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RESOURCE MANAGEMENT IN A TRADING ECONOMY*

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I. INTRODUCTION

With relatively few exceptions the extensive recent literature on resource depletion (reviewed in Dasgupta and Heal, 1977) has been concerned with analyzing depletion policies for closed economies—or, equivalently, for an economic system taken to be the world as a whole. Although the insights thus gained have often proved valuable, it is nevertheless true that this literature can provide very little help in answering a number of important policy questions. One such question, faced by every resource-rich country, concerns the extent to which resources should be exported rather than used domestically. Obviously, there is often a choice between using a natural resource as an input in a domestic industry or exporting it, and in this latter case the foreign exchange so generated may be used either to import capital to aid domestic capital formation, or to build up foreign assets on which interest may be earned. In the remaining sections of this paper, we shall present a model that enables us to analyze the optimal policy mix for an economy faced with such a range of options. It will be shown that, at least in the case of complete certainty about the future, the optimal depletion policy for such an economy may be determined by factors very different from those that are influential in the closed-economy case. Specifically, it will be shown that if the rate of return on foreign assets is exogenous, the depletion rate should be independent both of the discount rate and of the elasticity of marginal utility. Of course, in the no-trade case these two parameters play a very important role (as for example in equation (1.29) of Dasgupta and Heal, 1974), and in particular, the depletion policy is found to be very sensitive to the choice of discount rate. Given the considerable difficulties that arise in choosing an appropriate value for this parameter, the fact that an increase in the realism of the model leads to a sharp reduction in its importance may be a source of relief to those

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charged with extracting policy implications from this literature. In the final sections of the paper, we consider the effect of relaxing the assumptions of certainty and of exogenism of the foreign rate of return. Unfortunately, either move produces considerable complications.

II. THE MODEL

The production possibilities open to the economy we are considering are given by

$$Y_t = F(K_t,R_t,t),$$

where K_t and R_t are, respectively, the stock of domestic capital and the rate of utilization of the exhaustible resource in domestic production at time t. Because of the importance of other factors (such as labor and land) omitted from the model, $F(\ )$ is assumed to show decreasing returns to K and R. In addition to being used as a domestic input, the resource may be exported: if E_t is the rate of export at time t, it fetches an f.o.b. export price of $P(E_t)$, and the stock remaining at t satisfies

$$S_t = S_0 - \int_0^t (R_\tau + E_\tau)d\tau$$

or

$$\dot{S}_t = -(R_t + E_t). \tag{1}$$

The economy's total capital assets at time t are W_t, and consist of K_t of domestic capital and $(W_t - K_t)$ of foreign investments earning a rate of return r. Capital is malleable in the sense that domestic capital can be instantly and costlessly transformed into foreign capital, and vice versa. Hence, the basic accounting identity for this system is

$$\dot{W}_t = F(K_t,R_t,t) + r(W_t - K_t) + P(E_t)E_t - C_t, \tag{2}$$

where C_t is of course the rate of domestic consumption at time t. The overall planning problem facing the economy is now seen to be

$$\max \int_0^\infty u(C_t)e^{-\delta t}dt, \qquad \delta \geqq 0, \tag{3}$$

subject to (1), (2) and S_0 and W_0 given, $u'(0) = +\infty$, and the maximization is understood to be in the overtaking sense if the utility integral diverges. By introducing the Hamiltonian,

$$\mathcal{H} = u(C)e^{-\delta t} + \lambda e^{-\delta t}\{F(K,R,t)$$
$$+ r(W - K) + P(E)E - C\} - \mu e^{-\delta t}\{R + E\},$$

where λ and η are the shadow prices on total wealth W and the resource stock S, respectively, it is easily verified that necessary conditions for a solution to (3) are

$$u'(C) = \lambda \tag{4}$$

$$\lambda d/dE\{P(E)E\} = \mu \tag{5}$$

$$F_K \leqq r, \quad = \text{if } K > 0 \tag{6}$$

$$\lambda F_R \leqq \mu, \quad = \text{if } R > 0 \tag{7}$$

$$\dot{\lambda} - \delta\lambda = -\lambda r \tag{8}$$

$$\mu_t = \mu_0 e^{\delta t}. \tag{9}$$

It is reasonable to assume that

$$F(0,K,t) = F(K,0,t) = 0 \, \forall t, \tag{10}$$

in which case (6) will hold with equality if and only if (7) does: we investigate initially the case when both K and R are positive along an optimal path. In this case, (5) and (7) imply that

