ECON 4925 Autumn 2010
Resource Economics
Hydro power and thermal

Lecturer:
Finn R. Førsund

Thermal capacity

- Primary energy source coal, oil, gas, wood uranium
  - Water is heated up to produce steam that drives the turbines producing electricity
  - Burning gas directly like a jet engine: CCGT; combined cycle gas turbine
  - Emission of pollutants; acid rain, climate gases, nuclear waste
- Thermal capacity is power-restricted

Thermal capacity, cont.

- Variable cost of thermal electricity production as a function of output based on the consumption of primary fuel

$$c_{it} = c_i(e_{it}^{Th}),\ c_i' > 0, c_i'' > 0,\ e_{it}^{Th} \le \overline{e}_i^{Th}$$

c_{it} = variable cost of electricity production at unit i

e_{it}^{Th} = thermal electricity production at unit i

$\overline{e}_i^{Th}$ = capacity limit at unit i

Least cost combination of thermal plants

$$\min \sum_{i=1}^{M} c_i(e_{it}^{Th})$$

subject to

$$\sum_{i=1}^{M} e_{it}^{Th} \geq e_t^{Th}$$

$$e_{it}^{Th} \leq \overline{e}_i^{Th}$$

$$e_{it}^{Th} \geq 0$$

$$e_t^{Th}, \overline{e}_i^{Th} \text{ given}, t = 1,..,T, i = 1,..,M$$

The Lagrangian (formulating a maximisation problem)

$$L = -\sum_{i=1}^{M} c_i(e_{it}^{Th})$$
$$-\nu(-\sum_{i=1}^{M} e_{it}^{Th} + e_t^{Th})$$
$$-\sum_{i=1}^{M} \theta_i(e_{it}^{Th} - \overline{e}_i^{Th})$$

The Kuhn – Tucker conditions

$$\frac{\partial L}{\partial e_{it}^{Th}} = -c_i'(e_{it}^{Th}) + \nu - \theta_i \le 0 \;\; (= 0 \text{ for } e_{it}^{Th} > 0)$$

$$\nu \ge 0 (= 0 \text{ for } \sum_{i=1}^{M} e_{it}^{Th} > e_t^{Th})$$

$$\theta_i \ge 0 (= 0 \text{ for } e_{it}^{Th} < \overline{e}_i^{Th})$$

The three states of a plant

- Condition for not activating plants for a given level of total production

$$c_i'(0) \geq \nu \ (e_{it}^{Th} = 0)$$

- Conditions for activating plants
  - Interior solution for production

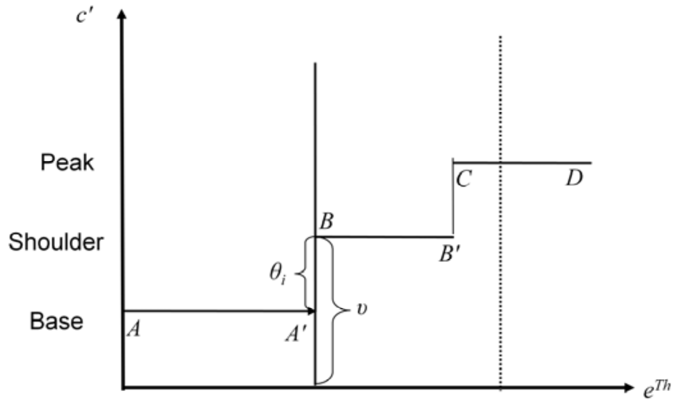
$$c_i'(e_{it}^{Th}) = \nu \ (0 < e_{it}^{Th} < \overline{e}_i^{Th})$$

  - Capacity-constrained production

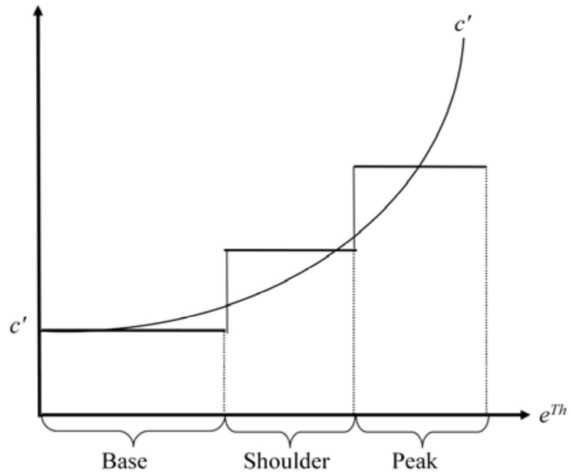
$$c_i'(\overline{e}_i^{Th}) = \nu - \theta_i \ (e_{it}^{Th} = \overline{e}_i^{Th})$$

