Lecture 3 – Non-renewable resources (stock externalities and taxes)

2.2 Unit extraction cost that depend on time

If extraction cost depend on time, the condition corresponding to (3) is: $p(t) = \lambda(t) + c \cdot d(t)$. From (4) we have $\dot{\lambda}(t) = r\lambda(t)$ or $\lambda(t) = \lambda_0 e^{rt}$. This implies that:

$$\dot{p} = \dot{\lambda} + c\dot{d} = r\lambda + c\dot{d} \tag{8}$$

The first term of the RHS of (8) is positive, but if costs are declining fast enough (the second term in the RHS outweighs the first term), the market price may decline.

2.3 Stock-dependent extraction cost

It is quite realistic to consider the case where the resource is not exhaustible in a strict physical sense, but in an economic sense: It is likely that the cost of extracting the last unit of a given resource exceed its marginal value. To formalize this, consider the case when the cost of extraction depend on accumulated extraction, i.e. $C = c(A)x$, where $A(t) = S_0 - S(t)$ and $\dot{A} = x$.

Importantly, assume $u'(0) = b$ and that $c(A)$ is positive, increasing in A and that $c(0) < b < c(S_0)$. The Hamiltonian in this case is:

$$\mathcal{H} = u(x) - c(S_0 - S)x - \lambda x \tag{9}$$

Necessary conditions for an optimum:

$$\frac{\partial \mathcal{H}}{\partial x} = u'(x) - c(S_0 - S) - \lambda = 0 \text{ for } x > 0 \tag{10}$$

$$\dot{\lambda} = r\lambda - \frac{\partial \mathcal{H}}{\partial S} = r\lambda - xc'(S_0 - S) \tag{11}$$

$$\lim_{t\to\infty} e^{-rt}\lambda(t)S(t) = 0 \tag{12}$$

[For a more rigorous discussion of this case, please refer to Hoel and Kverndokk (1996, sec.2 and Appendix A)]. It is "obvious" that the condition $c(S_0) > b$ implies that the constraint $S(t) \geq 0$ won't bind, but that there will be some $\bar{A}$ (which will be reached asymptotically), defined by $c(\bar{A}) = b$. From (12) we thus know that $\lim_{t\to\infty} \lambda(t) = 0$, i.e. the scarcity rent approaches zero. As long as $x > 0$, equation (11) tells us that $\dot{\lambda} < r\lambda$, but other than that we know not whether and when $\dot{\lambda}$ changes sign. The market price $p = u'(x) = c(A) + \lambda$ must however always rise, as (using $\dot{A} = x$ and (11)):

$$\dot{p} = c'(A) \cdot \dot{A} + \dot{\lambda} = c'(A) \cdot x + r\lambda - xc'(A) = r\lambda = r(p - c(A)) > 0 \tag{13}$$

3 Taxes

Read Hoel (2012, sections 1-4)

- Taxes may be used to raise revenue or to correct negative externalities
- Taxes may be levied on:
  - the resource stock
  - the flow of profits
  - the flow of the extracted resource (constant, or variable with time)
  - or they may come in the form of a mining concession
- Key questions are:
  - Will the entire resource stock be extracted?
  - Will there be changes to the extraction path?
- Political constraints on setting taxes

3.1 Constant unit cost

Consider the simple resource extraction problem given by (1) and denote the NPV (the integral of discounted profits) by Π. In equilibrium, supply must equal demand so that, at all times t, we have $x(t) = D(p(t))$, where D is the stationary demand function. Provided $D(c) > 0$, efficiency requires that total extraction is equal to the entire resource: $\int_0^\infty x(t)\mathrm{d}t = S_0$

Now suppose the tax $w(t)$ is so high that:

$$\int_0^\infty D(c + w(t))\mathrm{d}t < S_0 \tag{14}$$

In this case, the resource constraint is not binding, there is no scarcity rent and resource extraction is dictated by unit cost and the instantaneous tax rate, independent of the future tax rate (no green paradox, reduction of total emissions). A tax path that is so high is hardly realistic, but it may be realistic for high cost reserves (such as tar sand) in a model with heterogenous unit costs (see below).

Now consider a tax path that does not satisfy (14). Then all the resource will be extracted. Denote by Ω the present value of taxes:

$$\Omega = \int_0^\infty e^{-rt} w(t)x(t)\mathrm{d}t \tag{15}$$

Producers now maximize $\Pi - \Omega$. Assume that $w(t) = \tau e^{gt}$ (the tax rises at a constant rate g). Then we can write (15) as:

$$\begin{aligned} &= \int_0^\infty \tau e^{(g-r)t} x\mathrm{d}t = \tau \int_0^\infty \left[e^{(g-r)t}x + x - x\right]\mathrm{d}t = \tau \int_0^\infty x\mathrm{d}t + \tau \int_0^\infty \left[\left(e^{(g-r)t} - 1\right)x\right]\mathrm{d}t \\ &= \tau S_0 + \tau \int_0^\infty \left[\left(e^{(g-r)t} - 1\right)x\right]\mathrm{d}t \end{aligned}$$

This can be combined with (1), so that:

$$\Pi - \Omega = \Pi - \tau S_0 + \tau \int_0^\infty \left[\left(e^{(g-r)t} - 1 \right) x \right] \mathrm{d}t \tag{16}$$

This gives us three cases:

$g = r$ like a lump-sum tax (tax burden is constant over time), no change in extraction path

$g > r$ tax is burden is increasing with time, shift extraction forward

$g < r$ tax is burden is decreasing with time, delay extraction

3.2 Unit cost rising with accumulated extraction

The relevant model is the one given by equations (9)-(12) only that the tax rate $w(t)$ enters as an additional term. Accordingly updating (10) and using $u'(x) = p$ implies the following price path:

$$\dot{p} = r(p - c(A)) + [\dot{w} - rw] \tag{17}$$

Since $u'(0) = b$, the producers cannot demand a higher price than b. Hence, total extraction is now given by $c(A^*) = b - w(T^*)$, which means that (for $w > 0$ and $c'(A^*) < \infty$) total extraction will go down, no matter what the time-profile of the tax is. What about the near- and long-term emissions. Consider first the case that $\dot{w} \leq rw$. If the initial case would not increase in this case, it would never be higher than the original price path, implying that the consumer price would not reach the choke price. This cannot be optimal, hence the new initial price must be higher and near-term emissions are reduced. In fact, near-term emissions are reduced even if the tax rises at a (slightly) larger rate than r, namely when $\dot{w} \leq (r + \delta)w$ where δ is given by that margin that leads to the price path exactly hitting the choke price at time T.

References

Hoel, M. (2012). Carbon taxes and the green paradox. In Hahn, R. and Ulph, A., editors, *Climate Change and Common Sense: Essays in Honour of Tom Schelling*. Oxford University Press.

Hoel, M. and Kverndokk, S. (1996). Depletion of fossil fuels and the impacts of global warming. *Resource and Energy Economics*, 18(2):115–136.