Renewable Resources

Florian K. Diekert
(f.k.diekert@bio.uio.no)

Overview

This lecture (planned for two sessions) gives an introduction to renewable resources. The core theme will be the role of management institutions: A theoretical sole-owner would take the self-reproducible character of such a resource adequately into account and use it optimally, whereas open access to the resource is often associated with a "tragedy of the commons" (Hardin, 1968). Gordon (1954) was the first to make the malign effect of missing property rights in fisheries known to a wide public. However, largely unnoticed, the Danish economist Warming has described these mechanisms already in 1911 (translated by Andersen (1983)). An excellent account of the evolution of forestry economics is the paper by Samuelson (1976).

This note gives some rudimentary context and spells out all the mathematical models that will be lectured. It does not serve as a substitute for your readings in the text book (Perman et al., 2003, ch. 17 and 18) (mandatory) and the original articles (voluntary, but highly recommended; The bibliographic information is given below).

After the lecture, you should be able to discuss the following key concepts: *bio-economics; Gordon-Schaefer model; MSY; open-access; TAC; ITQ; stock collapse; rotation period; Faustmann rule.*

1 Introduction

Renewable resources are characterized by their ability to reproduce naturally. This process of reproduction can be completely outside the control of the resource user (such as a migrating stock of fish that every now and then visits the waters of a small fishing community), somewhat controllable by the resource users (when for example the harvesting effort of a commercial fishery in a given year determines to some extent the availability of fish in the next year), or it can be completely controlled (as in aquaculture). The first case is more like a problem of eating a cake of unknown size and will not be dealt with here. Rather, we will discuss the second case with respect to a fishery and the third case with respect to a forestry.

Bio-economics

The economics of renewable resources is essentially a bio-economic exercise since the process governing the reproduction of the resource stock is biological. Denote the size of the fish stock by S. Its growth function is given by $G(S)$. For fisheries, the growth function is most often presumed to be logistic:

$$G(S) = gS\left(1 - \frac{S}{S_{max}}\right) \tag{1}$$

It captures the idea that there is a maximum stock size S_{max}, determined by the carrying capacity of the ecosystem, so that $G(S_{max}) = 0$. Similarly, no fish fall from heaven (i.e. $G(0) = 0$). In between $S = 0$ and $S = S_{max}$, growth $G(S)$ will be positive and the fish stock will grow with the "intrinsic growth rate" g at the origin and it will grow strongest at $\frac{1}{2}S_{max}$. On the one hand allows the logistic growth function to describe the salient features of renewable resources in a very simple and plain manner. On the other hand does the logistic growth function not allow to analyze the internal dynamics of the stock (e.g. its age structure) and issues of minimum viable population sizes and stock collapse.

Denote the harvest by H. The stock dynamics of the renewable resource economy are then given by:

$$\dot{S} = G(S) - H \tag{2}$$

The harvest function

Harvest is commonly thought to depend on the amount of effort E that is applied to catch the fish and on the size of the fish stock S itself. Perhaps *the* canonical harvest function is

$$H(E,S) = qES, \tag{3}$$

Where harvest is proportional to effort and q is called the "catchability-coefficient", translating one unit of effort (e.g. measured in number of vessels) into one unit of harvest (mostly measured in kg). Together with the logistic-growth function, it is referred to as Gordon-Schaefer model after the seminal contributions of Gordon and Schaefer (both published in 1954). Note that this harvest function is a special form of the Cobb-Douglas production function where both $\alpha = 0$ and $\beta = 1$.

$$H(E,S) = qE^{1-\alpha}S^{\beta}. \tag{4}$$

First, $1 - \alpha$ is the effort-output elasticity, specifying by how much, *ceteris paribus*, the harvest increases when effort increases by one unit. Most often harvest is thought to be proportional to the amount of effort used and α is set to 0. Decreasing returns ($\alpha > 0$) could be thought of as representing a crowding externality. Second, β denotes the stock-output elasticity. The value of β tells how much, *ceteris paribus*, the harvest increases when the stock increases by one unit. If the fish stock follows an ideal free distribution ($\beta = 1$) always occupying a given area, the density of fish declines at the same rate as the stock gets depleted. Contrarily, for a perfect schooling species ($\beta = 0$), the fish density remains unchanged, but the area occupied by the fish declines as the stock gets depleted. Note that in this case, the harvest function is independent of the stock size! Note that for $\beta > 0$, the stock dependency makes it excessively costly to harvest the last fish in the ocean. This can be more clearly seen by an inspection of the cost function. Suppose that harvesting costs are proportional to effort: $c(E) = w \cdot E$ and use equation (4) with $0 < \alpha, \beta < 1$ to re-write the cost function in terms of harvest H and stock size S:

