ECON 4925 Autumn 2011
Electricity Economics
Lecture 3

Lecturer:
Finn R. Førsund

The bathtub diagram

- Only two periods
- Available water in the first period

  $$R_o + w_1$$

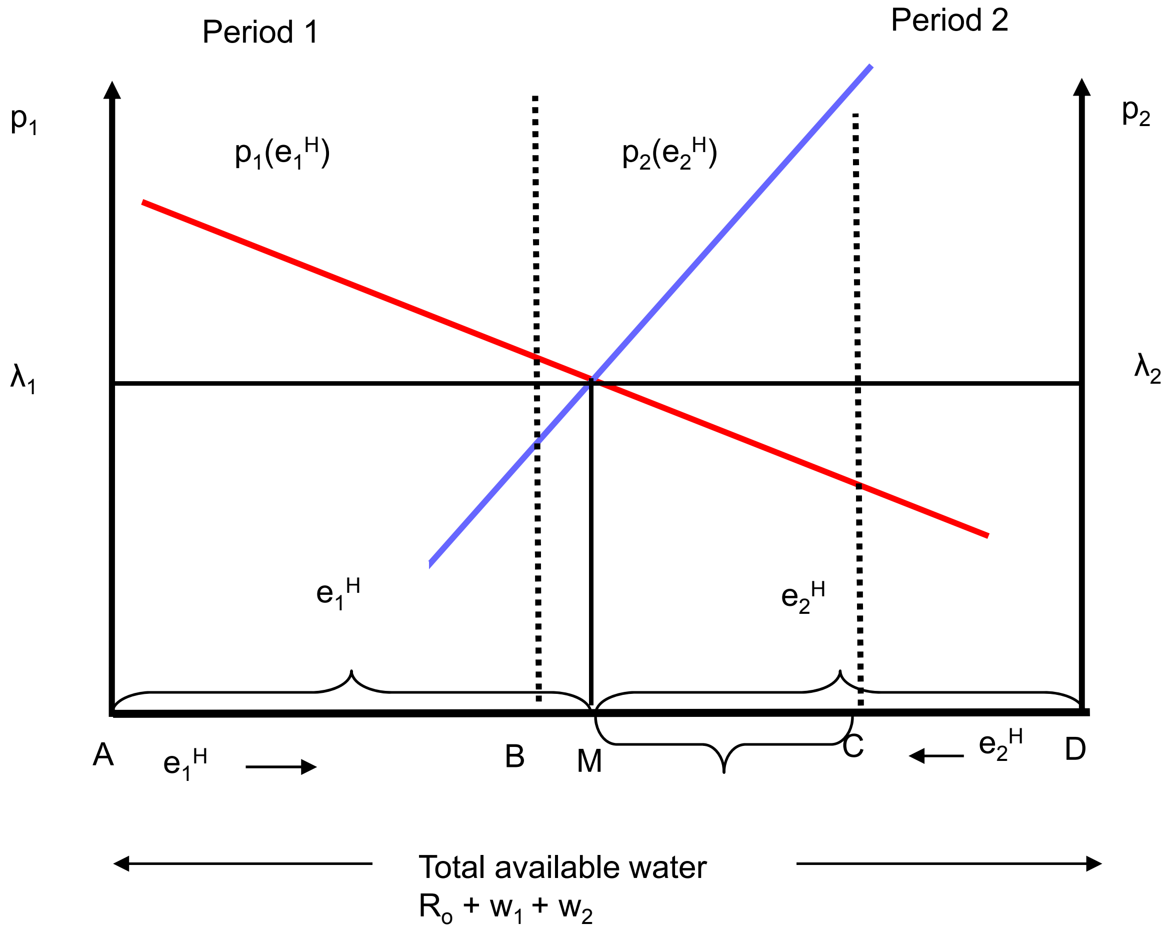
- Available water in the second period

  $$R_1 + w_2$$

- Adding together the two water-storage equations

  $$e_1^H + e_2^H = R_o + w_1 + w_2$$

The bathtub diagram for two periods



Explaining price change

- Binding reservoir constraint in period 1
- No overflow due to the non-satiation assumption

