ECON 4925 Spring 2011
Electricity Economics
Lecture 11

Lecturer:

Finn R. Førsund

Fundamental stochastic elements

- Inflows are stochastic
  - Plus minus 25 TWh within a 90% confidence interval for Norway
- Demand is stochastic
  - Demand for space heating is depending on outdoor temperature
- Availability of generating capacity and transmission capacity stochastic
  - Accidents occur with positive probability (but low values)

The general problem formulation with stochastic variables

$$\max \sum_{t=1}^{T} E\left\{ \int_{z=0}^{x_t} p_t(z)\,dz \right\}$$

subject to

$$x_t = \sum_{j=1}^{N} e_{jt}^{H},\ t = 1,..,T$$

$$R_{jt} \le R_{j,t-1} + w_{jt} - e_{jt}^{H}$$

$$R_{jt} \le \overline{R}_j$$

$$e_{jt}^{H} \le \overline{e}_j^{H}$$

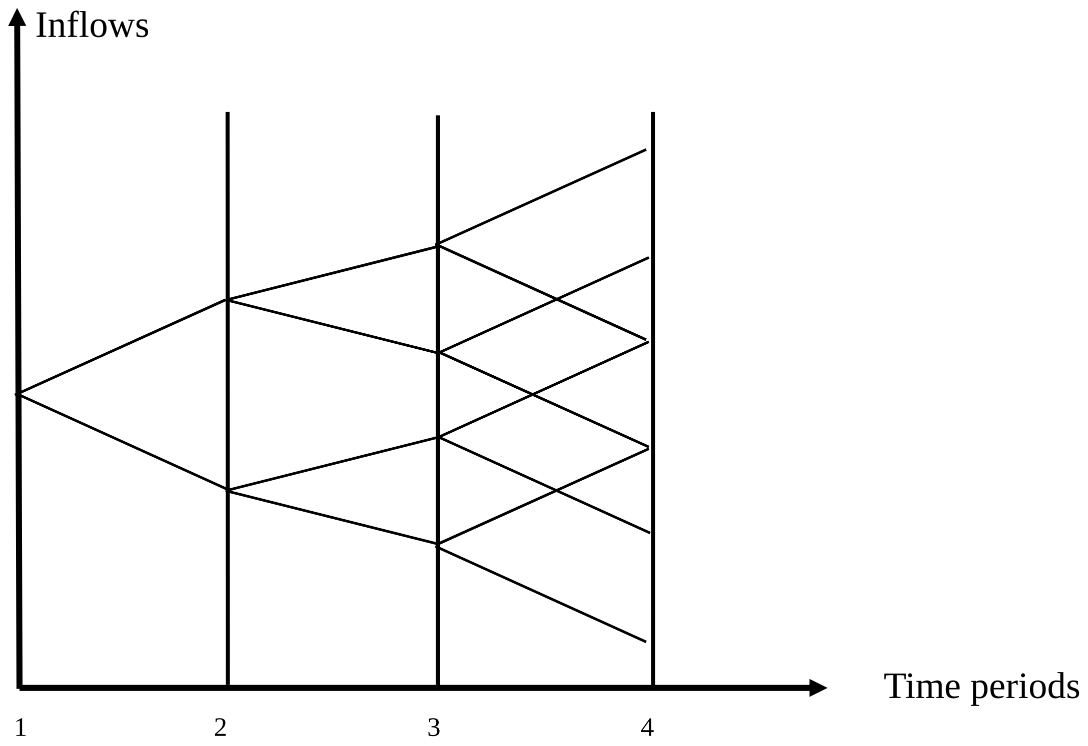
$$R_{jt}, w_{jt}, e_{jt}^{H} \ge 0$$

$$T, N, R_{jo}, \overline{R}_j, \overline{e}_j^{H} \text{ given, } R_{jT} \text{ free, } j = 1,..,N,\ t = 1,..,T$$