$$\begin{aligned}
H &= qE^{1-\alpha}S^{\beta} \\
E^{1-\alpha} &= \frac{H}{qS^{\beta}} \\
E(H,S) &= \left(\frac{H}{qS^{\beta}}\right)^{\frac{1}{1-\alpha}} \\
c(H,S) &= wE = w\left(\frac{H}{qS^{\beta}}\right)^{\frac{1}{1-\alpha}} \qquad (5)
\end{aligned}$$

Under these assumptions, costs are decreasing in S, increasing in H and E, and strictly convex. In the following, we will use this specification of the technology (equation 4 and 5). However, many empirical applications use an exponential harvest function (equation 6). This function has not only an intuitive interpretation as the probability of harvesting one fish over an interval of one time unit, it has also the property that it is by design impossible to harvest more than the stock.[1]

$$H(E,S) = \left(1 - e^{-qE}\right)S. \qquad (6)$$

2 Optimal harvesting

Consider again the dynamics of the renewable resource:

$$\dot{S} = G(S) - H \qquad (2)$$

Clearly, there is an indefinite number of equilibria in this system. In other words, all harvest levels that take out as much as regrows are sustainable. The question thus is which sustainable harvest level is the optimum?

Biologists and stock managers often used to call for “maximum sustainable yield” (MSY). Obviously, the objective of harvesting as much as can maximally be sustained places no concern whatsoever on economic criteria. Instead, economists have therefore called for “maximum economic yield” (MEY), often interpreted as the sustainable stock size which gives the highest surplus (calculated as revenue over cost). Perman et al. (2003, p.573) call this the “static private-property steady state”. It is static because the

[1] In contrast, under technology (4) and (3) the constraint $H_{max} \leq S$ has to be artificially introduced.

objective of maximizing equilibrium profits neglects both the dynamics of approaching steady state and discounting. It does however serve as an interesting benchmark case and we will return to it below.

Let us then consider the "correct" control-theoretic problem of optimal harvesting. Assume there is a sole-owner of a fishery whose dynamics are given by (2). Her objective is to maximize the net-present value of this fishery, i.e. the discounted (at the rate r) sum of revenues from harvesting (pH) minus the cost of harvesting ($c(H,S)$). Cost depend on the amount harvested H and the stock size S with the usual properties $c_H > 0, c_{HH} \geq 0, c_S < 0, c_{SS} \leq 0$ (for example when the cost of effort are linear and the harvest function is of the Cobb-Douglas type).

$$\begin{gathered}\max_H \; \Pi = \int_0^\infty \left[pH - c(H,S)\right] e^{-rt} \mathrm{d}t \\ \text{subject to:} \quad \dot{S} = G(S) - H; \quad S(0) = S_0, S \geq 0; \quad 0 \leq H \leq H_{max}\end{gathered} \tag{7}$$

The current value Hamiltonian is then:

$$\mathcal{H} = pH - c(H,S) + \mu\left(G(S) - H\right), \tag{8}$$

and necessary conditions for optimality include

$$p - c_H(H,S) - \mu \leq 0 \quad (= 0 \text{ for } H > 0), \tag{9}$$

and the shadow price must obey:

$$\dot{\mu} = \mu\left(r - G'(S)\right) + c_S(H,S). \tag{10}$$

From equation (9) we see that the shadow price μ has an interpretation as resource rent, i.e. the net of price over marginal cost. Equation (10) can then be re-written as "Hotelling rule" for renewable resources:

$$r = \frac{\dot{\mu}}{\mu} - \frac{c_S(H,S)}{\mu} + G'(S) \tag{10'}$$

The resource owner must thus be indifferent between taking out one unit of stock and investing its value in the financial market or leaving it in the ocean, where in addition to the proportionate growth in net price, it yields a proportionate reduction in harvesting cost (due to the marginal increase in stock size) and an increase in natural growth.