$$p_1(e_1^H) = \lambda_1 \ (e_1^H > 0),$$

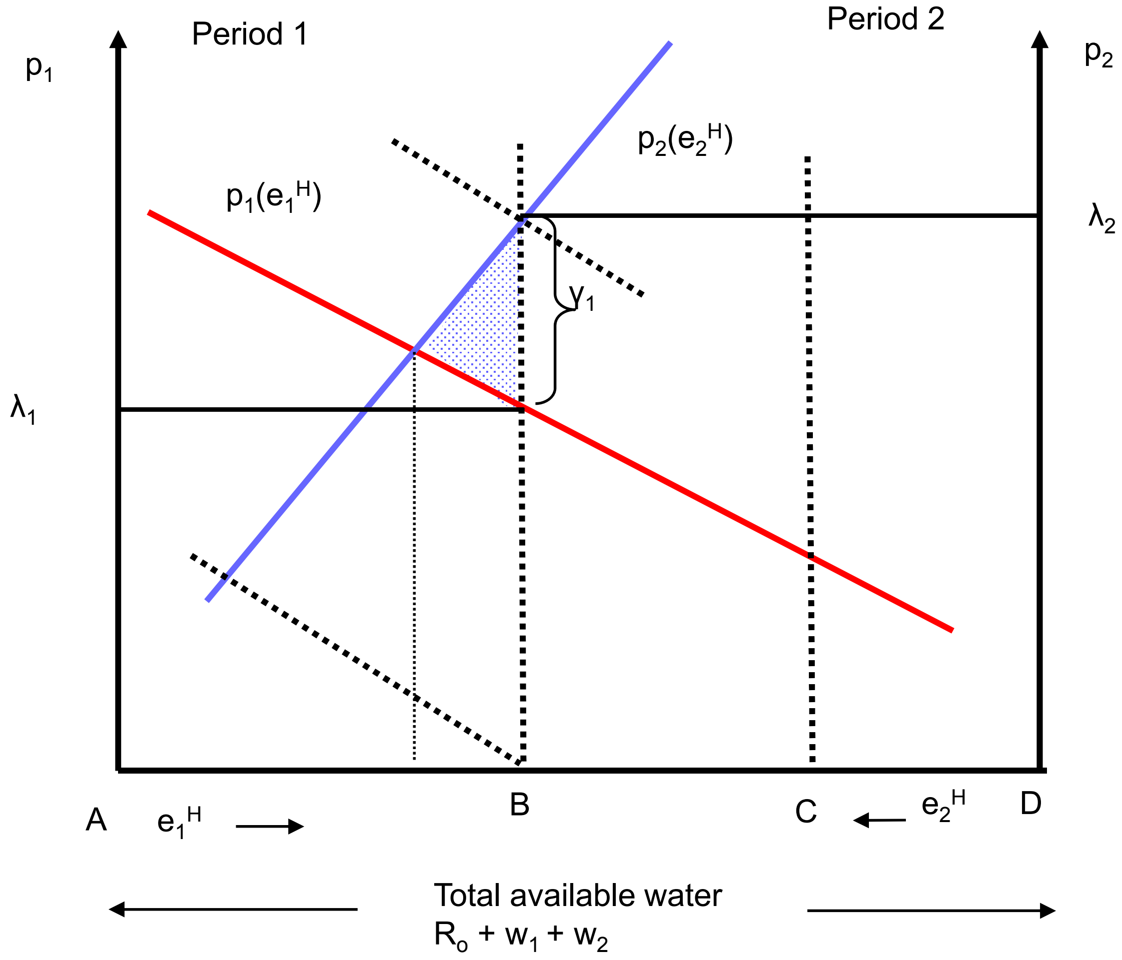
$$-\lambda_1 + \lambda_2 - \gamma_1 \leq 0 \ (= 0 \text{ if } R_1 > 0)$$

