ECON 4930 Spring 2011
Electricity Economics
Lecture 10

Lecturer:

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Restricting spill

$$\max \sum_{t=1}^{T} p_t(e_t^H) \cdot e_t^H$$

subject to

$$R_t = R_{t-1} + w_t - e_t^H$$

$$R_t \leq \overline{R}$$

$$R_t, e_t^H \geq 0 \; , \; t = 1,..,T$$

$$T, w_t, R_o, \overline{R} \text{ given}, R_T \text{ free}$$

- The Kuhn – Tucker conditions

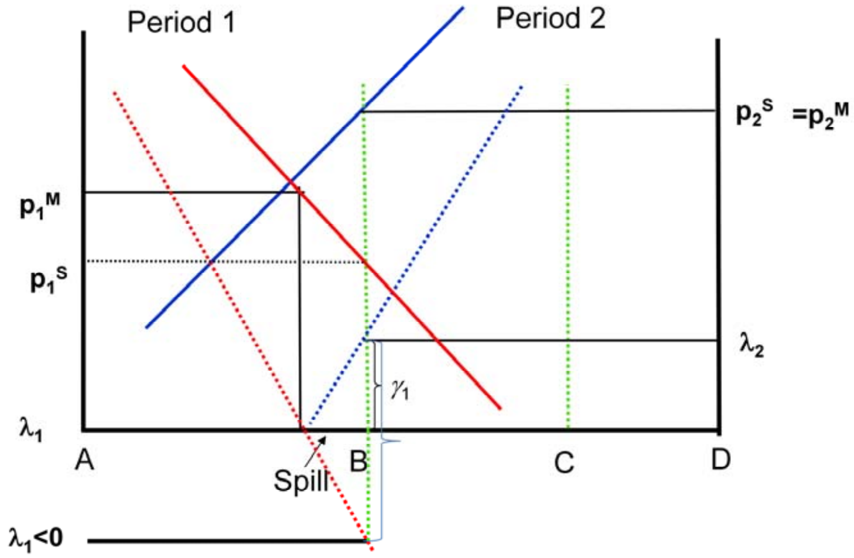
$$\frac{\partial L}{\partial e_t^H} = p_t'(e_t^H)e_t^H + p_t(e_t^H) - \lambda_t \leq 0 \; (= 0 \text{ for } e_t^H > 0)$$

$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \leq 0 \; (= 0 \text{ for } R_t > 0)$$

$$\gamma_t \geq 0 \; (0 = \text{for } R_t < \overline{R}) \,, \qquad t = 1,..,T$$

- The reservoir constraint is now an equality constraint and λt is free in sign
- The price flexibility becomes less than -1, and the water value negative for period 1
- We get the social solution for water use and prices

Illustration of monopoly solution with reservoir and spill



Monopoly and trade

- Optimisation problem

$$\max \sum_{t=1}^{T} (p_t(x_t)x_t + p_t^{XI} e_t^{XI})$$

subject to

$$x_t = e_t^H - e_t^{XI}$$

$$-\overline{e}^{XI} \le e_t^{XI} \le \overline{e}^{XI},$$

$$R_t \le R_{t-1} + w_t - e_t^H$$

$$R_t \le \overline{R}$$

$$x_t, e_t^H, R_t \ge 0$$

$$T, \overline{R}, \overline{e}^{XI}, p_t^{XI} \text{ given, } e_t^{XI} \text{ free, } t = 1,..,T$$

- The Lagrangian

$$L = \sum_{t=1}^{T} p_t(e_t^H - e_t^{XI}) \cdot (e_t^H - e_t^{XI}) + p_t^{XI} e_t^{XI}$$

$$-\sum_{t=1}^{T} \lambda_t (R_t - R_{t-1} - w_t + e_t^H)$$

$$-\sum_{t=1}^{T} \gamma_t (R_t - \overline{R})$$

$$-\sum_{t=1}^{T} \alpha_t (e_t^{XI} - \overline{e}^{XI})$$

$$-\sum_{t=1}^{T} \beta_t (-e_t^{XI} - \overline{e}^{XI})$$

- The first-order conditions

$$\frac{\partial L}{\partial e_t^H} = p_t'(e_t^H - e_t^{XI})(e_t^H - e_t^{XI}) + p_t(e_t^H - e_t^{XI}) - \lambda_t \le 0$$

$$(= 0 \text{ for } e_t^H > 0)$$

$$\frac{\partial L}{\partial e_t^{XI}} = -p_t'(e_t^H - e_t^{XI})(e_t^H - e_t^{XI}) - p_t(e_t^H - e_t^{XI}) - \alpha_t + \beta_t + p_t^{XI} = 0$$