- Stochastic elements: $p_t(e_t^H), w_{jt}$

A decision tree diagram

- Impossible to solve from period 1



The general planning problem

$$\max \left[\int_{z=0}^{e_1^H} p_1(z)\,dz + \sum_{t=2}^{T} E\left\{ \int_{z=0}^{e_t^H} p_t(z)\,dz \right\} \right]$$

subject to

$$R_t = \min\left[\overline{R}, R_{t-1} + w_t - e_t^H\right],\ R_t \in \left[0, \overline{R}\right]$$

$$R_t, w_t, e_t^H \geq 0,\ t = 1,..,T$$

$$T, R_o, \overline{R} \text{ given, } R_T \text{ free}$$

The two-period problem

$$\max \left[\int_{z=0}^{e_1^H} p_1(z)\,dz + E\left\{ \int_{z=0}^{e_2^H} p_2(z)\,dz \right\} \right]$$

subject to

$$R_t = \min\left[\bar{R}, R_{t-1} + w_t - e_t^H \right],\ R_t \in \left[0, \bar{R} \right]$$

$$R_t, w_t, e_t^H \geq 0,\ t = 1,2$$

$$R_o, \bar{R} \text{ given, } R_2 \text{ free}$$

Rewriting inequality constraints

- Assumption: no spill in period 1
  - Follows from the general assumption of no satiation of demand
- Implication of a binding reservoir constraint

$$R_1 = \overline{R} = R_o + w_1 - e_1^H$$

- Implication for the production in period 2

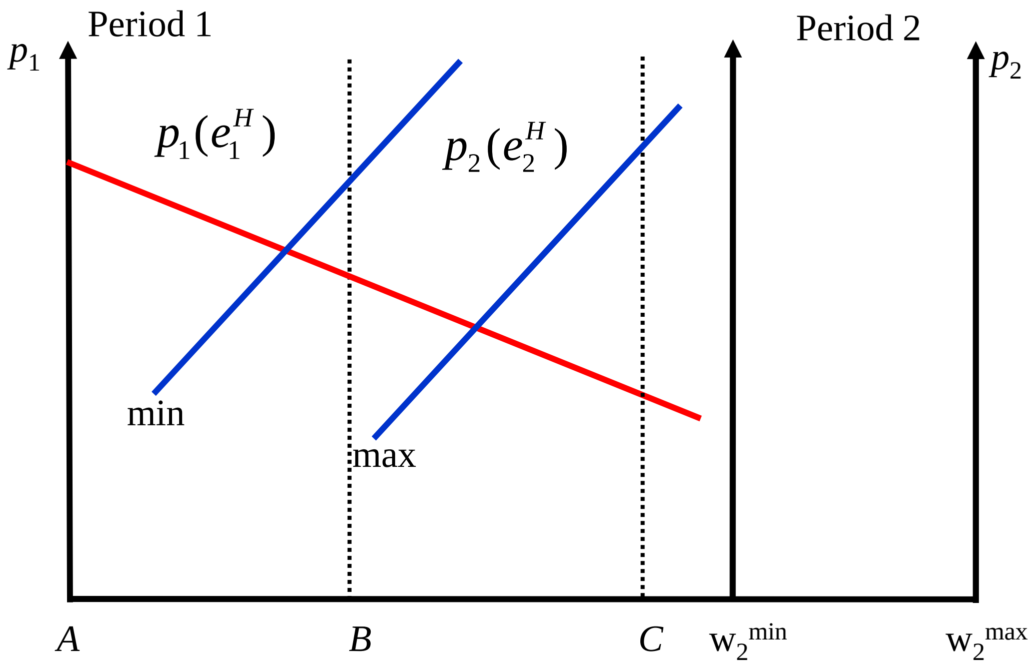
$$e_2^H = R_1 + w_2, \ R_1 \in [0, \overline{R}],$$

$$R_1 = R_o + w_1 - e_1^H \Rightarrow e_2^H = R_o + w_1 - e_1^H + w_2$$

  - With no spill e_2^H is a unique function of e_1^H

Bathtub with stochastic inflow in period 2

- Inflow known in period 1



The planning problem, cont.