- Implication of binding reservoir constraint

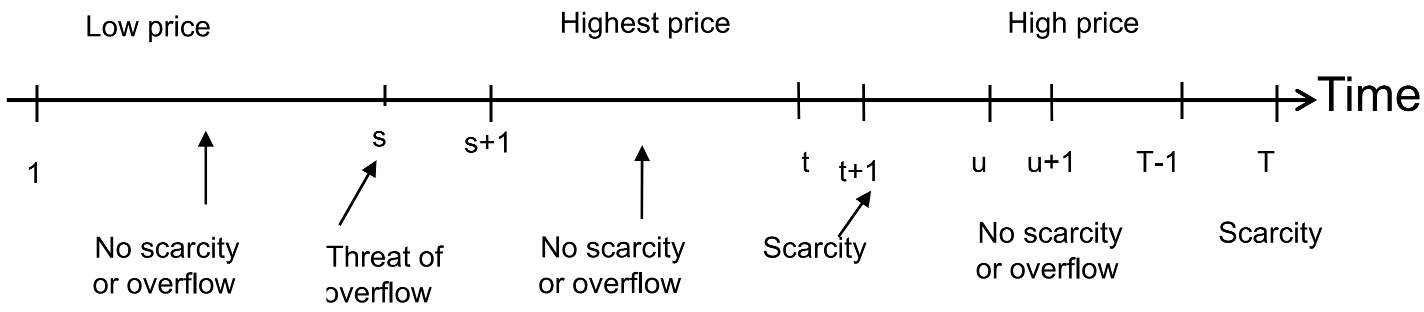
$$R_1 = \overline{R} \Rightarrow -\lambda_1 + \lambda_2 - \gamma_1 = 0 \ , \ \gamma_1 \geq 0$$