$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \ (= 0 \text{ for } R_t > 0)$$

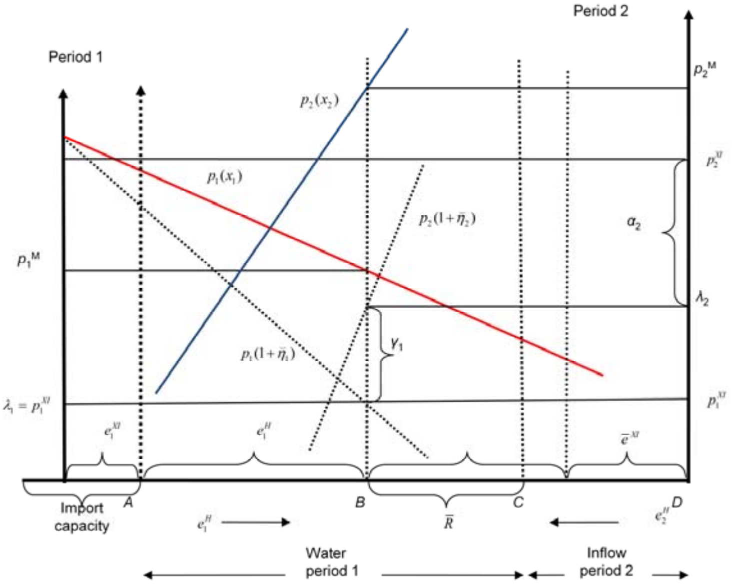
$$\lambda_t \ge 0 \ (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$

$$\gamma_t \ge 0 \ (= 0 \text{ for } R_t < \overline{R})$$

$$\alpha_t \ge 0 \ (= 0 \text{ for } e_t^{XI} < \overline{e}^{XI})$$

$$\beta_t \ge 0 \ (= 0 \text{ for } e_t^{XI} < -\overline{e}^{XI})$$

Monopoly and trade, figure



Monopoly with thermal plants

- Optimisation problem

$$\max \sum_{t=1}^{T} p_t(x_t) \cdot x_t - c(e_t^{Th})$$

subject to

$$x_t = e_t^H + e_t^{Th}$$

$$R_t \leq R_{t-1} + w_t - e_t^H$$

$$R_t \leq \overline{R}$$

$$e_t^{Th} \leq \overline{e}^{Th}$$

$$R_t, e_t^H, e_t^{Th} \geq 0 \ , \ t = 1,..,T$$

$$T, w_t, R_o, \overline{R}, \overline{e}^{Th} \text{ given}, R_T \text{ free}$$

- The Lagrangian

$$L = \sum_{t=1}^{T} [p_t(e_t^H + e_t^{Th})(e_t^H + e_t^{Th}) - c(e_t^{Th})]$$
$$-\sum_{t=1}^{T} \theta_t (e_t^{Th} - \overline{e}^{Th})$$
$$-\sum_{t=1}^{T} \lambda_t (R_t - R_{t-1} - w_t + e_t^H)$$
$$-\sum_{t=1}^{T} \gamma_t (R_t - \overline{R})$$

- Necessary first-order conditions

$$\frac{\partial L}{\partial e_t^H} = p_t'(e_t^H + e_t^{Th})(e_t^H + e_t^{Th}) + p_t(e_t^H + e_t^{Th}) - \lambda_t \leq 0$$

$$(= 0 \text{ for } e_t^H > 0)$$

$$\frac{\partial L}{\partial e_t^{Th}} = p_t'(e_t^H + e_t^{Th})(e_t^H + e_t^{Th}) + p_t(e_t^H + e_t^{Th}) - c'(e_t^{Th}) - \theta_t \leq 0$$

$$(= 0 \text{ for } e_t^{Th} > 0)$$

$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \leq 0 \ (= 0 \text{ for } R_t > 0)$$