$$d/dE\{P(E)E\} = F_R, \tag{11}$$

and (6) and (7) imply that

$$\dot{F}_R/F_R = r = F_K, \tag{12}$$

while (4) and (8) lead to

$$u''\dot{C}/u' = \delta - r. \tag{13}$$

Equations (11), (12), and (13) are very obvious conditions. Equation (11) is merely a necessary condition for static efficiency, requiring equality of marginal export revenue and marginal productivity of the resource. Equation (12) is a necessary condition for dynamic efficiency: there are three assets in the model—domestic and foreign capital and the resource—and the rates of return to these are, respectively, F_K, r, and $\dot{F}_R/F_R$. Efficiency requires that these be equated. Equation (13) is the usual condition for optimality of an intertemporal consumption profile, requiring the marginal utility of consumption to fall at a rate equal to the net rate of return—or, equivalently, that the consumption rate of discount $(\delta - u''\dot{C}/u')$ equals the rate of return.

In order to pursue the analysis further, we shall adopt specific forms for the various functional relationships encountered: this will simplify matters and allow explicit solutions to be reached, without detracting noticeably from their generality. We therefore assume that

$$(14) \qquad -u''(C)C/u'(C) = \eta, \qquad \text{constant}$$

$$(15) \qquad P(E) = \gamma E^{\gamma-1}, \qquad 0 < \gamma < 1$$

$$(16) \qquad F(K,R,t) = e^{\beta t} K^{\alpha_1} R^{\alpha_2}, \qquad \alpha_1 + \alpha_2 < 1.$$

Equations (13) and (14) imply that

$$(17) \qquad C_t = C_0 e^{[(r-\delta)/\eta]t}.$$

Equations (11), (12), and (15) give

$$(18) \qquad E_t = E_0 e^{[r/(\gamma-1)]t}.$$

From (12),

$$F_R(t) = F_R(0)e^{rt}$$

so that

$$(19) \qquad K_t{}^{\alpha_1} R_t{}^{\alpha_2-1} = K_0{}^{\alpha_1} R_0{}^{\alpha_2-1} e^{(r-\beta)t},$$

and

$$(20) \qquad K_t = [\alpha_1/r]^{1/(1-\alpha_1)} R_t{}^{\alpha_2/(1-\alpha_1)} e^{[\beta/(1-\alpha_1)]t}.$$

Using (20) and (19), we see that

$$\begin{aligned} R_t = {} & [\alpha_1/r]^{\alpha_1/(1-\alpha_1-\alpha_2)} K_0{}^{-\alpha_1(1-\alpha_1)/(1-\alpha_1-\alpha_2)} \\ & \times R_0{}^{(1-\alpha_1)(1-\alpha_2)/(1-\alpha_1-\alpha_2)} \\ (21) \qquad & \times \exp\{[\beta - (1-\alpha_1)r]/(1-\alpha_1-\alpha_2) \cdot t\}, \end{aligned}$$

and

$$\begin{aligned} K_t = {} & [\alpha_1/r]^{1/(1-\alpha_1)+\alpha_1\alpha_2/(1-\alpha_2)(1-\alpha_1-\alpha_2)} \\ & \times K_0{}^{-\alpha_1\alpha_2/(1-\alpha_1-\alpha_2)} R_0{}^{\alpha_2(1-\alpha_2)/(1-\alpha_1-\alpha_2)} \\ (22) \qquad & \times \exp\{(\beta - \alpha_2 r)/(1-\alpha_1-\alpha_2) \cdot t\}. \end{aligned}$$

At this point it is important to distinguish between two cases:

$$(23) \qquad \begin{cases} \beta < (1-\alpha_1)r & \dot{R} < 0 \\ \beta \geqq (1-\alpha_1)r & \dot{R} \geqq 0. \end{cases}$$

In the former case, the declining rate of resource use means that there will typically be paths satisfying (21) and the integral constraint (1) on resource consumption. In the latter case, however, this is not so: any solution to (21) involves infinite resource consumption and hence resource imports: while these are not explicitly ruled out in the sense that E is not constrained to be nonnegative, the demand function (15) is not defined for negative E, and indeed a solution involving continually increasing imports is in many ways unappealing. We therefore concentrate below on the case where $\beta < (1 - \alpha_1)r$.

Note that

$$\dot{K} \geqq 0 \qquad \text{iff } \beta \geqq \alpha_2 r, \tag{24}$$

so that if $\beta < \alpha_2 r$, the scale of domestic production shrinks asymptotically to zero. (Of course, $\alpha_2 r < (1 - \alpha_1)r$.) The possibility that domestic production may be gradually phased out on an optimal path is easily explained: the dynamic efficiency condition (12) implies that

$$F_K = r.$$

Now, F_K is a declining function of K/R: on any feasible path, R tends to zero, so that to maintain F_K constant, K must also fall. But with technical progress, F_K has an exogenous upward trend, which may be sufficiently great to permit F_K to remain constant even when K is nondecreasing. Hence we find an inequality such as (24).