Merit-order ranking

- No intersection of marginal cost curves



Merit-order aggregation of cost curves



Thermal and hydro with a reservoir constraint

- The social planning problem

$$\max \sum_{t=1}^{T} [\int_{z=0}^{x_t} p_t(z)\,dz - c(e_t^{Th})]$$

subject to

$$x_t = e_t^H + e_t^{Th}$$

$$R_t \le R_{t-1} + w_t - e_t^H$$

$$R_t \le \overline{R}$$

$$e_t^{Th} \le \overline{e}^{Th}$$

$$x_t, e_t^H, e_t^{Th}, R_t \ge 0, \quad t = 1,..,T$$

$$T, w_t, R_o, \overline{R}, \overline{e}^{Th} \text{ given}, R_T \text{ free}$$

The Lagrangian

$$L = \sum_{t=1}^{T}[\int_{z=0}^{e_t^H + e_t^{Th}} p_t(z)\,dz - c(e_t^{Th})]$$

$$-\sum_{t=1}^{T}\lambda_t(R_t - R_{t-1} - w_t + e_t^H)$$

$$-\sum_{t=1}^{T}\gamma_t(R_t - \overline{R})$$

$$-\sum_{t=1}^{T}\theta_t(e_t^{Th} - \overline{e}^{Th})$$

The Kuhn – Tucker conditions

$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H + e_t^{Th}) - \lambda_t \le 0 \;(= 0 \text{ for } e_t^H > 0)$$

$$\frac{\partial L}{\partial e_t^{Th}} = p_t(e_t^H + e_t^{Th}) - c'(e_t^{Th}) - \theta_t \le 0 \;(= 0 \text{ for } e_t^{Th} > 0)$$

$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \;(= 0 \text{ for } R_t > 0)$$

$$\lambda_t \ge 0 \;(= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$

$$\gamma_t \ge 0 \;(= 0 \text{ for } R_t < \overline{R})$$

$$\theta_t \ge 0 \;(= 0 \text{ for } e_t^{Th} < \overline{e}^{Th}), \quad t = 1,..,T$$

The use of hydro and thermal

- Thermal will not be used in period *t* if

$$c'(0) > p_t(x_t) = \lambda_t \ (e_t^{Th} = 0, \theta_t = 0, x_t = e_t^H)$$

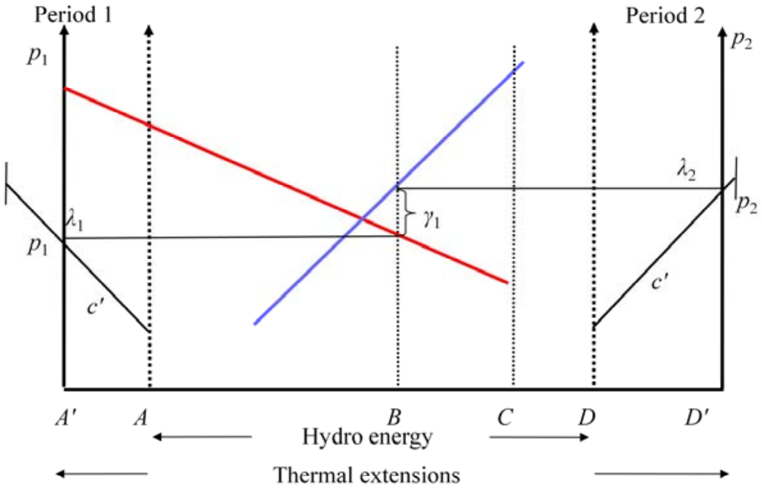
- Hydro will not be used in period *t* if

$$\lambda_t > p_t(x_t) = c'(e_t^{Th}) + \theta_t \ \ (\theta_t \geq 0, e_t^H = 0, \ x_t = e_t^{Th})$$

- Both thermal and hydro in use

$$p_t(x_t) = \lambda_t = c'(e_t^{Th}) + \theta_t \, (\theta_t \geq 0, x_t = e_t^H + e_t^{Th})$$

Bathtub diagram with thermal and hydro with reservoir constraint



Adding renewables without current costs

$$\max \sum_{t=1}^{T} [\int_{z=0}^{x_t} p_t(z)\,dz - c(e_t^{Th})]$$

subject to

$$x_t = e_t^H + e_t^{Th} + e_t^W$$

$$R_t \leq R_{t-1} + w_t - e_t^H$$

$$R_t \leq \overline{R}$$

$$e_t^{Th} \leq \overline{e}^{Th}$$

$$e_t^W \leq \overline{e}^W$$

$$x_t, e_t^H, e_t^{Th}, e_t^W, R_t \geq 0, \quad t = 1,..,T$$

$$T, w_t, R_o, \overline{R}, \overline{e}^{Th},\ e_t^W, \overline{e}^W \text{ given}, R_T \text{ free}$$

