ECON 4925 Spring 2011
Electricity Economics
Lecture 12

Lecturer:

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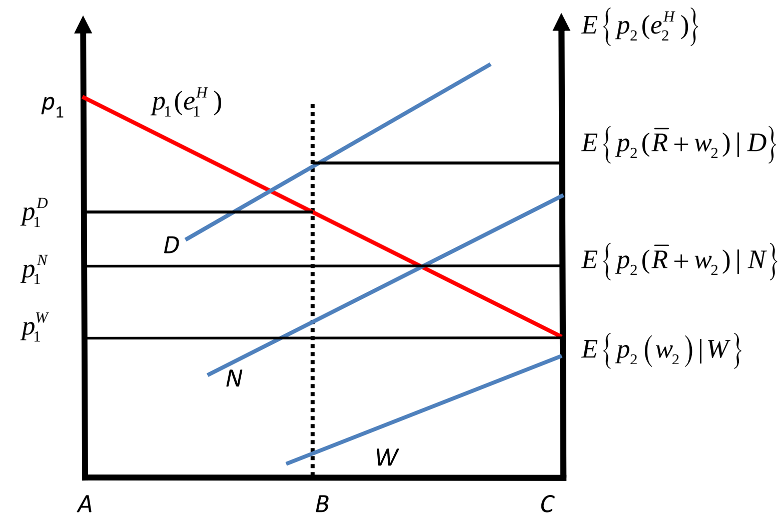
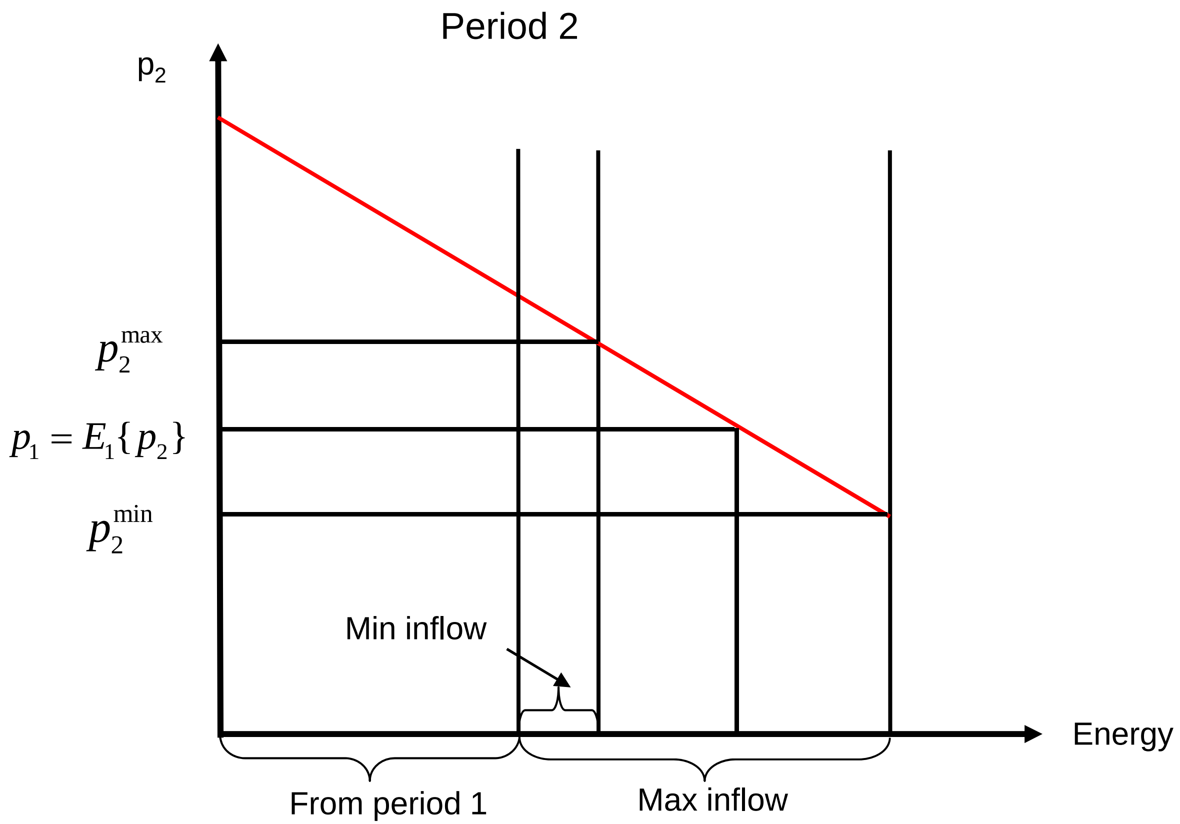
Stochastic inflow in period 2 follows three different distributions



