ECON 4925 Autumn 2011
Electricity Economics
Lecture 2

Lecturer:
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Non-linear programming

- Problem formulation (Sydsæter et al., 1999)

$$\max f(x_1,...,x_n) \text{ subject to } g_j(x_1,...,x_n) \le b_j,\ j=1,..,m$$

$$x_i \ge 0, i=1,..,n$$

- The Lagrangian function

$$L(x) = f(x) - \sum_{j=1}^{m} \lambda_j (g_j(x) - b_j)$$

- Kuhn – Tucker necessary conditions

$$\frac{\partial L(x)}{\partial x_i} = \frac{\partial f(x)}{\partial x_i} - \sum_{j=1}^{m} \lambda_j \frac{\partial g_j(x)}{\partial x_i} \le 0 (= 0 \text{ for } x_i > 0)$$

$$\lambda_j \ge 0 (= 0 \text{ for } g_j(x) < b_j),\ j=1,..,m$$

Applying Kuhn - Tucker

- The Lagrangian

$$L = \sum_{t=1}^{T} \int_{z=0}^{e_t^H} p_t(z)\,dz$$

$$-\sum_{t=1}^{T} \lambda_t (R_t - R_{t-1} - w_t + e_t^H)$$

$$-\sum_{t=1}^{T} \gamma_t (R_t - \overline{R})$$

Applying Kuhn – Tucker, cont.

- First-order necessary conditions

$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H) - \lambda_t \leq 0 \ (= 0 \text{ for } e_t^H > 0)$$

$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \leq 0 \ (= 0 \text{ for } R_t > 0)$$

$$(\text{constraint for period } t+1 : -\lambda_{t+1}(R_{t+1} - R_t - w_{t+1} + e_{t+1}^H)$$

$$\lambda_t \geq 0 (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$

$$\gamma_t \geq 0 (= 0 \text{ for } R_t < \bar{R}) \ , \qquad t = 1,..,T$$

Interpreting the Lagrangian parameters

- Shadow prices on the constraints
  - A shadow price tells us the change in the value of the objective function (value function) when the corresponding constraint is changed marginally
- λ_t : shadow price on stored water, water value, the benefit of one unit more of water, envelope theorem: $\partial(\sum_{t=1}^{T}\int_{z=0}^{e_t^H} p_t(z)\,dz) / \partial w_t = \partial L / \partial w_t = \lambda_t$
- γ_t : shadow price on the reservoir constraint, the benefit of one unit increase in capacity

$$\partial(\sum_{t=1}^{T}\int_{z=0}^{e_t^H} p_t(z)\,dz) / \partial \bar{R} = \partial L / \partial \bar{R} = \gamma_t$$

Interpreting the Kuhn – Tucker conditions

- Inequality in the first condition
  - Social price lower than the shadow price on stored water
  - Nothing will be produced $\longrightarrow$
    - Must have positive production in the pure hydro case
- Inequality in the second condition
  - Water value in the next period lower than the sum of water value in the present period and the shadow price on the reservoir constraint $\longrightarrow$
    - May empty the reservoir $\longrightarrow$
    - shadow price zero on reservoir constraint

Qualitative interpretations

- Reasonable assumption
  - Positive production in each period →
    - Social price equal to the water value

$$p_t(e_t^H) = \lambda_t$$

  - Assuming reservoir capacity is not reached→
    - Water value in the present period will be equal to the water value in the next period

