Resource extraction, uncertainty, and learning

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This paper analyzes resource extraction when the true size of the reserves and the future costs of extraction are uncertain. The effects of increased uncertainty are investigated under the assumption that the extraction process yields information about the variables which are uncertain at the initial planning time. Mean-stock-preserving and mean-cost-preserving increases in uncertainty affect the initial resource extraction rate differently. Mean-utility-preserving increases in uncertainty do not affect the initial rate of extraction.

1. Introduction

■ Several economists have analyzed the extraction of a nonrenewable resource when the true size of the initial resource stock is uncertain (Cropper, 1976; Gilbert, 1976; Kemp, 1976, ch. 23; Loury, 1976). Most of this literature ignores the possibility that the extraction process reveals useful information about the true size of the resource stock.

The present paper provides an analysis of resource extraction when the true reserves and the costs of future extraction are uncertain. A very simple learning process is assumed, and the effects of uncertainty are investigated. We assume that there are two deposits: A and B. Because of technical considerations, deposit A must be completely exhausted before extraction from deposit B can take place. (For example, deposit B may lie deeper in the mine than deposit A.) Weitzman (1976) has analyzed resource extraction under certainty when such technical constraints on the sequence of extraction are present.

Furthermore, we assume that the stock and extraction costs of deposit A are known with certainty, while the corresponding variables for deposit B are uncertain at the initial planning time. However, information which may change the decisionmaker's subjective probability distributions of these variables is obtained through the process of extracting deposit A. We shall use the following drastically simplified representation of this learning process: As long as some of deposit A remains unextracted, no changes are made in the probability distribution of the stock and extraction costs of deposit B. At the instant deposit A is completely exhausted, the information obtained through this extraction process gives the decisionmaker perfect knowledge of the true stock and extraction costs of deposit B.

The following sections give a brief outline of the formal model and present the results. For a more detailed derivation of the results, see Hoel (1978a).

2. The formal model

■ The decisionmaker wishes to maximize the expected present value of utility minus the extraction costs. The utility function is assumed to be an increasing concave function of resource extraction, and unit extraction costs are constant. The discount factor r is constant and positive. At the initial point in time ($t = 0$), the decisionmaker decides the exhaustion date ($t = T$) for deposit A. For a given T, the maximal present value of the utility minus the cost of extracting deposit A is denoted by $F(T)$. At time T the maximal present value of the utility minus cost of extracting deposit B is denoted by $g(R,b)$, where R is the stock and b is the extraction cost of deposit B. Notice that under our assumptions R and b are known at time T. Obviously $g_R > 0$ and $g_b < 0$. Furthermore, it can be shown that g is strictly concave in R and strictly convex in b (Hoel, 1978a).

At the initial planning time ($t = 0$), R and b are not known with certainty. We shall assume that the decisionmaker has a subjective probability distribution of R and b, with R and b independently distributed. The total expected present value of utility minus costs at $t = 0$ is therefore given by

$$EW = F(T) + e^{-rT}Eg(R,b), \tag{1}$$

where $Eg(R,b)$ is the expected value of $g(R,b)$ for the subjective probability distribution of R and b the decisionmaker holds at $t = 0$. The optimal choice of T is that value which maximizes (1), i.e., satisfying

$$F'(T) - re^{-rT}Eg(R,b) = 0, \tag{2}$$

$$\frac{\partial}{\partial T}[F'(T) - re^{-rT}Eg(R,b)] < 0. \tag{3}$$

3. The effects of increased uncertainty

■ Holding the distribution of b unchanged, consider a mean-stock-preserving increase in the uncertainty of R, as defined by Diamond and Stiglitz (1974) and Rothschild and Stiglitz (1970). Since $g(R,b)$ is concave in R, such an increase in the spread of R will reduce $Eg(R,b)$. But from (2) and (3) this implies that T will increase. Furthermore, it is easily verified that the rate of extraction of deposit A must be lower for all $t < T$, the higher is T. We therefore have the following conclusion:

Proposition 1: For any distribution of future extraction costs, a mean-stock-preserving increase in the uncertainty of the size of deposit B will reduce the rate of extraction of deposit A for all $t < T$ and delay the date of exhaustion of deposit A.