It is now very easy to demonstrate that the optimum time profiles of K_t, R_t, and E_t (and, therefore, the optimum rate of extraction $R_t + E_t$) are each independent of preferences (i.e., independent of η and δ). Note first that we shall have demonstrated this if it can be shown that K_0, R_0, and E_0 are independent of η and δ. Equations (18), (21), and (22) will then directly confirm this claim. Now, for a policy to be optimal we must have

$$\int_0^\infty (R_t + E_t)dt = S_0. \tag{25}$$

Moreover, equation (12) implies that

$$r = \alpha_1 K_0{}^{\alpha_1 - 1} R_0{}^{\alpha_2}, \tag{26}$$

and equation (11) implies that

$$\alpha_2 K_0^{\alpha_1} R_0^{\alpha_2 - 1} = \gamma^2 E_0^{\gamma - 1}. \tag{27}$$

If we now use equations (18) and (21) in equation (25), this last and equations (26) and (27) are three equations in three unknowns, K_0, R_0, and E_0. The unique solution is independent of preferences. Along the optimum, domestic capital stock rises at the rate $(\beta - \alpha_2 r)/(1 - \alpha_1 - \alpha_2)$; resource export falls at the rate $r/(1 - \gamma)$; and domestic resource use falls at the rate $(r(1 - \alpha_1) - \beta)/(1 - \alpha_1 - \alpha_2)$. This independence result is similar to a standard result in static trade theory, to the effect that an open economy's optimum production point is independent of its preferences, and determined entirely by world prices.

In fact, the present result is slightly stronger than the static result in the following sense. The static result implies that domestic production, but not of course trade, is independent of preferences. We have shown that both domestic production and trade (that is, E) are independent of preferences. The reason is, of course, that export policy is determined just by the efficiency conditions (11) and (12).

We have so far obtained explicit solutions for the time paths of C, E, K, and R. This leaves only W, an explicit solution for which may obtained by substituting the foregoing into (2) and integrating. This is left as an exercise for the (very) interested reader. Because C depends on δ and η, it is clear that W will also.

In the following sections we consider some modifications of the model that lead to rather different results. However, before tackling these, we shall mention briefly a change that allows a useful gain in realism at little extra cost. The external rate of return has so far been taken as exogenous and constant: exogenism is surely an acceptable proxy to certain situations, but one might worry about the constancy. If the exhaustible resource were an essential input in all countries, one would expect the foreign rate of return to decline as the worldwide capital-resource ratio rises over time. One could model this by replacing the constant r by the variable $r(t)$ defined by

$$r(t) = r^* + \gamma e^{-\epsilon t},$$

where

$$r^*, \gamma, \epsilon \geqq 0.$$

The foreign rate of return now declines exponentially toward a nonnegative lower limit. The effect of this is merely to replace r by $r(t)$ in equations (1) to (23). If $r^* > 0$, the only difference worthy of comment is that, because one may have $r^* < \delta < r(0)$, the optimal consumption profile may be single-peaked, first rising and then falling as the foreign return drops below the discount rate. The avoidance of continually increasing imports now requires that

$$(23') \qquad \beta < (1 - \alpha_1) r(t) \qquad \text{for all } t,$$

and it is clear that this condition cannot be satisfied if $r^* = 0$ and $\beta > 0$. This is thus a special case for which, in the framework of the model and functional forms involved, there is no reasonable optimal solution: but for r^* positive, though arbitrarily small, the behavior of the solution is not altered significantly. Of course, if $r^* = 0$ and

$\beta = 0$, the problem is asymptotically equivalent to the optimal depletion problem under autarky (discussed in Dasgupta and Heal, 1974), and so both domestic resource use and capital stock fall to zero in the long run.

III. VARIABLE RETURN ON FOREIGN INVESTMENT

If the return on foreign investment is written as $r(W - K)$, $r' < 0$, $r'' \neq 0$, then it is easily verified that the necessary conditions for optimality take the form

$$u'(C) = \lambda \tag{4'}$$

$$\lambda d/dE\{P(E)E\} = \mu \tag{5'}$$

$$F_K \leqq r'(W - K)(W - K) + r, \qquad = \text{if } K > 0 \tag{6'}$$

$$\lambda F_R \leqq \mu, \qquad = \text{if } R > 0 \tag{7'}$$

$$\dot{\lambda} - \delta\lambda = -\lambda\{r'(W - K)(W - K) + r\} \tag{8'}$$

$$\mu_t = \mu_0 e^{\delta t}. \tag{9'}$$

These equations can be given exactly the same efficiency and optimality interpretations as (4) to (9), namely,

$$d/dE\{P(E)E\} = F_R \tag{11'}$$

$$\dot{F}_R/F_R = r'(W - K)(W - K) + r = F_K \tag{12'}$$

$$u''\dot{C}/u' = \delta - r'(W - K)(W - K) - r. \tag{13'}$$

However, the appearance of W in (12′) and (13′) means that the equations are much more interdependent than they were previously and cannot now be integrated one at a time. In general, the time path of any variable will depend on all the parameters of the system, and the previous independence results are no longer valid.

