with renewable and non-renewable natural resources. The simplest example of the latter case is the economy treated by Hartwick [1] and Solow [5]. Here there is one produced capital good with stock K, one natural resource with stock S and current use R ($= -\dot{S}$) and one consumption good C. Labour growth and technical progress are ignored, so that (1) in this case may be written as

$$(3) \qquad C + \dot{K} - \phi(K,R) \leqq 0.$$

Notice that S does not enter (3), but the non-negativity constraint $S(t) \geqq 0$ gives a restriction on what time paths of R(t) are feasible.

A more general economy with natural resources than illustrated by (3) is an economy with several produced capital and consumption goods, several exogenous factors of production (various types of labour, technology levels etc.) and several natural resources, some renewable and some non-renewable. For all types of natural resources we may have $F_{ki} < 0$, this will be the case if the resource stock affects the production possibilities, cf. [3] and [6]. For non-renewable resources we have the restriction $u_i \leqq 0$. Whatever goods k and c consist of we have the non-negativity constraints $k \geqq 0$ and $c \geqq 0$. If costs of extraction are allowed for in the case of natural resources, we may not have $F_{ui} > 0$ for alle values of u_i. However, in efficient solutions the value of the marginal productivity of using the resource will exceed the marginal cost of extraction, implying that $F_{ui} > 0$.

The efficiency prices (i.e. prices prevailing in a competitive economy without production externalities) of capital good i and consumption good j are F_{ui} and F_{cj}, respectively. Similarly, $-F_{ki}$ is the rental rate of capital good i. As always, it is only the *relative* values of these prices which are of any interest.

In the absence of effective boundaries on the value of u, intertemporal efficiency can easily be shown to require that

$$(4) \qquad \frac{\dot{F}_{ui} - F_{ki}}{F_{ui}} = \frac{\dot{F}_{u1} - F_{k1}}{F_{u1}}, \qquad i=2,\ldots,m.$$

With the price interpretation given above, equation (4) is recognized as the well-known condition that the own rates of return on all capital goods must be equal. The generalization to cases with $u_i(t)$ on a boundary, for instance $u_i(t) \geqq 0$ for any t such that $k_i(t) = 0$, is straightforward. With the boundary condition $u_i(t) \geqq 0$ we get "$\leqq$" instead of "=" in (4), with a stict inequality implying $u_i(t) = 0$.

For the special case given by (3), the efficiency condition (4) simply becomes $\dot{\phi}_R/\phi_R = \phi_K$, which is identical to the well-known Hotelling [4] condition.

3. *Consumption development with zero investment.*

We now introduce the investment condition used by Hartwick. In our general case this condition states that the value sum of the net investment of all capital goods is zero, when the valuation is done in efficiency prices. Formally, this requirement states that

$$(5) \qquad F_u u = 0.$$

Notice that this is *not* the same as requiring the value of the capital stock to be constant over time, which would mean

$$(6) \qquad \frac{d}{dt}(F_u k) = F_u u + \dot{F}_u k = 0.$$

For the special case given by (3) the investment condition (5) becomes $\dot{K} = \phi_R R$, which is identical to equation (1) in [1].

We are interested in studying the growth of consumption. More precisely, with several consumption goods we mean the value sum of the changes in all consumption goods, when the valuation is done in efficiency prices. Formally, this growth, denoted by g, is given by

$$(7) \qquad g = F_c \dot{c}.$$

Notice that this is *not* the same as the growth of the value of total consumption, which is given by

$$(8) \qquad \tilde{g} = \frac{d}{dt}(F_c c) = F_c \dot{c} + \dot{F}_c c.$$

However, if we only have one consumption good and we choose this good as numeraire, we of course get $\tilde{g} = g$.