$$\Rightarrow \lambda_1 \leq \lambda_2 \Rightarrow p_1 \leq p_2$$

Threat of overflow

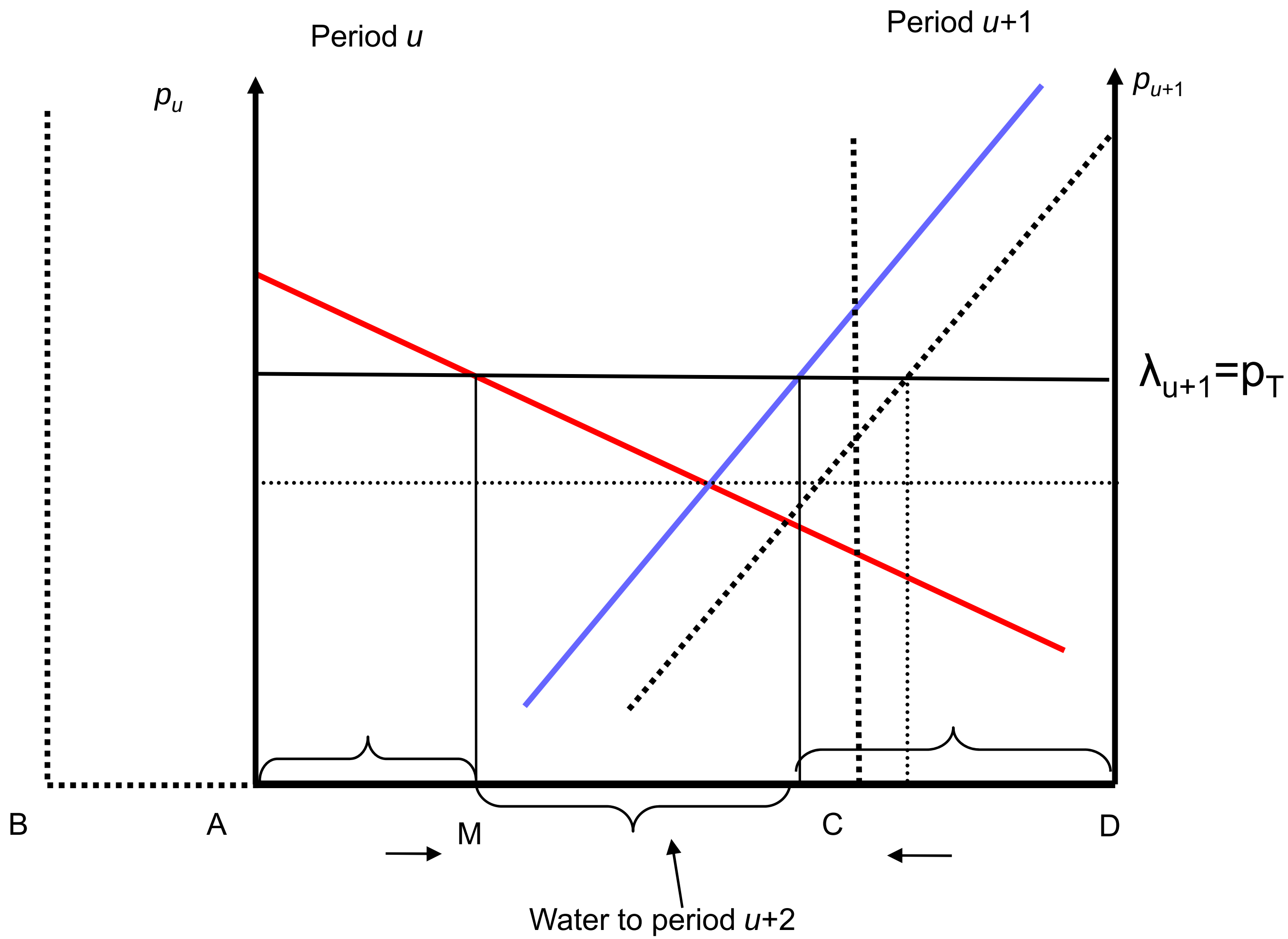


Price-determining events, general case

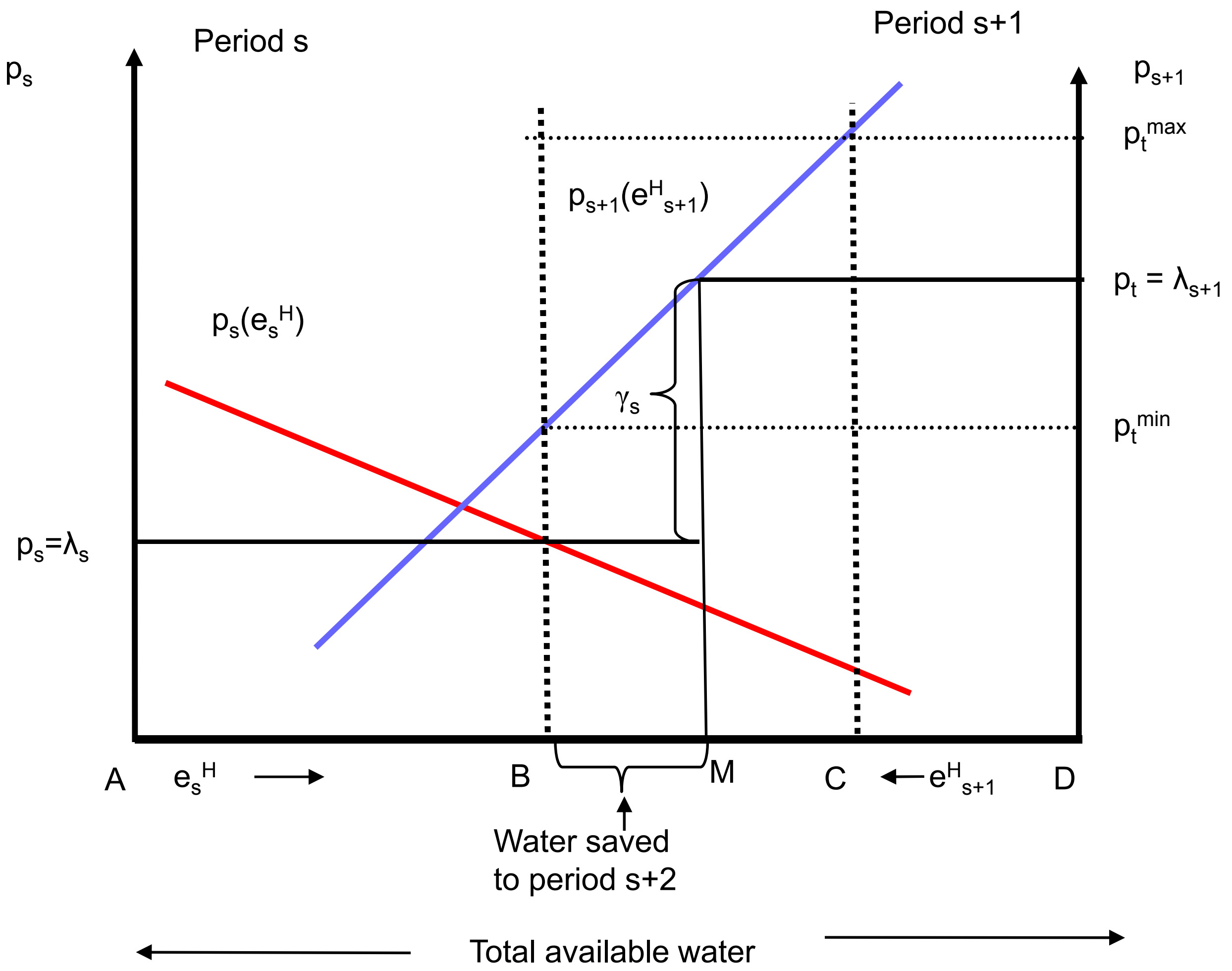


- Price-determining events
  - Scarcity and threat of overflow

In between empty and full



Reservoir full in period s



Hydropower constraints

Variable	Constraint type	Expression
R_t : reservoir at end of t	Max reservoir: $\overline{R}_t$	$R_t \leq \overline{R}_t$
	Environmental concerns, min reservoir: $\underline{R}_t$	$R_t \geq \underline{R}_t$
e_t^H : hydropower during t	Max power capacity: $\overline{e}^H$	$e_t^H \leq \overline{e}^H$
	Max transmission capacity: $\overline{e}_t^H$	$e_t^H \leq \overline{e}_t^H$
r_t : release of water during t	Water flows, environment: $\underline{r}_t$ = min, $\overline{r}_t$= max	$\underline{r}_t \leq r_t \leq \overline{r}_t$
	Environment: ramping up: r_t^u	$r_t - r_{t-1} \leq r_t^u$
	ramping down: r_t^d	$r_{t-1} - r_t \leq r_t^d$

Production constraint

- A constraint on power (turbines, dimension of pipes)
  - Implies a constraint on energy production

$$e_t^H \leq \overline{e}^H$$

  - Constraint independent of period
- A production constraint may become binding when
  - Preventing overflow
  - Trying to satisfy peak demand