- Inserting for the inflow in period 2

$$\max_{e_1^H} \left[\int_{z=0}^{e_1^H} p_1(z)\,dz + E\left\{ \int_{z=0}^{R_o+w_1+w_2-e_1^H} p_2(z)\,dz \right\} \right]$$

subject to

$$e_1^H \in \left[\max(0, R_o + w_1 - \overline{R}), R_o + w_1 \right]$$

- The problem is a function of the production in period 1 only

The optimality conditions

- The interior solution

$$p_1(e_1^H) = E\left\{p_2(R_o + w_1 + w_2 - e_1^H)\right\}$$

$$\text{for } e_1^H \in \left(\max(0, R_o + w_1 - \overline{R}), R_o + w_1\right)$$

- The corner solutions

$$E\left\{p_2(\overline{R} + w_2)\right\} \geq p_1^{\max} \equiv p_1(R_o + w_1 - \overline{R}) \Rightarrow$$

$$e_1^H = R_o + w_1 - \overline{R}$$

$$E\left\{p_2(w_2)\right\} \leq p_1^{\min} \equiv p_1(R_o + w_1) \Rightarrow e_1^H = R_o + w_1$$

Interpreting the optimality conditions

- Jensen's inequality

$$E\left\{p_2(R_o + w_1 + w_2 - e_1^H)\right\} \geq p_2(R_o + w_1 + E\left\{w_2\right\} - e_1^H)$$

- Increased uncertainty
  - Mean-preserving spread (Rothchild and Stiglitz, 1970)

$$E\left\{p_2(R_o + w_1 + w_2 - e_1^H)\right\}$$

increases when uncertainty increases

Calculating the expected price in period 2

- Discretising the probability distribution for inflow
  - Introducing the frequency for the inflow in period 2 for each yearly observation i (e.g. K=70 years) of period 2

$$E\left\{p_2(e_2^H)\right\} = \sum_{i=1}^{K} \phi_i p_2(R_o + w_1 + w_{2i} - e_1^H)$$

