ECON 4925 Autumn 2007
Electricity Economics
Lecture 4

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Multiple plants

- N plants with reservoirs, a single aggregate consumer, x_t = total consumption
- The energy balance

$$x_t = \sum_{j=1}^{N} e_{jt}^{H}, \quad j = 1,..,N,\ t = 1,..,T$$

- The energy balance has to hold as an equality
- Reservoir constraints only

The social planning problem

$$\max \sum_{t=1}^{T} \int_{z=0}^{x_t} p_t(z)\,dz$$

subject to

$$x_t = \sum_{j=1}^{N} e_{jt}^H$$

$$R_{jt} \le R_{j,t-1} + w_{jt} - e_{jt}^H$$

$$R_{jt} \le \overline{R}_j$$

$$R_{jt}, x_t, e_{jt}^H \ge 0$$

$$T, w_{jt}, R_{jo}, \overline{R}_j \text{ given}, R_{jT} \text{ free}, \; j = 1,..,N, \; t = 1,..,T$$

The Lagrangian function

- Eliminating consumption by inserting the energy balance

$$L = \sum_{t=1}^{T} \int_{z=0}^{\sum_{j=1}^{N} e_{jt}^{H}} p_t(z)\,dz$$

$$-\sum_{t=1}^{T} \sum_{j=1}^{N} \lambda_{jt} (R_{jt} - R_{j,t-1} - w_{jt} + e_{jt}^{H})$$

$$-\sum_{t=1}^{T} \sum_{j=1}^{N} \gamma_{jt} (R_{jt} - \bar{R}_j)$$

The Kuhn – Tucker conditions

$$\frac{\partial L}{\partial e_{jt}^{H}} = p_t(\sum_{j=1}^{N} e_{jt}^{H}) - \lambda_{jt} \le 0 (= 0 \text{ for } e_{jt}^{H} > 0)$$

$$\frac{\partial L}{\partial R_{jt}} = -\lambda_{jt} + \lambda_{j,t+1} - \gamma_{jt} \le 0 (= 0 \text{ for } R_{jt} > 0)$$

$$\lambda_{jt} \ge 0 (= 0 \text{ for } R_{jt} < R_{j,t-1} + w_{jt} - e_{jt}^{H})$$

$$\gamma_{jt} \ge 0 (= 0 \text{ for } R_{jt} < \bar{R}_j) , \qquad t = 1,..,T , j = 1,..,N$$

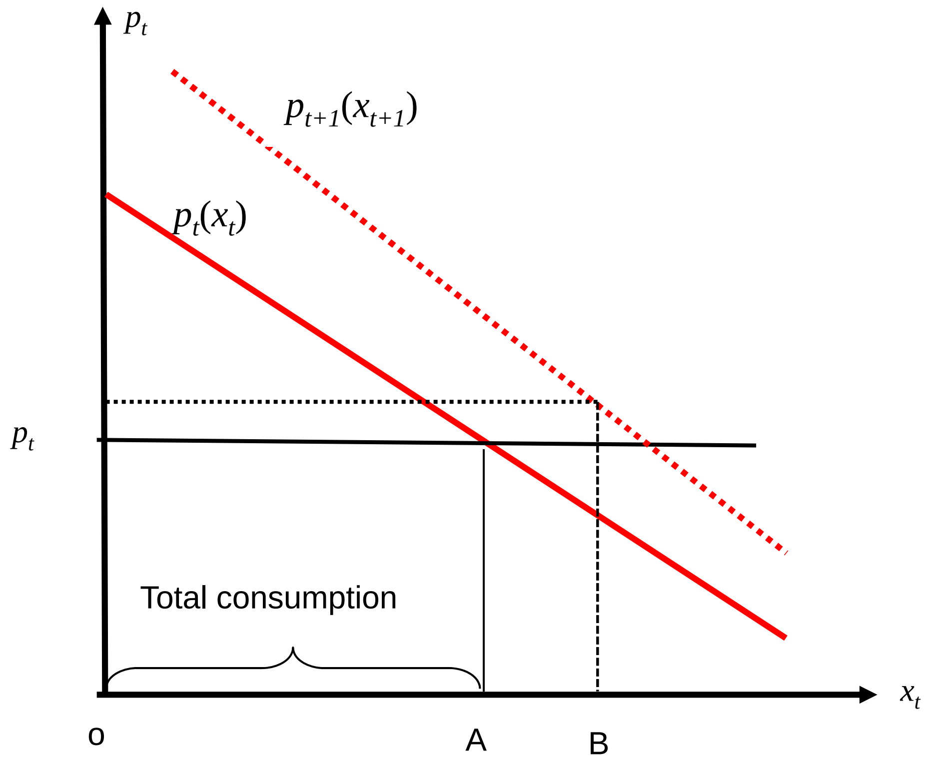
Backwards induction

- The terminal period: No satiation of demand → all plants use up all water in the terminal period

$$p_T(\sum_{j=1}^{N} e_{jT}^H) = \lambda_{jT} > 0 \Rightarrow \lambda_{jT} = \lambda_T$$

- If all reservoirs are in between full and empty in period *T* -1: the plants will face the same price equal to the water value in the terminal period also in period *T* -1

Indeterminacy of plant quantities



Equality of water values

- For a water value to change either the reservoir constraint must be binding or the reservoir is emptied
- Assume that the social price for periods t and $t+1$ are equal. Can it then be optimal for a plant to have a full reservoir in period t?
  - Yes it may be optimal, but the shadow price on the reservoir constraint will be zero
  - The reservoir must be full when the social price increases

Equality of water values, cont.

- Assume that the social price is the same in period t and $t+1$. Can it then be optimal for a plant to empty the reservoir in period t ?
  - Yes, but the water value in period t and $t+1$ must be the same
  - The reservoir must be full when the social price increases
  - The reservoir must be emptied when the social price decreases

The case of pure accumulation

- If the water value is greater than the social price, then production is zero and all inflows are accumulated
- A plant may accumulate water and produce zero for many periods
- The water value will be equal for all zero production periods and equal to the social price in the first period with positive production

Hveding's conjecture

- Assume independent hydropower plants with one limited reservoir each, and perfect manoeuvrability of reservoirs, but plant-specific inflows
- The plants can be regarded as a single aggregate plant and the reservoirs can be regarded as a single aggregate reservoir when finding the social optimal solution for operating the hydropower system.