Realised price in period 2



Backwards induction

- Starting with the terminal period T
  - All water will be used

$$e_T^H = R_{T-1} + w_T \,,\; p_T = p_T(R_{T-1} + w_T) > 0$$

- Period T-1 optimisation problem

$$\max_{e_{T-1}^H} \left[\int_{z=0}^{e_{T-1}^H} p_{T-1}(z)\,dz + E\left\{ \int_{z=0}^{R_{T-2}+w_{T-1}+w_T-e_{T-1}^H} p_T(z)\,dz \right\} \right]$$

subject to

$$e_1^H \in \left[\max(0, R_{T-2} + w_{T-1} - \bar{R}), R_{T-2} + w_{T-1} \right]$$

$$R_{T-1} \in \left[0, \bar{R} \right]$$

- Interior solution and corner solutions

$$p_{T-1}(e_{T-1}^H) = E\left\{p_T(e_T^H)\right\} = E\left\{p_T(R_{T-1} + w_T)\right\}$$

$$\text{for } e_{T-1}^H \in \left(\max(0, R_{T-2} + w_{T-1} - \bar{R}), R_{T-2} + w_{T-1}\right)$$

$$p_{T-1}(e_{T-1}^H) \geq E\left\{p_T(\bar{R} + w_T)\right\}$$

$$\text{for } e_{T-1}^H = R_{T-2} + w_{T-1} - \bar{R} > 0 (R_{T-1} = \bar{R})$$

$$p_{T-1}(e_{T-1}^H) \leq E\left\{p_T(w_T)\right\}$$

$$\text{for } e_{T-1}^H = R_{T-2} + w_{T-1} (R_{T-1} = 0)$$

- The two-period model
  - Insert T=2

- Solution for period t

$$p_t(e_t^H) = E\left\{p_{t+1}(e_{t+1}^H)\right\} = E\left\{p_{t+1}(R_t - R_{t+1} + w_{t+1})\right\}$$

$$\text{for } e_t^H \in \left(\max(0, R_{t-1} + w_t - \bar{R}), R_{t-1} + w_t\right)$$

$$p_t(e_t^H) \geq E\left\{p_{t+1}(\bar{R} - R_{t+1} + w_{t+1})\right\}$$

$$\text{for } e_t^H = R_{t-1} + w_t - \bar{R} > 0 \ (R_t = \bar{R})$$

$$p_t(e_t^H) \leq E\left\{p_{t+1}(w_{t+1} - R_{t+1})\right\}$$

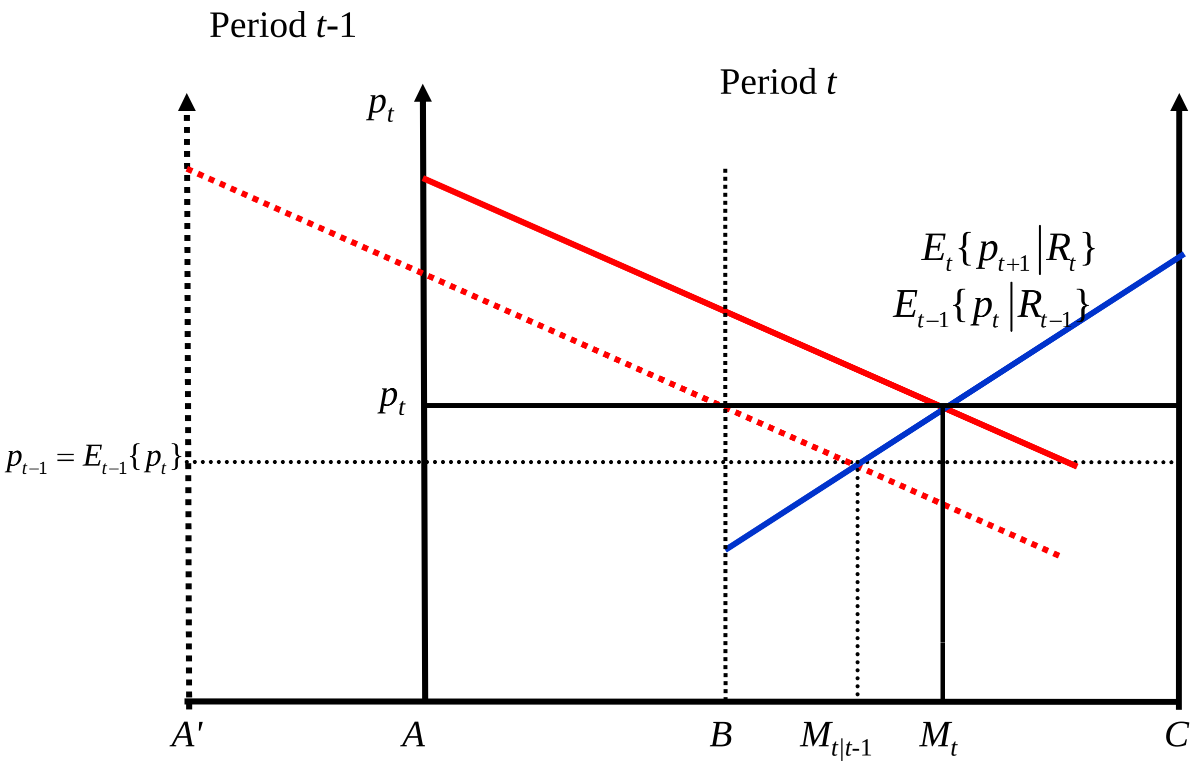
$$\text{for } e_t^H = R_{t-1} + w_t \ (R_t = 0)$$