The optimal development of the renewable resource economy is to steer the stock from its initial value S_0 to the optimal equilibrium S^* and continue harvesting $H^* = G(S^*)$ forevermore. When the economy is described by the traditional Gordon-Schaefer function (3), we see that the Hamiltionian (8) is linear in the control H yielding the bang/bang solution $H^* = 0$ if $S < S^*; H^* = H_{max}$ if $S > S^*$. The approach dynamics are therefore characterized by the "most rapid approach path" (MRAP).

A characterisation of the harvest level H^*

Let us give a further characterisation of the optimal harvest level, and consider harvesting costs of the form $c(S,H) = w\left(H/qS^\beta\right)^{\frac{1}{1-\alpha}}$ (equation 5). From the (economic) first-order condition (9) it follows that if harvesting is profitable at all, the optimal harvest H^* can be expressed as a function of stock size:

$$\begin{aligned}
c_H(H^*,S) &= p-\mu \\
\frac{1}{1-\alpha} wH^{\frac{1}{1-\alpha}-1}\left(\frac{1}{qS^\beta}\right)^{\frac{1}{1-\alpha}} &= p-\mu \\
H^{\frac{1}{1-\alpha}-1}\left(\frac{1}{qS^\beta}\right)^{\frac{1}{1-\alpha}} &= \frac{(p-\mu)(1-\alpha)}{w} \\
H^{\frac{\alpha}{1-\alpha}} &= \frac{(p-\mu)(1-\alpha)}{w}\left(qS^\beta\right)^{\frac{1}{1-\alpha}} \\
H^* &= \underbrace{\left(\frac{(p-\mu^*)(1-\alpha)}{w}\right)^{\frac{1-\alpha}{\alpha}} q^{\frac{1}{\alpha}}}_{A} S^{\frac{\beta}{\alpha}} \quad = \quad AS^{\frac{\beta}{\alpha}} \qquad (11)
\end{aligned}$$

The (biological) equilibrium condition $\dot{S} = 0 = H(S) - G(S)$ then allows to draw both growth and harvest function in one diagram to read off the steady state stock value S^*.

Comparative statics

To gain some further insight, let us look at the situation once steady state has been reached, i.e. a situation where $S = S^*$ and $H = G(S)$. There are basically three measures of interest: effort, harvest, and stock. How their values change when the parameter values change will often depend on the specific model specification.

A formal investigation for the stock size S^* would require to differentiate the equilibrium condition $0 = H(S) - G(S)$ wrt the parameter of interest e.g.

$$0 = \frac{\partial H(S)}{\partial S}\frac{\partial S}{\partial w} + \frac{\partial H(S)}{\partial w} - \frac{\partial G(S)}{\partial S}\frac{\partial S}{\partial w}$$

Which – provided that $\frac{\partial G(S)}{\partial S} < \frac{\partial H(S)}{\partial S}$ – can be solved for:

$$\frac{\partial S}{\partial w} = \frac{\frac{\partial H(S)}{\partial w}}{\frac{\partial G(S)}{\partial S} - \frac{\partial H(S)}{\partial S}} > 0$$

A rough and simple way is to see how A changes with prices, costs, and technology. It is then relatively straightforward to see that the equilibrium stock will increase with the resource price (a higher p) and technology (a higher q), and it will decrease when marginal effort costs w are higher.

The role of discounting however is not explicit, and we will thus take a closer look. Due to equilibrium, we have $\dot{\mu} = 0$, and $\mu(t) = \mu^* = p - c_H(H^*, S^*)$. Then, equation (10') can be written as (12) below.

$$\begin{aligned} r\mu^* &= \mu^* G'(S^*) - c_S(H^*, S^*) \\ \mu^* &= \frac{c_S(H^*, S^*)}{G'(S^*) - r} \end{aligned} \tag{12}$$

Recall that $c_S(H^*, S^*) < 0$ and $G'(S^*) < r$, so that the optimal shadow value – the resource rent – is the larger, the smaller is r. Finally, when $r = 0$, (12) collapses to:

$$\mu^* G'(S^*) = c_S(H^*, S^*) \tag{13}$$

stating that profits are maximized when the marginal revenue with respect to stock changes equals the marginal cost (with respect to stock changes), which is exactly the "static private-property steady state" mentioned above. For $r > 0$, (12) can be written in present-value terms:[2]

$$p - c_H(H^*, S^*) = \frac{\Pi'(S^*)}{r} \tag{14}$$

where the left-hand-side is the present value of catching one more unit of the stock today while the right-hand-side give the cost in terms of the present value of the reduced permanent income due to the changed steady state. Now as the discount rate increases, the future income from the stock has less and less present value. In the limit, as $r \to \infty$, the right-hand-side tends to zero and the optimal action is to deplete the resource until the point where marginal revenue just equals marginal price, hence no profits are made.