$$\lambda \geq 0 \ (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$

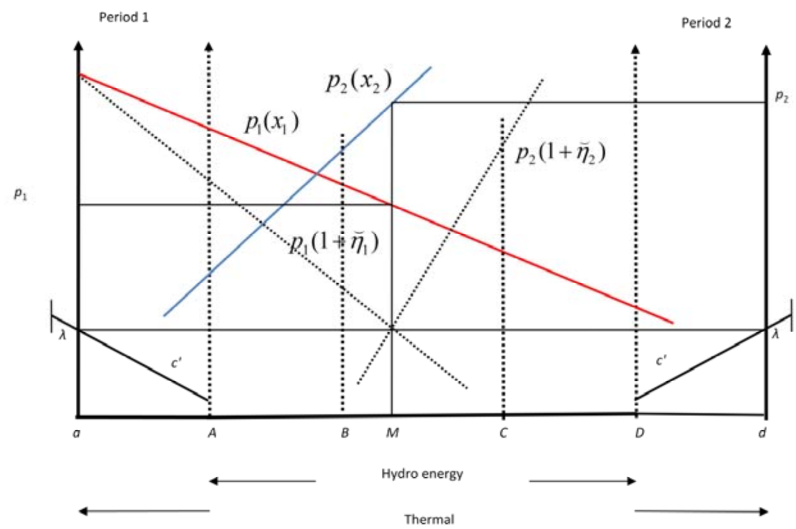
$$\theta_t \geq 0 \ (= 0 \text{ for } e_t^{Th} < \overline{e}^{Th})$$

$$\gamma_t \geq 0 \ (0 = \text{for } R_t < \overline{R})$$

- Using both hydro and thermal in the same period the marginal revenue substitutes for the marginal willingness to pay in the social optimal solution

$$p_t(x_t)(1+\breve{\eta}_t) = \lambda_t = c'(e_t^{Th}) + \theta_t$$

Hydro and thermal plants



Dominant hydro with a thermal competitive fringe

- Optimisation problem

$$\max \quad \sum_{t=1}^{T} p_t(x_t)e_t^H$$

subject to

$$x_t = e_t^H + e_t^{Th}$$

$$R_t \le R_{t-1} + w_t - e_t^H$$

$$R_t \le \overline{R}$$

$$p_t(x_t) = c'(e_t^{Th})$$

$$x_t, e_t^H, e_t^{Th} \ge 0, \quad t = 1,..,T$$

$$T, \overline{R} \text{ given}, R_T \text{ free}$$

- The fringe response to change in hydro production
  - Differentiation of the fringe 1. order condition

$$p_t(e_t^H + e_t^{Th}) = c'(e_t^{Th})(t = 1,..,T)$$

$$p_t'(e_t^H + e_t^{Th})(de_t^H + de_t^{Th}) = c''(e_t^{Th})de_t^{Th} \Rightarrow$$

$$\frac{de_t^{Th}}{de_t^H} = \frac{-p_t'(e_t^H + e_t^{Th})}{p_t'(e_t^H + e_t^{Th}) - c''(e_t^{Th})} < 0 \quad (t = 1,..,T)$$

  - The relationship between the fringe output and hydro output

$$e_t^{Th} = f_t(e_t^H), f_t' < 0 \ (t = 1,..,T)$$

- The Lagrangian

$$L = \sum_{t=1}^{T} p_t(e_t^H + f_t(e_t^H))e_t^H$$

$$-\sum_{t=1}^{T} \lambda_t (R_t - R_{t-1} - w_t + e_t^H)$$

$$-\sum_{t=1}^{T} \gamma_t (R_t - \overline{R})$$

- First-order order conditions

$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H + e_t^{Th}) + p_t'(e_t^H + e_t^{Th})e_t^H(1 + \frac{de_t^{Th}}{de_t^H}) - \lambda_t \le 0$$

$$(= 0 \text{ for } e_t^H > 0)$$

$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \ (= 0 \text{ for } R_t > 0)$$

$$\lambda_t \ge 0 \ (0 = \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$

$$\gamma_t \ge 0 \ (0 = \text{for } R_t < \overline{R}) , \qquad t = 1,..,T$$

- Inserting the fringe output reaction

$$1 + \frac{de_t^{Th}}{de_t^H} = 1 + \frac{-p_t'(e_t^H + e_t^{Th})}{p_t'(e_t^H + e_t^{Th}) - c''(e_t^{Th})} = \frac{-c''(e_t^{Th})}{p_t'(e_t^H + e_t^{Th}) - c''(e_t^{Th})} > 0$$

- The conditional marginal revenue function

$$MR_{t|p_t'=c'} = p_t(1+\breve{\eta}_t \frac{e_t^H}{e_t^H + e_t^{Th}}) + p_t' \frac{de_t^{Th}}{de_t^H} e_t^H,\ t = 1,..,T$$

Competitive thermal fringe