  - Extreme-value estimation

The expected water value table

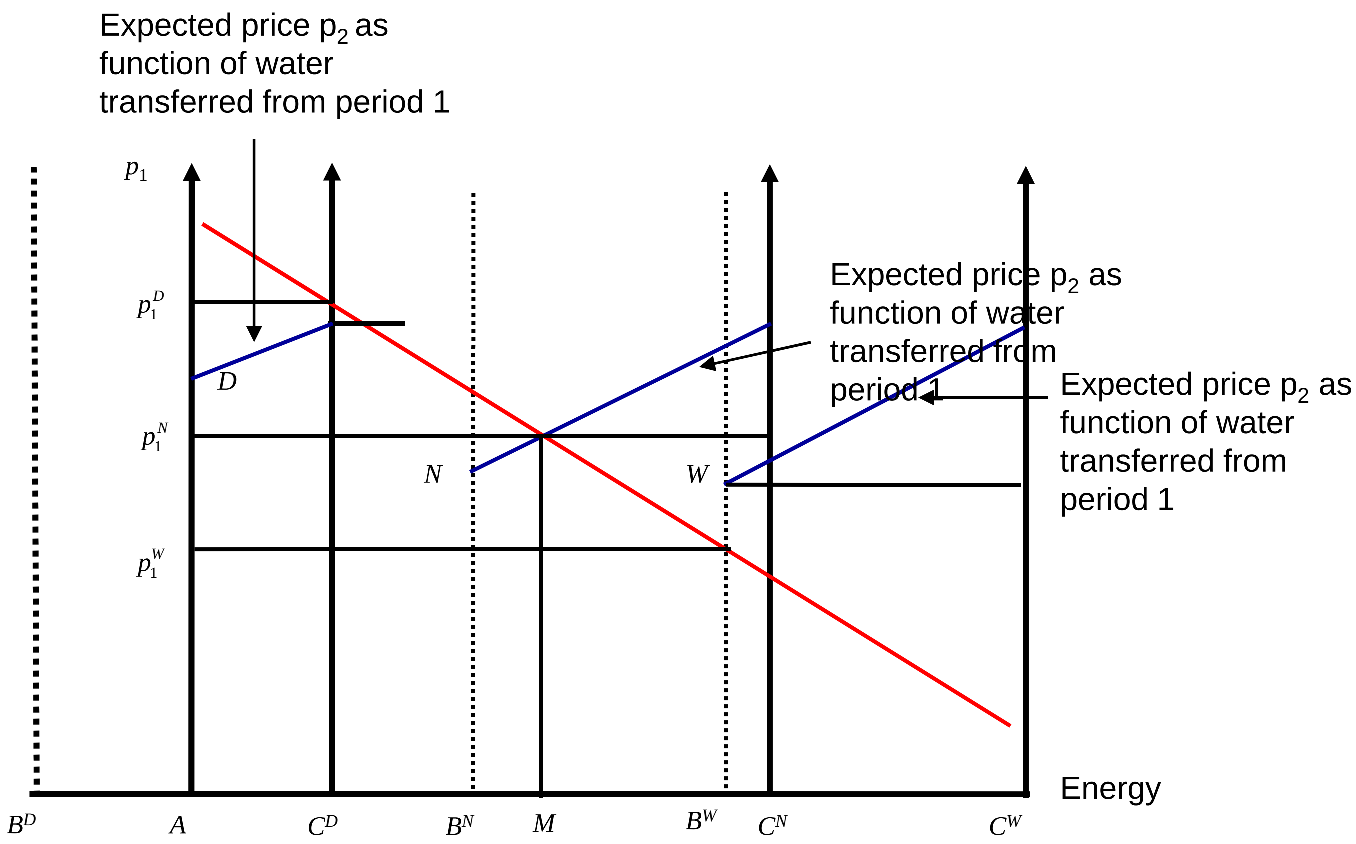
- The expected price in period 2, i.e., the water value, is a function of the amount of water transferred to period 2 from period 1

$$E\left\{p_2(R_o + w_1 + w_2 - e_1^H)\right\} = E\left\{p_2(R_1 + w_2)\right\} \equiv E\left\{p_2 \middle| R_1\right\}$$

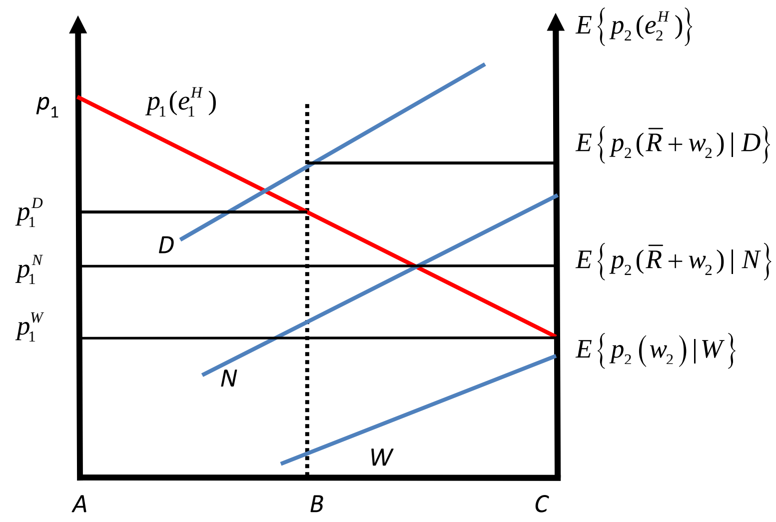
- For each value of R_1 we get a value for the expected price in period 2
  - The range of R_1