3 Competitive harvesting

Open access

Scenarios of unregulated competitive harvesting come in many different forms, but they all share one aspect: the resource users do not fully take the self-reproducible character of the resource into account: The gains from harvesting one more fish are private, while

[2] Where we have made use of $G(S^*) = H^*$, $\mu^* = p - c_H(H^*, S^*)$, and consequently write current profits as: $\Pi(S^*) = pG(S^*) - c(G(S^*), S^*)$. Hence (12) can be written as: $r\mu^* = \mu^* G'(S^*) - c_S(H^*, S^*) \Rightarrow r(p - c_H(H^*, S^*)) = pG'(S^*) - c_H(G(S^*), S^*)G'(S^*) - c_S(G(S^*), S^*) \Rightarrow r(p - c_H(H^*, S^*)) = pG'(S^*) - c_H(G(S^*), S^*)G'(S^*) - c_S(G(S^*), S^*)$.

the cost (in terms of a reduced stock size) are borne by all. We will consider one especially stark form of unregulated competitive harvesting: open access when costs are proportional to effort.

Under open access effort will enter the fishery until all rents are dissipated. The economic equilibrium condition is therefore $pH(E,S) = c(E)$. Using equations (4) and (5) as above, this can be re-written as:

$$\begin{aligned} pH &= w\left(\frac{H}{qS^{\beta}}\right)^{\frac{1}{1-\alpha}} \\ H^{\frac{-\alpha}{1-\alpha}} &= \frac{w}{p} q^{\frac{-1}{1-\alpha}} S^{\frac{-\beta}{1-\alpha}} \\ H &= \left(\frac{w}{p}\right)^{\frac{1-\alpha}{-\alpha}} q^{\frac{1}{\alpha}} S^{\frac{\beta}{\alpha}} \\ H^{OA} &= \underbrace{\left(\frac{p}{w}\right)^{\frac{1-\alpha}{\alpha}} q^{\frac{1}{\alpha}}}_{\tilde{A}} S^{\frac{\beta}{\alpha}} \quad = \tilde{A} S^{\frac{\beta}{\alpha}} \end{aligned} \tag{15}$$

Compare this to (11):

$$H^* = \underbrace{\left(\frac{(p-\mu^*)(1-\alpha)}{w}\right)^{\frac{1-\alpha}{\alpha}} q^{\frac{1}{\alpha}}}_{A} S^{\frac{\beta}{\alpha}} \quad = \quad AS^{\frac{\beta}{\alpha}}$$

Clearly since $0 < \alpha < 1$ and $\mu^* > 0$ we have that $H^* < H^{OA}$ and consequently $S^* > S^{OA}$.

In other words, open access can – as the optimal control problem with an infinite discount rate – yield only a short-term profit as the fish stock is harvested down to the open-access level S^{OA}. This is of course the root of the fishermen's myopia: given that someone else will harvest any fish that they leave in the ocean as an investment into the future, they have no incentive to do so. Note that the open-access stock level will be inefficiently low but positive, if there is stock-dependency in the harvesting cost. Conversely, this implies that the fishery will be extirpated when the stock size has no influence on the harvesting cost.

Fisheries management

Fisheries management is an important real-world problem, not the least because fisheries could significantly contribute to food security for the world's growing population. However, this is a very tricky issue as demonstrated by one of the most common regulatory instruments in developed fisheries, the "total allowable catch" quota (TAC): An overall limit for the harvest in a given season is specified and the fishery is closed whenever this limit is reached. While such a regulation effectively safeguards the remaining stock, it does not avoid rent-dissipation. Rather, such a regulation invites a “race to fish” where every fisherman tries to catch a large as possible share of the total quota. Any other regulatory instrument has its own drawbacks and far too often one can observe how the inefficiency of competitive harvesting sneaks in through the backdoor.