- Water transferred from period t+1 to t+2 enters in the expectation formation

- At period t we are looking forward; the past resolution of uncertainty has no influence on the solution for period t; nothing is learned as time goes by
- The expected water value table used at t is a function of all remaining stochastic variables

$$p_t(e_t^H) = E\left\{ p_{t+1}(R_{t-1} + \sum_{i=t}^{T} w_i - e_t^H - \sum_{i=t+2}^{T} e_i^H) \right\} \ (t = 1,..,T-2)$$

A general case for period t



Hydro and thermal with uncertain inflow

- Two periods only, period 1 known, period 2 uncertain inflow
- The social planning problem

$$\max \left[\int_{z=0}^{x_1} p_1(z)\,dz - c(e_1^{Th}) + E\left\{ \int_{z=0}^{x_2} p_2(z)\,dz - c(e_2^{Th}) \right\} \right],$$

$$x_t = e_t^H + e_t^{Th}, t = 1,2$$

$$e_1^H \in \left[\max(0, R_o + w_1 - \overline{R}), R_o + w_1 \right]$$

$$e_2^H \in \left[w_2, \overline{R} + w_2 \right]$$

$$e_t^{Th} \in \left[0, \overline{e}^{Th} \right], t = 1,2$$

Reformulating the planning problem

- Eliminating total consumption, using the water accumulating equation for period 2

$$\max_{e_1^H, e_1^{Th}} \left[\int_{z=0}^{e_1^H + e_1^{Th}} p_1(z)\,dz - c(e_1^{Th}) + E\left\{ \int_{z=0}^{R_o + w_1 + w_2 - e_1^H + e_2^{Th}} p_2(z)\,dz - c(e_2^{Th}) \right\} \right]$$

$$e_1^H \in \left[\max(0, R_o + w_1 - \overline{R}), R_o + w_1 \right]$$

$$e_t^{Th} \in \left[0, \overline{e}^{Th} \right], t = 1, 2$$

First-order conditions

$$p_1(e_1^H + e_1^{Th}) - E\left\{p_2(R_o + w_1 + w_2 - e_1^H + e_2^{Th})\right\} = 0$$