$$R_1 \in \left[0, \bar{R}\right]$$

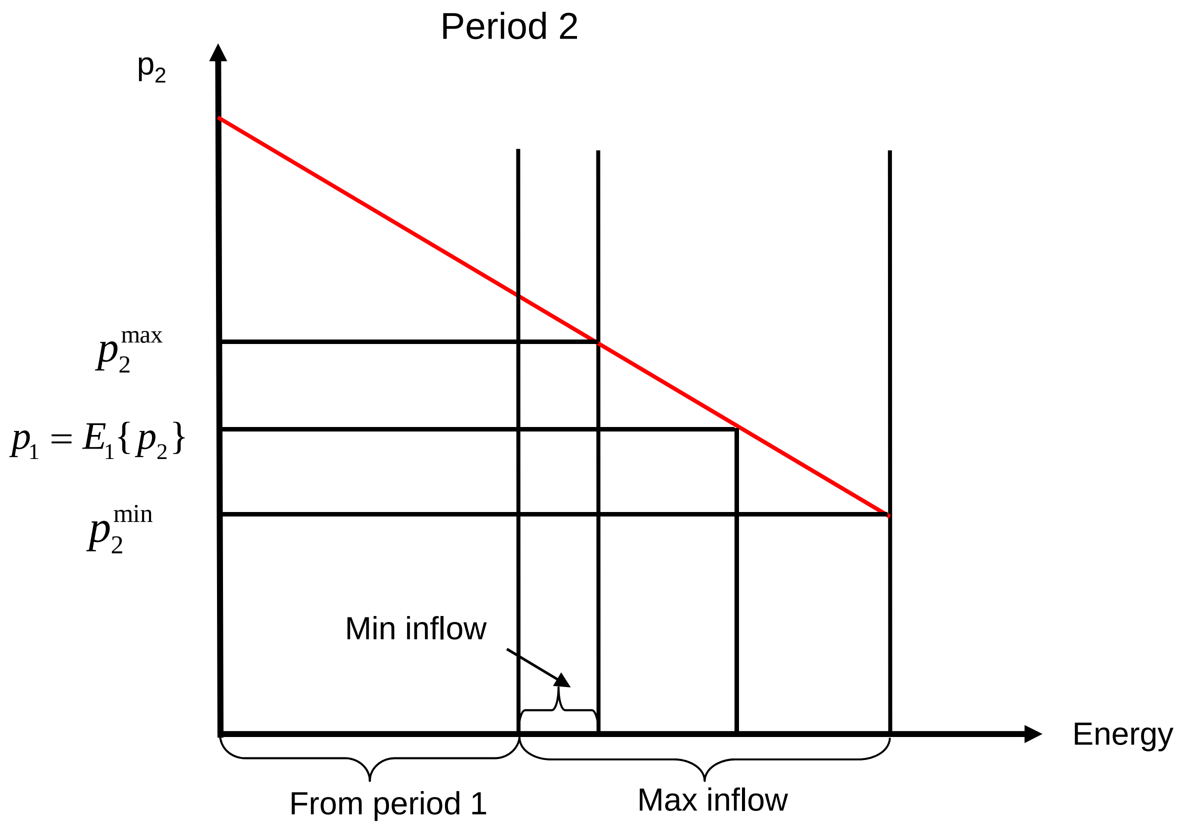
Illustration of decision in period 1
Use of water value table