Theoretically however, the optimal harvesting solution could be easily established by a Pigouvian tax τ on harvesting: Fishermen would take this into account and under open access, we would have $(p-\tau)H = c(E)$ which would lead to the following harvest function:

$$H^{tax} = \left(\frac{p-\tau}{w}\right)^{\frac{1-\alpha}{\alpha}} q^{\frac{1}{\alpha}} S^{\frac{\beta}{\alpha}}$$

Compare this to (11):

$$H^{*} = \left(\frac{(p-\mu^{*})(1-\alpha)}{w}\right)^{\frac{1-\alpha}{\alpha}} q^{\frac{1}{\alpha}} S^{\frac{\beta}{\alpha}}$$

It is then plain to see that the optimal tax must be $\tau = \alpha p + (1-\alpha)\mu^{*}$, correcting both for the stock and the crowding externality.

However, such a tax will be informationally expensive, and – to the best of my knowledge – no real world fishery is managed by this instrument. In contrast, individual transferable quotas (ITQ) are increasingly being used, also in Norway. Theoretically they work very similar to a Pigouvian tax: every fishermen has to buy a quota at price ρ per unit of harvest (alternatively, he has to pay the opportunity cost of not selling the quota unit), and by adequately setting the total allowable catch (i.e. by determining quota supply in the market), it can be ensured that $\rho = \tau$. Effectively, an ITQ-system therefore establishes private property rights to a common pool resource, thereby overcoming the “tragedy of the commons”. Accordingly, there is an argument that ITQs establish stewardship incentives, but this is much disputed in the literature.

4 Extension: Resource exhaustion

There are many perceivable extensions to this model. First and foremost, the sea is an unknown place: Even with the most advanced stock assessment methods, no one really knows how many fish there are in the ocean. Similarly, no fish is an island: accounting for multi-species and ecosystem interactions is an important and challenging task. Thirdly, size matters: in spite of the fact that the overall biomass in many industrialized fisheries is reasonably well managed, there are dramatic changes in the size-distribution within the fish stock. Due to the selective properties large fish are over-proportionally exposed to harvesting pressure, leading not only to economic waste, but also to evolutionary effects. In fact, managing growth overfishing could be more important than managing reproductive overfishing (provided of course that the population size remains sufficiently large so that positive reproduction takes place).

Let us consider this last aspect, the breakdown of the reproduction process leading to the exhaustion of the renewable resource. This could happen “accidentally” when the growth function shows critical depensation. This commonly referred to as stock collapse, meaning a situation where the population abundance has dropped to a mere fraction of its previous size. The idea is that there is a minimum viable population size above which the ecosystem functions normally, but below which the ecosystem changes its state[3] so that the population cannot recover. Now if there are large uncertainties in the system and the natural environment is very volatile it seems to be wise to follow a precautionary approach to stay away from this threshold.

However, resource exhaustion could in fact be an optimal policy in a completely deterministic world, namely if (a) the discount rate is higher than maximum natural growth rate and (b) the cost of catching the last fish are less than its price. Technically, the condition for resource exhaustion is that the harvest function $H(S)$ cuts the growth function $G(S)$ “from above", that is $H(\bar{S}) = G(\bar{S})$ and $\frac{\partial H^{OA}(\bar{S})}{\partial S} < \frac{\partial G(\bar{S})}{\partial S}$.

“Accidental” resource exhaustion could also be the result of hyperbolic discounting (for the original paper see Hepburn et al., 2010). Under hyperbolic discounting, the planner will harvest more in the near future and reduce harvesting pressure later on. If the planner could commit to such a strategy, then no problem would arise, even if the stock level would drop below critical value at which resource exhaustion would be the optimal thing. If the planner cannot make such a commitment, and re-optimizes along the harvesting path, he will find it optimal to deplete the stock once the critical threshold has passed. In fact, it is a common feature of real-world recovery plans that they include an early phase of little effort reduction but promise a much tougher control on harvesting effort later on.

5 Controllable Reproduction

In this section, we will be concerned with natural resources where the process of reproduction can be controlled, as, for example, in aquaculture or in a forestry. In this setting we are left with a question of timing, i.e. when to harvest the resource (as opposed to how much to harvest). To fix ideas, we will switch from fish to forests, not only because the basic concepts and considerations of forestry belong to the education of a resource economist, but also because our scratching on the surface might be an invitation to probe deeper into this fascinating subject area (see Perman et al., 2003, ch.18). An excellent account of the evolution of the field is the paper by Samuelson (1976).

[3] For example by changing the multi-species interactions so that e.g. a now large population of crustaceans eat up almost all cod larvae whereas earlier adult cod were keeping the crustacean population in check.