The manoeuvrability of the system

- Perfect manoeuvrability:
  - All available water can be produced within the period
- Limited manoeuvrability:
  - Takes more than a period to empty the reservoir
  - Key parameter: the minimum number of periods t^o it takes to empty a full reservoir

$$t^o = \min t \text{ such that } t\,\overline{e}^H \geq \overline{R}$$

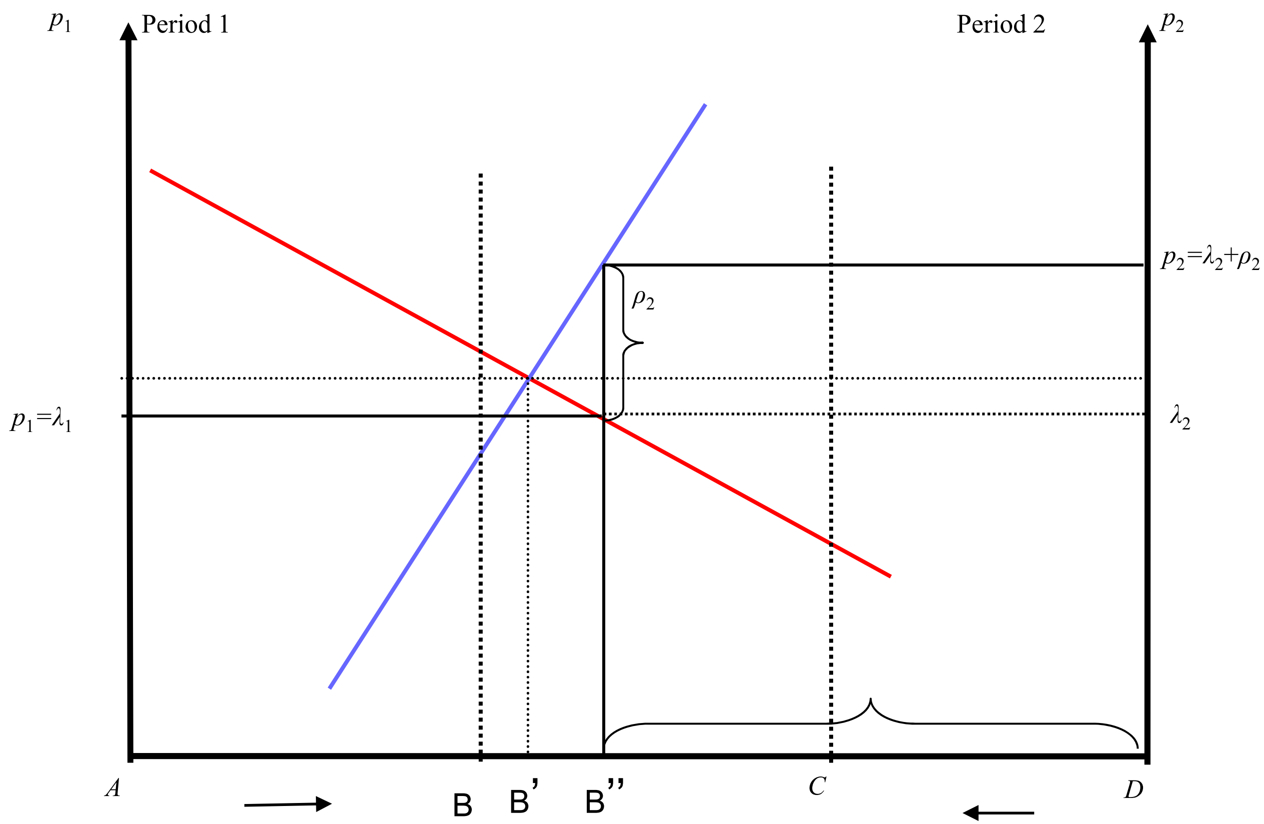
  - Manoeuvrability depends on the choice of time period here, but is a real time problem

Locking in of water

- Unavoidable loss of water due to a production constraint
  - Starting with an empty reservoir the inflows are such that even processing maximal water in each period overflow is in the end unavoidable

$$R_{t'} = 0, \sum_{t=t'}^{t''} w_t - (t'' - t' + 1)\overline{e}^H > \overline{R}$$

A bathtub illustration:
constraint in period 2: peak demand



A bathtub illustration:
constraint in period 1:threat of overflow