$$\gamma_t = 0 (R_t < \bar{R}) \Rightarrow \lambda_t = \lambda_{t+1}$$

Qualitative conclusion

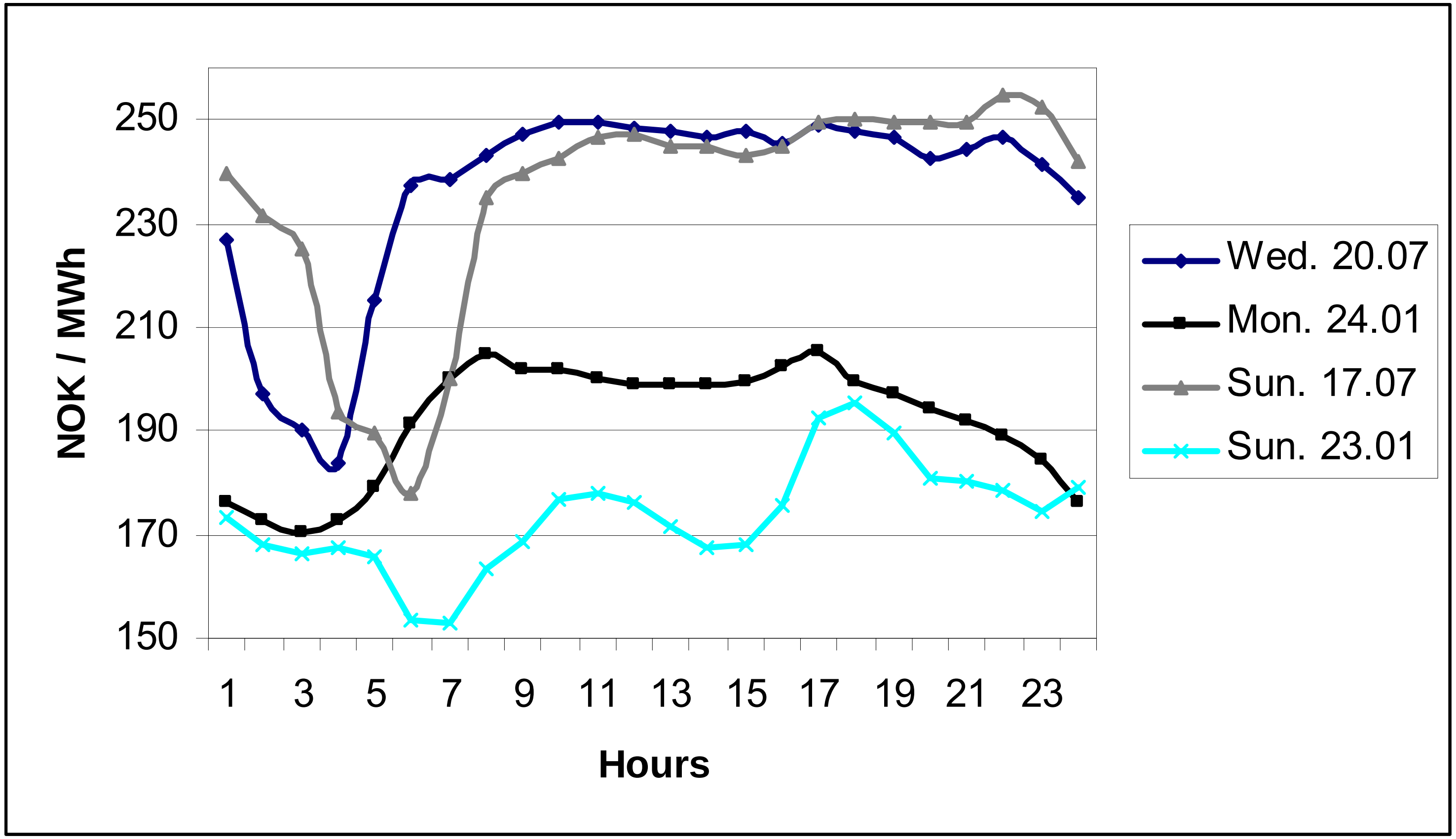
- For consecutive periods with the reservoir constraint not binding the water values are all the same