Notice that this proposition also covers the case in which b is known with certainty, and the comparison is between the case in which R is known with certainty and the case in which R is uncertain. The proposition implies that uncertainty with respect to the true size of the resource stock tends to reduce the optimal initial extraction rate. This result accords with Gilbert (1976) for the case in which no real learning takes place.

Let us now turn to the case in which the distribution of R is held unchanged, and consider a mean-cost-preserving increase in the uncertainty of b. Since $g(R,b)$ is convex in b, such an increase in the spread of b will increase $Eg(R,b)$.

But from (2) and (3) this implies that T will decline. From what we said above, we can therefore draw the following conclusion:

Proposition 2: For any distribution of the size of deposit B, a mean-cost-preserving increase in the uncertainty of the cost of extracting deposit B will increase the rate of extraction of deposit A for all $t < T$ and advance the date of exhaustion of deposit A.

The proposition implies that uncertainty with respect to the future extraction cost tends to increase the optimal initial extraction rate. Notice that the proposition is also valid for the special case in which $R = +\infty$, i.e., deposit B is a "backstop technology" (Nordhaus, 1973). A case similar to the latter case is treated in Hoel (1978b). There it is assumed that the date at which the production cost b becomes known is given and independent of the accumulated extraction level. For that case, Hoel (1978b) shows that increased cost uncertainty usually will tend to increase the initial rate of extraction.

It may seem surprising that the effect of increased uncertainty of the extraction cost is the opposite of the effect of increased uncertainty of the size of the resource stock. The reason for the difference is clearly that the two types of uncertainty have different effects on $Eg(R,b)$. An implication of this difference is that while increased uncertainty of R reduces EW, an increase in the uncertainty of b will increase EW (cf. (1)).

We shall call any change in the probability distribution of R and b which leaves the maximized value of EW unchanged a mean-utility-preserving change in the probability distribution (Diamond and Stiglitz, 1974). The reasoning above suggests that any such change in the probability distribution will leave T unchanged. This can be proved as follows. Consider a change in the probability distribution represented by a change in a parameter α. From (1) we have

$$\frac{dEW}{d\alpha} = [F'(T) - re^{-rT}Eg(R,b)]\,\frac{dT}{d\alpha} + e^{-rT}\,\frac{dEg(R,b)}{d\alpha}\,. \tag{4}$$

From (2) and (4) we see that $dEW/d\alpha = 0$ implies $dEg(R,b)/d\alpha = 0$. But then from (2) it follows that $dT/d\alpha = 0$. We therefore have the following proposition:

Proposition 3: A mean-utility-preserving change in the probability distribution of the stock and extraction cost of deposit B will have no effect on the rate of extraction of deposit A for any t or on the date of exhaustion of deposit A.

This proposition of course also covers the comparison between the case of certainty and the case of uncertainty. In a model without any explicit learning, Loury (1976) has shown that uncertainty of the resource stock does not affect the initial resource extraction as long as the expected discounted utility is the same in the case with uncertainty as it is in the case without uncertainty. The proposition above confirms this result for the case with explicit learning and also extends the result to the case in which it is the future extraction cost which is uncertain.

4. Concluding comments

■ The simple way we have introduced explicit learning in the present paper does not change the results of Gilbert (1976) and Loury (1976) about how uncertainty affects the optimal initial rate of resource extraction.

A second important conclusion is that uncertainty with respect to the size of the resource stock may affect the extraction path in a way which differs from uncertainty with respect to the future extraction cost. Economic models with a fixed and finite supply of a homogeneous natural resource are sometimes regarded as a simplified representation of the more realistic case where the natural resource is gradually becoming more costly to extract as it is depleted, but is never completely exhausted. Our results in Propositions 1 and 2 indicate that when uncertainty is treated, the two descriptions of natural resource scarcity have different implications.

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