When to cut a tree?

Let us start simple and suppose that you are given one tree and that you are solely interested in maximizing the profit from selling the wood of that tree. Denote the standing volume of timber by $x(t)$. It is a function of time (the tree grows up to a certain point but will eventually deteriorate). Timber price P and felling cost c (assumed to be proportional to timber volume) are constant in real terms. As usual, the discount rate is given by r. The problem of maximizing the net present value is then to find the time T that solves

$$\max_T \quad (P-c)x(T)e^{-rT}. \tag{16}$$

The first order condition of this problem is then:

$$(P-c)x'(T)e^{-rT} - r(P-c)x(T)e^{-rT} = 0 \tag{17}$$

$$\Leftrightarrow$$

$$r = \frac{x'(T)}{x(T)}. \tag{18}$$

This shows that in this simple model with constant net prices, the optimal time to cut a tree depends solely on the discount rate. The tree should be felled when the growth rate of its value equals the discount rate.

Infinite rotation forestry

Now suppose that you are not given one tree but one plot of land with a lot of trees of all the same age. You are still only interested in maximizing the net present value of profits from your forestry, but now you have to take into account that new trees can be planted (at constant planting cost k) every time old trees have been cut, or alternatively, that you can sell your land at its opportunity cost. Presume again that net prices $p = P - c$ remain constant over time and that the environment does not change. As a consequence, the optimal rotation period (the time passed between planting and cutting of trees) is constant as well. The problem is then to find the rotation period T that solves:

$$\max_T \Pi = px(T)e^{-rT} - k + e^{-rT}[px(T)e^{-rT} - k] + e^{-2rT}[px(T)e^{-rT} - k] + \dots \tag{19}$$

$$= px(T)e^{-rT} - k + e^{-rT}\Pi \tag{20}$$

$$= \frac{px(T)e^{-rT} - k}{1 - e^{-rT}} \tag{21}$$

The resulting first-order-condition[4] is known as the "Faustmann rule":

$$\frac{px'(T)}{px(T)-k} = \frac{r}{1-e^{-rT}} \tag{22}$$

which, after re-arranging, can be written as:

$$px'(T) = rpx(T) + r\Pi \tag{23}$$

or:

$$r = \frac{px'(T)}{px(T)+\Pi} \tag{24}$$

First note that in contrast to the single-tree problem above, planting costs and net prices do play a role for the determination of the optimal length of the rotation period. T is only defined implicitly in equation (22), but it can be shown that the rotation period decreases as the rate of interest r increases, increases as the planting cost k increase, and decreases with increasing net prices. Equation (23), which is an alternative formulation of the Faustmann rule, gives some intuition for the choice of the optimal rotation period: Its length is chosen such that the gain from letting the timber grow for one additional instant (the left-hand-side) exactly equals the cost of doing so. These cost consist both of the money lost from not harvesting the timber and putting the money in the bank (the first term on the right-hand-side) and the money lost from not starting a new growing cycle (or selling the land at its current site value). Finally, the Faustmann rule as written in equation (24) takes the form of an Hotelling rule, where the marginal return on the resource is adjusted by the land value.

[4] To derive the first-order-condition, it is useful to first rewrite (21):

$$\Pi = \frac{px(T)e^{-rT}-k}{1-e^{-rT}} \Leftrightarrow \frac{px(T)-ke^{rT}}{e^{rT}-1} \Leftrightarrow \frac{px(T)-ke^{rT}}{e^{rT}-1} + k\frac{e^{rT}-1}{e^{rT}-1} - k \Leftrightarrow \frac{px(T)-k}{e^{rT}-1} - k$$

which upon differentiating and setting equal to zero yields:

$$\frac{\partial \Pi}{\partial T} = \frac{px'(T)(e^{rT}-1)-(px(T)-k)re^{rT}}{(e^{rT}-1)^2} = \frac{px'(T)}{e^{rT}-1} - \frac{(px(T)-k)re^{rT}}{(e^{rT}-1)^2} = 0$$

$$\Leftrightarrow$$

$$px'(T) = \frac{(px(T)-k)re^{rT}}{e^{rT}-1} \Leftrightarrow \frac{px'(T)}{px(T)-k} = \frac{re^{rT}}{e^{rT}-1} = \frac{r}{1-e^{-rT}}$$

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