$$\lambda_t = \lambda_{t+1} = \lambda,\ t \in T_{\gamma_{t=0}}$$

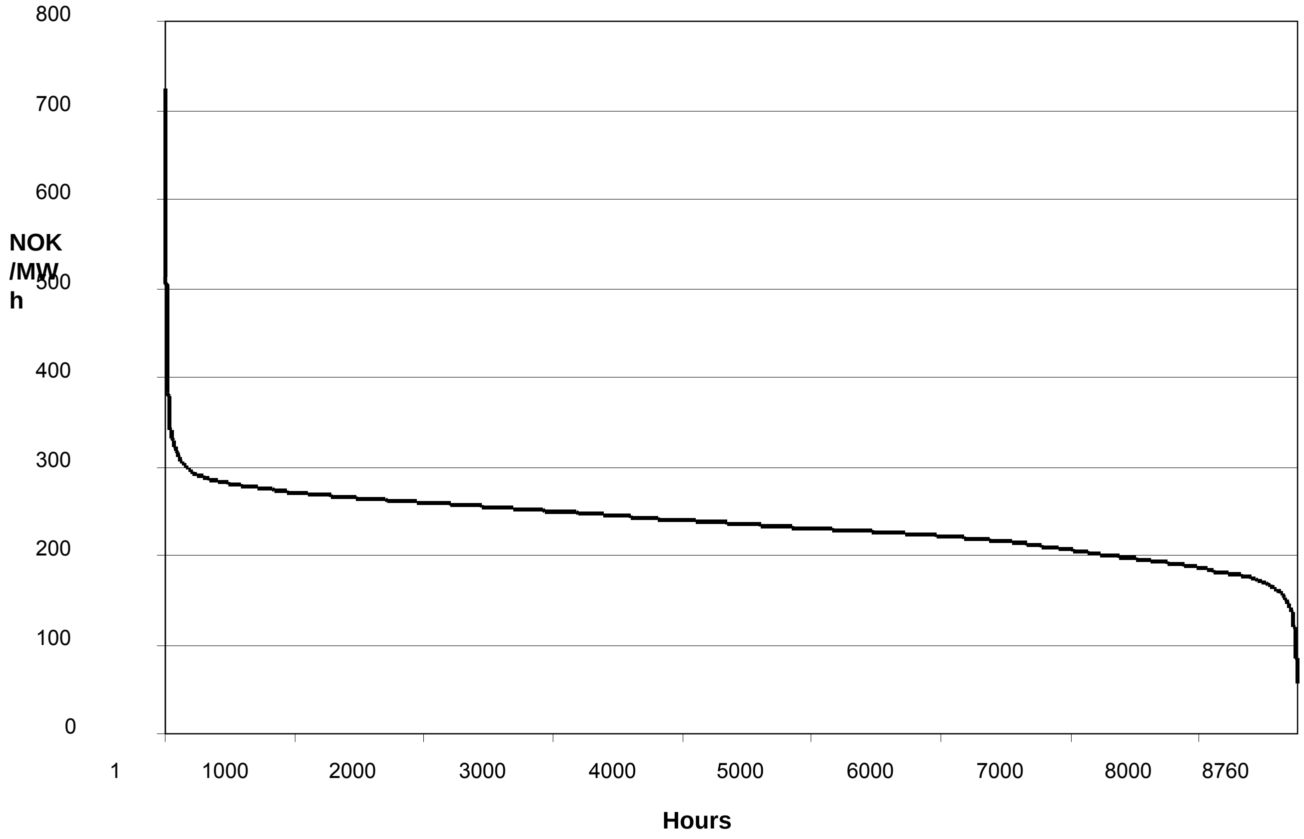
- The social price is the same for all periods with non-binding reservoir constraints and equal to the common water value