$$p_1(e_1^H + e_1^{Th}) - c'(e_1^{Th}) = 0$$

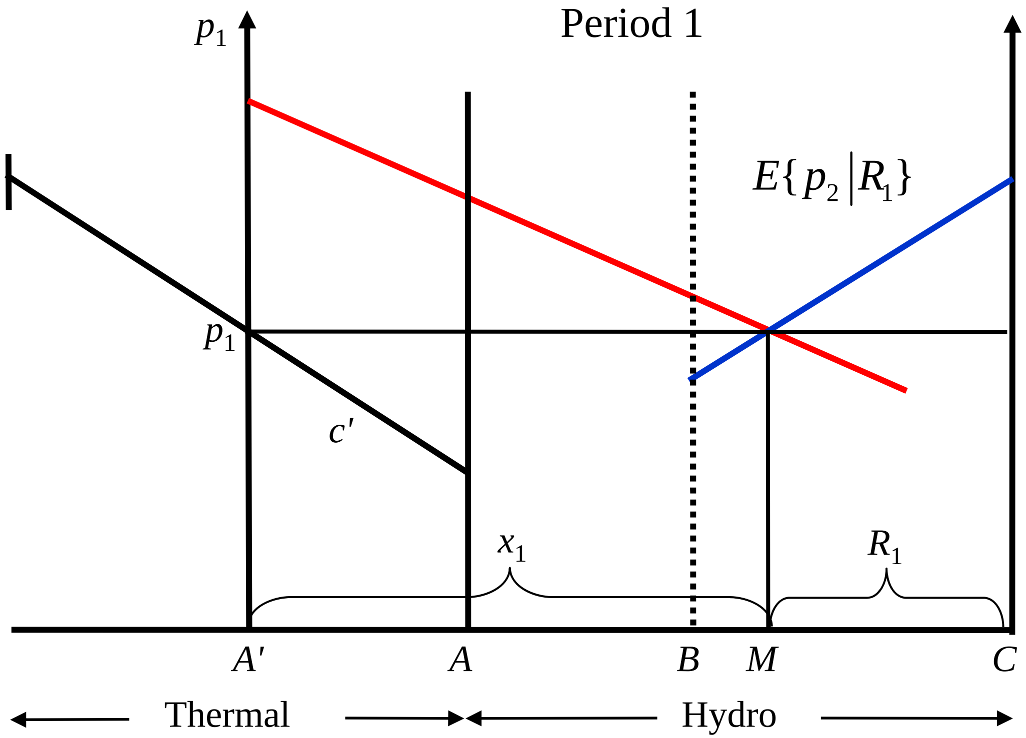
$$E\left\{p_2(R_o + w_1 + w_2 - e_1^H + e_2^{Th})\right\} - E\left\{c'(e_2^{Th})\right\} = 0$$

- The role of thermal in period 2

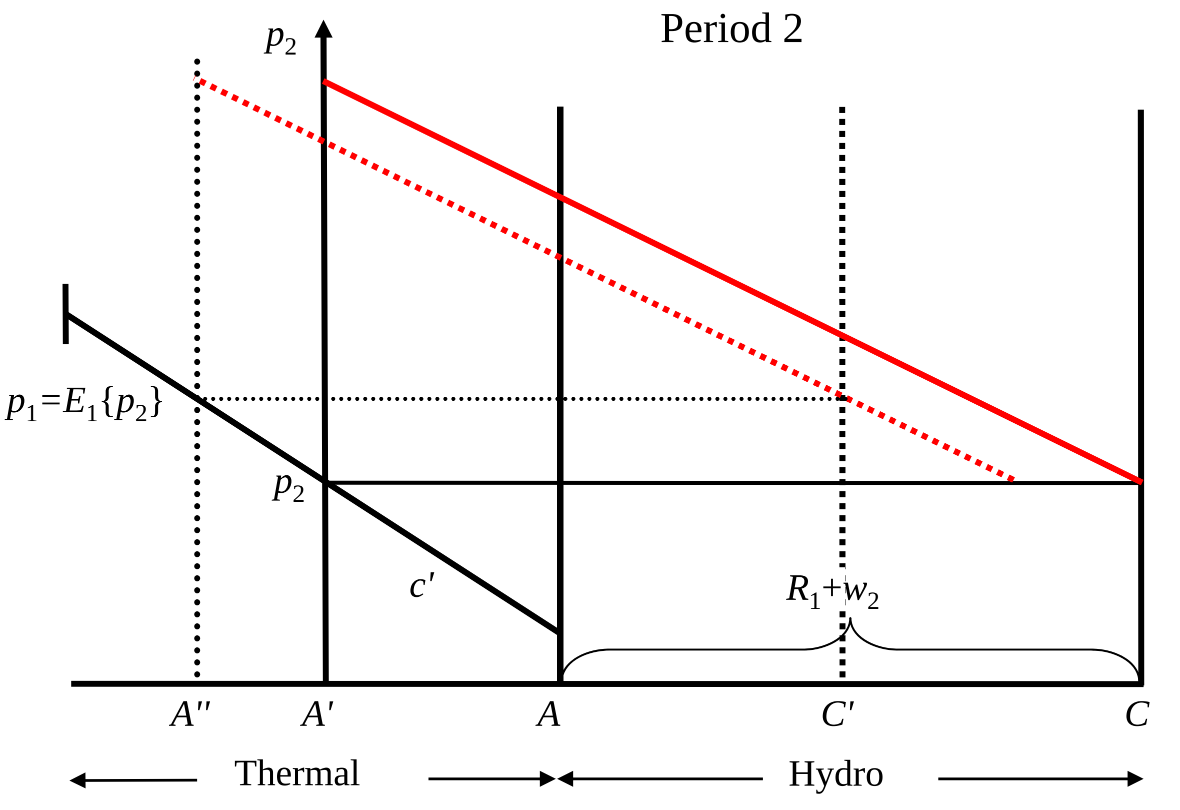
$$E\left\{p_2(R_o + w_1 + w_2 - e_1^H + e_2^{Th})\right\} = E\left\{p_2(R_1 + w_2 + e_2^{Th})\right\} = E\left\{c'(e_2^{Th})\right\}$$