To find the value of g, differentiate (1) with respect to time. Together with (7) and $\dot{k} = u$ this gives us

$$(9) \qquad g = -F_z \dot{z} - F_k u - F_u \dot{u}.$$

From (6) we obtain

$$(10) \qquad F_u \dot{u} = -\dot{F}_u u,$$

which inserted into (9) gives

(11) $$g = - F_z \dot{z} - F_k u + \dot{F}_u u.$$

But our efficiency condition (4) gives us

(12) $$F_k = \dot{F}_u - \frac{\dot{F}_{u1} - F_{k1}}{F_{u1}} F_u.$$

Inserting (12) into (11) and using (5) finally gives us

(13) $$g = -F_z \dot{z} + \frac{\dot{F}_{u1} - F_{k1}}{F_{u1}} F_u u - \dot{F}_u u + \dot{F}_u u = (-F_z)\dot{z}.$$

In other words, the growth of the consumption goods as defined by g is determined completely by the growth of the exogenous factors of production. With zero growth of these factors, as Hartwick assumes, we find $g = 0$, i.e. zero consumption growth.

4. *The feasibility of solutions with zero investment.*

So far, we have only shown the consequence for consumption growth of using the investment rule given by (5). We shall now give a brief discussion of the feasibility of using such an investment rule over an infinite horizon. A development is feasible if the constraints $k(t) \geqq 0$, $c(t) \geqq 0$ and possible constaints on $u(t)$ are satisfied for all t. Consider first the case in which all capital goods consist of producible capital. Ruling out perverse specifications of (1), $u(t) = 0$ will give feasible solutions, namely $k(t) = k(0)$ (where $k(0)$ is the historically determined initial capital stock) and $c(t)$ developing as determined by (13). Obviously, $c(t)$ is not uniquely determined for $n > 1$ unless we make additional assumptions about how the composition of consumption goods should develop. If $\dot{z}(t) = 0$ the solution $\dot{c}(t) = 0$ will be feasible and may be efficient, while $\dot{z}(t) \neq 0$ will usually imply that $k(t) = k(0)$ and the corresponding possible developments of $c(t)$ are inefficient, although feasible.

If some of the capital goods are non-renewable resources with $k_i(0) > 0$ and $F_{ki} = 0$, a solution with $u(t) = 0$ will clearly be inefficient, since the resource stock will not be utilized. Furthermore, $u(t) = 0$ may imply $c(t) = 0$, which is the case for the specification (3) if $\Phi(\cdot)$ is a Cobb-Douglas function. Let us therefore see if it is possible in such cases to choose $u_i(t)$ (for a natural resource for which $u_i(t) \leqq 0$ and $F_{ki} = 0$) such that

$$(14) \qquad - \int_0^\infty u_i(t)\, dt = k_i(0)$$

and giving $c(t) > 0$ for all t. The condition (14) is necessary for intertemporal efficiency as long as $F_{ki} = 0$, $F_{ui} > 0$ and $F_{cj} > 0$.

To simplify our discussion, let $n = 1$ (i.e. only one consumption good) and $\dot{z}(t) = 0$. Then (1), (4) and (5) give 2m-1 differential equations in $k(t)$ and $u_1(t),\ldots,u_{m-1}(t)$ ($u_m(t)$ is a function of $k(t)$ and the other $u_i(t)$'s, $i \neq m$, defined by (5) when $c(t)$ is inserted from (1)). The initial vector $k(0)$ is historically given. To solve the differential equations we must know $u_1(0),\ldots,u_{m-1}(0)$. These initial values must be chosen so that the resulting solution satisfies the constraints $k(t) \geqq 0$ as well as the constraints (14) for all i with $F_{ki} = 0$. There *may* be several initial values satisfying these values and giving $c(t) = c(0) > 0$, but only the one with the highest value of $c(0)$ will be efficient. On the other hand, there may exist *no* initial values satisfying these constraints and giving $c(0) > 0$. For instance, it follows from Solow's [5] analysis that for the case described by (3) where $\Phi(\cdot)$ is a Cobb-Douglas function, *no* $R(0)$ satisfying a condition of the type (14) and giving $K(t) \geqq 0$ will give $C(t) = C(0) > 0$ if the marginal elasticity of capital is lower than or equal to the marginal elasticity of the natural resource.

With the general transformation function (1), it is not possible to give simple rules of the type mentioned above about when feasible solutions with $c(t) > 0$ derived from the investment rule (5) will exist.

References:

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