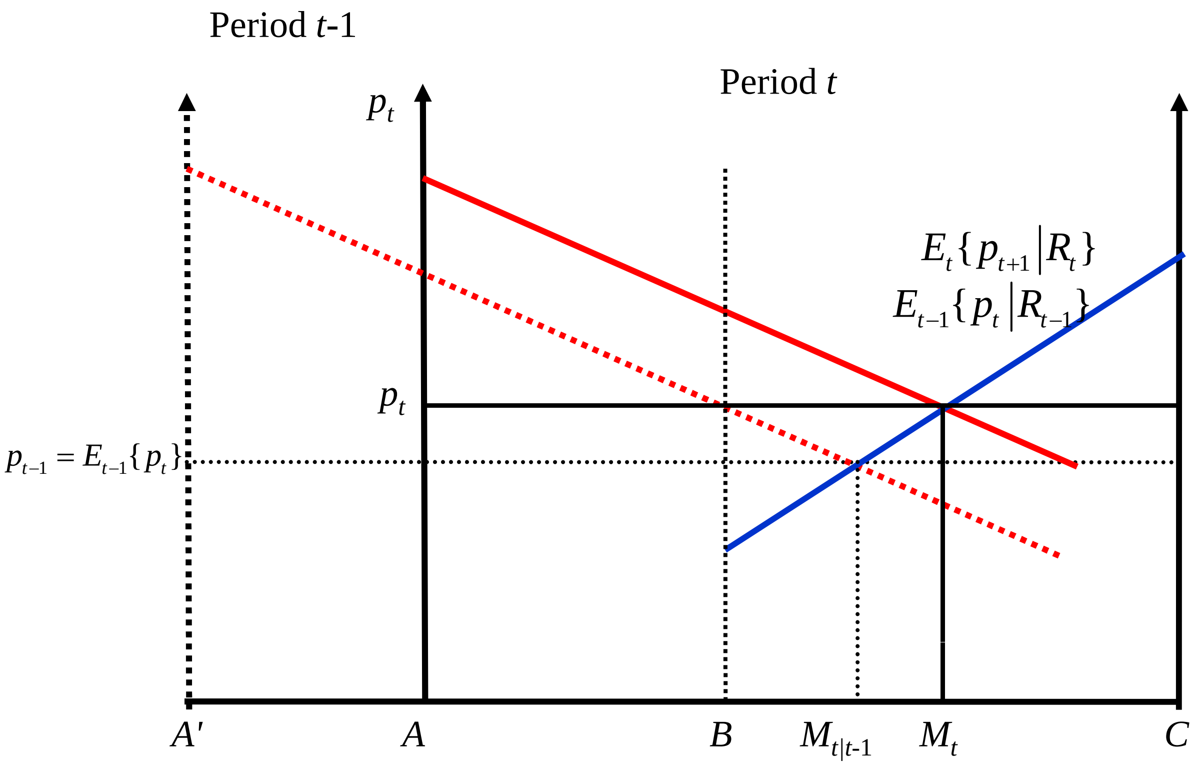
Stochastic inflow in period 2 follows three different distributions



Realised price in period 2



A general case for period t



Hydro and thermal with uncertain inflow

- Two periods only, period 1 known, period 2 uncertain inflow
- The social planning problem

$$\max \left[\int_{z=0}^{x_1} p_1(z)\,dz - c(e_1^{Th}) + E\left\{ \int_{z=0}^{x_2} p_2(z)\,dz - c(e_2^{Th}) \right\} \right],$$

$$x_t = e_t^H + e_t^{Th}, t = 1,2$$

$$e_1^H \in \left[\max(0, R_o + w_1 - \overline{R}), R_o + w_1 \right]$$

$$e_2^H \in \left[w_2, \overline{R} + w_2 \right]$$

$$e_t^{Th} \in \left[0, \overline{e}^{Th} \right], t = 1,2$$

Reformulating the planning problem

- Eliminating total consumption, using the water accumulating equation for period 2

$$\max_{e_1^H, e_1^{Th}} \left[\int_{z=0}^{e_1^H + e_1^{Th}} p_1(z)\,dz - c(e_1^{Th}) + E\left\{ \int_{z=0}^{R_o + w_1 + w_2 - e_1^H + e_2^{Th}} p_2(z)\,dz - c(e_2^{Th}) \right\} \right]$$