The Kuhn – Tucker conditions with renewables

$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H + e_t^{Th} + e_t^W) - \lambda_t \le 0 \ (= 0 \text{ for } e_t^H > 0)$$

$$\frac{\partial L}{\partial e_t^{Th}} = p_t(e_t^H + e_t^{Th} + e_t^W) - c'(e_t^{Th}) - \theta_t \le 0 \ (= 0 \text{ for } e_t^{Th} > 0)$$

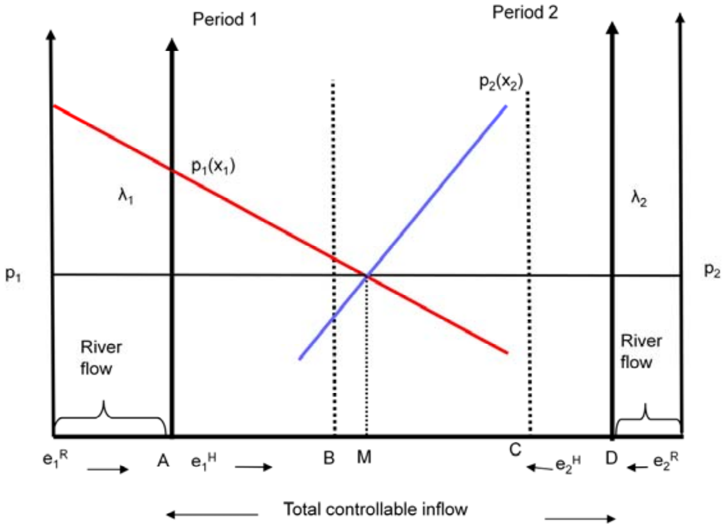
$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \ (= 0 \text{ for } R_t > 0)$$

$$\lambda_t \ge 0 \ (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$

$$\gamma_t \ge 0 \ (= 0 \text{ for } R_t < \overline{R})$$

$$\theta_t \ge 0 \ (= 0 \text{ for } e_t^{Th} < \overline{e}^{Th}) , \quad t = 1,..,T$$

Must take: Run-of-river and wind power



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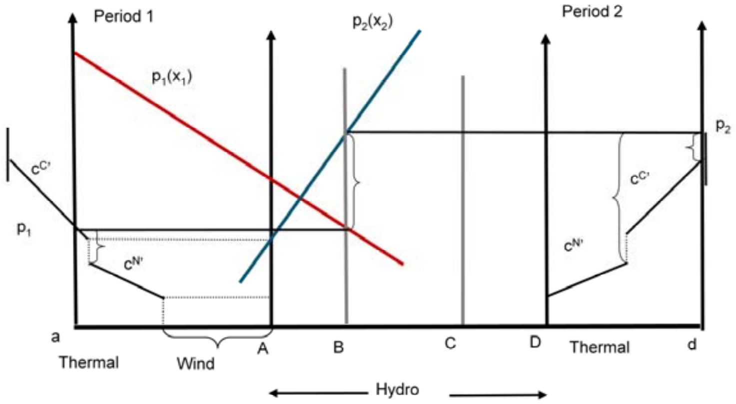
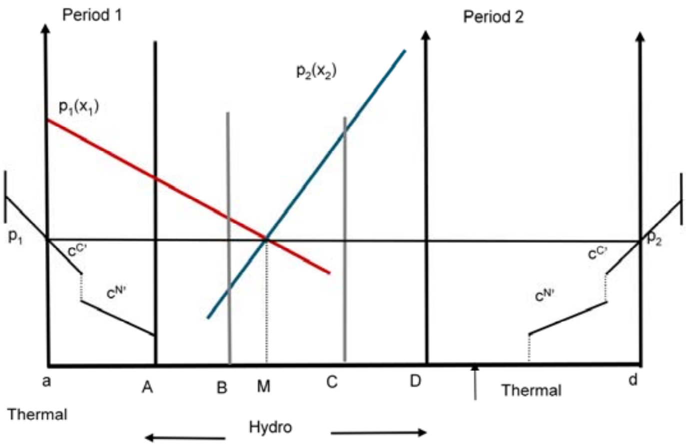
Hydro power and other technologies
Wind only in period 1



Wind only in period 2



No use of hydro in period 1