IV. UNCERTAIN FUTURE DEMAND

Another aspect of our original model, which one might reasonably wish to relax, is the one that postulates that future demand conditions on export markets are known with certainty. Obviously, one of the main risks faced by any resource-rich country is that new technologies or resource discoveries will bring a sharp change in demand conditions in the future. Such an event cannot, however, be anticipated with certainty. To model this phenomenon, we suppose that at some future date T there will be a sharp change in foreign demand conditions for the resource. For illustrative purposes, one might imagine that at T the export demand function changes from $P(E)$ to $P^*(E)$, $P^*(E) < P(E)$ for all E. Although this eventuality is anticipated in principle, its precise timing is unknown: T is thus seen as a random variable with density function $W(T)$. We now define a real-valued function $V[W_T,S_T]$ of the stocks of wealth and resources available at time T as

$$V[W_T,S_T] = \max \int_T^{\infty} u(C_t)e^{-\delta(t-T)}dt,$$

subject to all constraints operative from T onward.

If the change in demand occurs at some date T_1, then the total payoff is

$$\int_0^{T_1} u(C_t)e^{-\delta t}dt + V[W_{T_1},S_{T_1}]e^{-\delta T_1},$$

so that the expected payoff is

$$\int_0^{\infty} W(T_1)\left\{\int_0^{T_1} u(C_t)e^{-\delta t}dt + V[W_{T_1},S_{T_1}]e^{-\delta T_1}\right\} dT_1,$$

which, defining

$$\Omega(T_1) = \int_{T_1}^{\infty} W(t)dt,$$

may be rewritten as

$$\int_0^{\infty} e^{-\delta t}\{\Omega(t)u(C) + W(t)V(W_t,S_t)\}dt. \tag{28}$$

The planning problem is now one of maximizing (28) subject to the earlier constraints (1) and (2): necessary conditions are

$$u'(C)\Omega(t) = \lambda \tag{4''}$$

$$\lambda d/dE\{P(E)E\} = \mu \tag{5''}$$

$$F_K \leqq r, \qquad = \text{if } K > 0 \tag{6''}$$

$$\lambda F_R \leqq \mu, \qquad = \text{if } R > 0 \tag{7''}$$

$$\dot{\lambda} - \delta\lambda = -\lambda r - W(t)V_W \tag{8''}$$

$$\dot{\mu} - \delta\mu = -W(t)V_S. \tag{9''}$$

Positivity of domestic production still implies the static efficiency condition,

$$(11'') \qquad d/dE\{P(E)E\} = F_R,$$

but the analogue of the dynamic efficiency condition is now

$$(12'') \qquad \dot{F}_R/F_R + W\{V_S/\mu - V_W/\lambda\} = r = F_K,$$

and the optimality condition is

$$(13'') \qquad u''\dot{C}/u' = \delta - r - WV_W/\lambda + W/\Omega.$$

Once again, W and K appear in the dynamic efficiency and optimality conditions, giving a degree of interdependence too great to permit integration in the general case. Once again, therefore, the independence of our first simple case is missing. There is, however, one special case in which it reappears. Suppose that

$$(29) \qquad V_S/V_W = \mu/\lambda.$$

Then the dynamic efficiency conditions are as in Section II; and if one adopts the functional forms (14) to (16), it is possible to obtain explicit solutions for E, R, and K identical to those given in (18), (21), and (22). Equation (29) can be given a natural economic interpretation: V_S and V_W are the shadow prices of S and W after the change in demand conditions, and μ and λ are the same prices before. Equation (29) is therefore satisfied if the random event leaves the relative shadow prices of the two types of wealth unchanged.

V. CONCLUSIONS

Opening a resource-rich economy to trade can clearly change its depletion policies considerably. In the simplest case, which is perhaps not a bad approximation to the world around us, choosing an optimal depletion policy becomes a very simple asset management policy because there is one asset whose return is given exogenously. When this return is not exogenous, the problem is not separable in this way, and matters are more complex. Dropping the certainty assumption, but maintaining that of a constant foreign return, again produces complications. As there is still an asset with a constant rate of return, an optimal policy again consists of aligning the risk-adjusted expected rates of return on others with this. But of course the appropriate risk adjustment depends on the parameters of the preference function.

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