  - For a given R_1 we get a relationship between e_2^{Th} and w_2 via the expected price. Changing R_1 shifts this relationship
  - The water value table contingent upon both the realised R_1 and the adjustment rule for thermal: $E\{p_2 | R_1\}$

Hydro and thermal in period 1



Hydro and thermal in period 2



Monopoly and uncertainty

- The optimisation problem two periods

$$\max\left[\, p_1(e_1^H)\cdot e_1^H + E\left\{ p_2(e_2^H)e_2^H \right\}\right]$$

subject to

$$R_t \leq R_{t-1} + w_t - e_t^H$$

$$R_t \leq \overline{R}$$

$$R_t, e_t^H \geq 0 \;,\; t = 1,2$$

$$T, w_t, R_o, \overline{R} \text{ given}, R_2 \text{ free}$$

- Getting rid of the constraint format

$$e_2^H = R_1 + w_2, \ \ R_1 \in [0, \overline{R}],$$

$$R_1 = R_o + w_1 - e_1^H \Rightarrow e_2^H = R_o + w_1 - e_1^H + w_2$$

- The optimisation problem

$$\max \begin{bmatrix} p_1(e_1^H) \cdot e_1^H + \\ E\left\{ p_2(R_o + w_1 - e_1^H + w_2)(R_o + w_1 - e_1^H + w_2) \right\} \end{bmatrix}$$

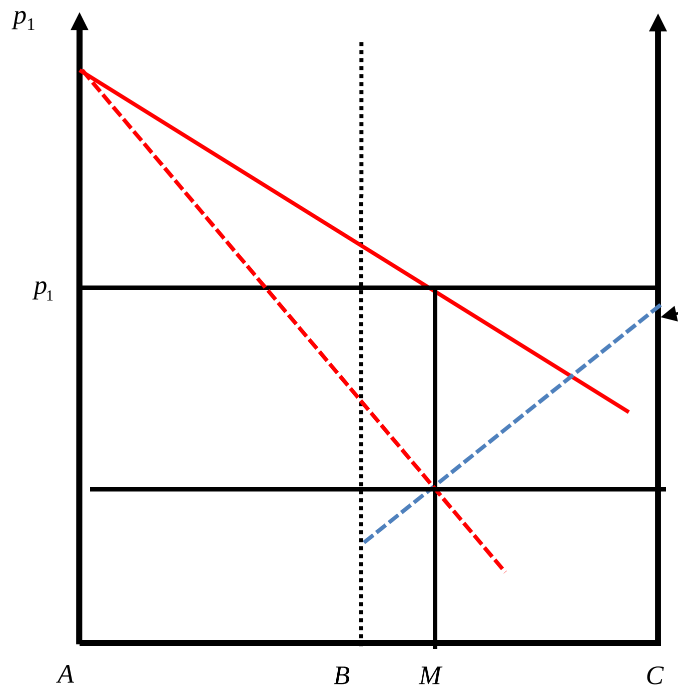
- First-order condition

$$p_1'(e_1^H)e_1^H + p_1(e_1^H) = E\{p_2'(e_2^H)e_2^H + p_2(e_2^H)\} \Rightarrow$$

$$p_1(e_1^H)(1+\breve{\eta}_1) = E\{p_2(e_2^H)(1+\breve{\eta}_2)\big|R_1\}$$

- The flexibility-corrected price in period 1 is set equal to the expected flexibility-corrected price in period 2 conditional on the transfer of water from period 1 to 2, i.e. the monopolist’s water value table.
- If convex marginal revenue functions: increasing uncertainty will lead to less water being used in period 1

Illustration of decision in period 1
Use of " water value table"



Expected flexibility-corrected price p_2 as function of water transferred from period 1