$$p_t(e_t^H) = p_{t+1}(e_{t+1}^H) = \lambda,\ t \in T_{\gamma_t=0}$$

Hourly price variation four days in 2005



Price-duration curve Norway 2005



The bathtub diagram

- Only two periods
- Available water in the first period

$$R_o + w_1$$

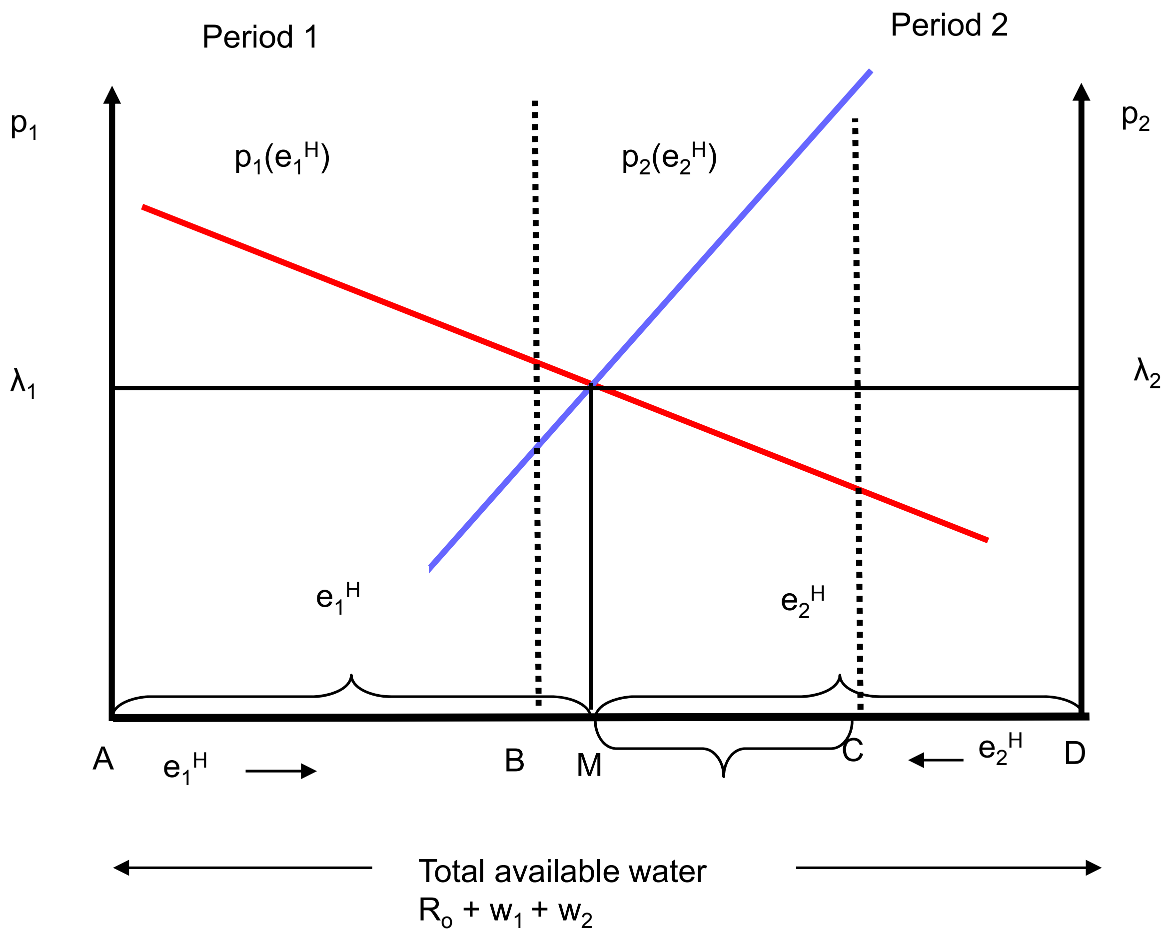
- Available water in the second period

$$R_1 + w_2$$

- Adding together the two water-storage equations

$$e_1^H + e_2^H = R_o + w_1 + w_2$$

The bathtub diagram for two periods



Backwards induction, Bellman

- Start with the terminal period

$$p_2(e_2^H) = \lambda_2 \, (e_2^H > 0),$$
$$-\lambda_2 - \gamma_2 \leq 0 \; (= 0 \text{ if } R_2 > 0)$$

- Assumption: no satiation of demand

$$p_2(w_2 + Max\, R_1) = p_2(w_2 + \overline{R}) > 0$$

- Implication

$$R_2 = 0, \; \gamma_2 = 0, \; p_2(e_2^H) = \lambda_2 > 0$$

- No threat of overflow in period 1

$$R_1 < \overline{R} \Rightarrow \gamma_1 = 0, \; p_1(e_1^H) = \lambda_1 = \lambda_2 > 0$$