$$e_1^H \in \left[\max(0, R_o + w_1 - \overline{R}), R_o + w_1 \right]$$

$$e_t^{Th} \in \left[0, \overline{e}^{Th} \right], t = 1, 2$$

First-order conditions

$$p_1(e_1^H + e_1^{Th}) - E\left\{p_2(R_o + w_1 + w_2 - e_1^H + e_2^{Th})\right\} = 0$$

$$p_1(e_1^H + e_1^{Th}) - c'(e_1^{Th}) = 0$$

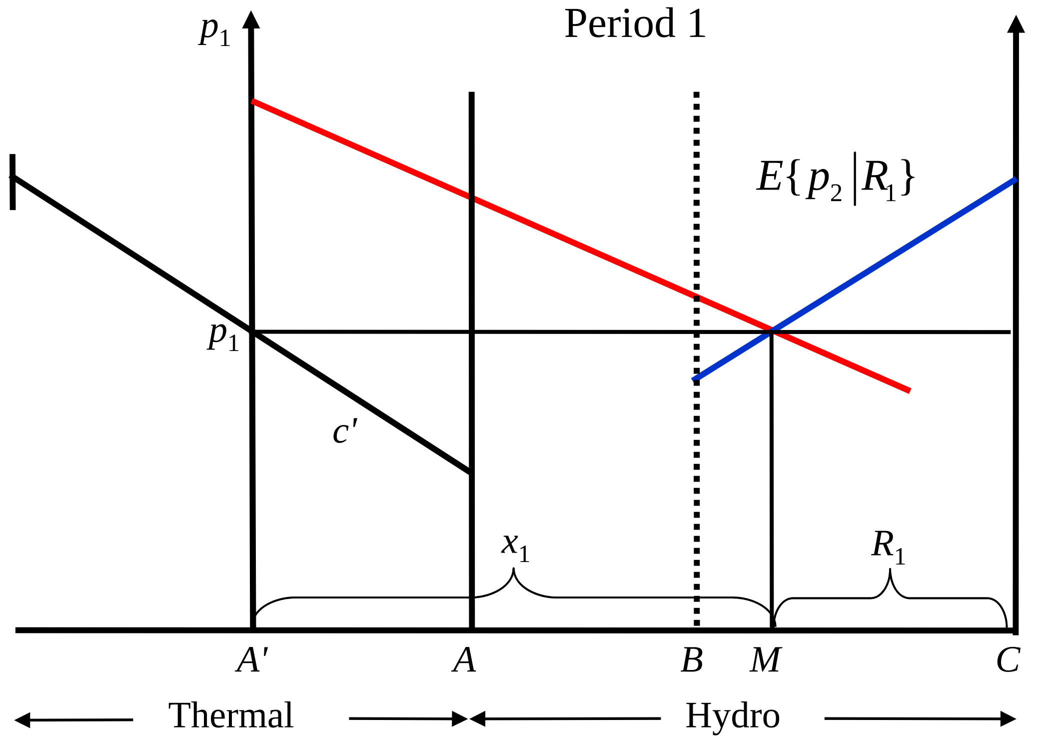
$$E\left\{p_2(R_o + w_1 + w_2 - e_1^H + e_2^{Th})\right\} - E\left\{c'(e_2^{Th})\right\} = 0$$

- The role of thermal in period 2

$$E\left\{p_2(R_o + w_1 + w_2 - e_1^H + e_2^{Th})\right\} = E\left\{p_2(R_1 + w_2 + e_2^{Th})\right\} = E\left\{c'(e_2^{Th})\right\}$$

  - For a given R_1 we get a relationship between e_2^{Th} and w_2 via the expected price. Changing R_1 shifts this relationship
  - The water value table contingent upon both the realised R_1 and the adjustment rule for thermal: $E\{p_2 | R_1\}$

Hydro and thermal in period 1



Hydro and thermal in period 2