ECON 4925 Spring 2011
Electricity Economics
Lecture 5 and 6

Lecturer:
Finn R. Førsund

Thermal capacity

- Primary energy source coal, oil, gas, wood uranium
  - Water is heated up to produce steam that drives the turbines producing electricity
  - Burning gas directly like a jet engine: CCGT; combined cycle gas turbine
  - Emission of pollutants; acid rain, climate gases, nuclear waste
- Thermal capacity is power-restricted

Thermal capacity, cont.

- Variable cost of thermal electricity production as a function of output based on the consumption of primary fuel

$$c_{it} = c_i(e_{it}^{Th}),\ c_i' > 0, c_i'' > 0,\ e_{it}^{Th} \leq \overline{e}_i^{Th}$$

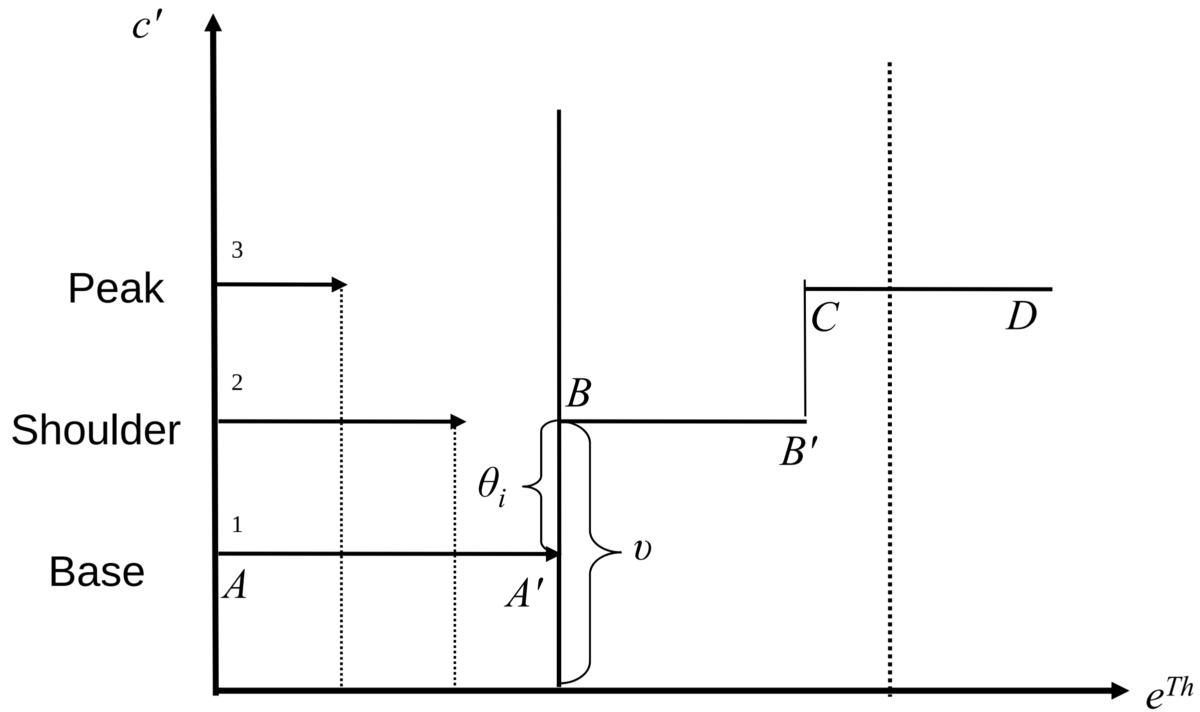
$$c_{it} = \text{variable cost of electricity production at unit } i$$

$$e_{it}^{Th} = \text{thermal electricity production at unit } i$$

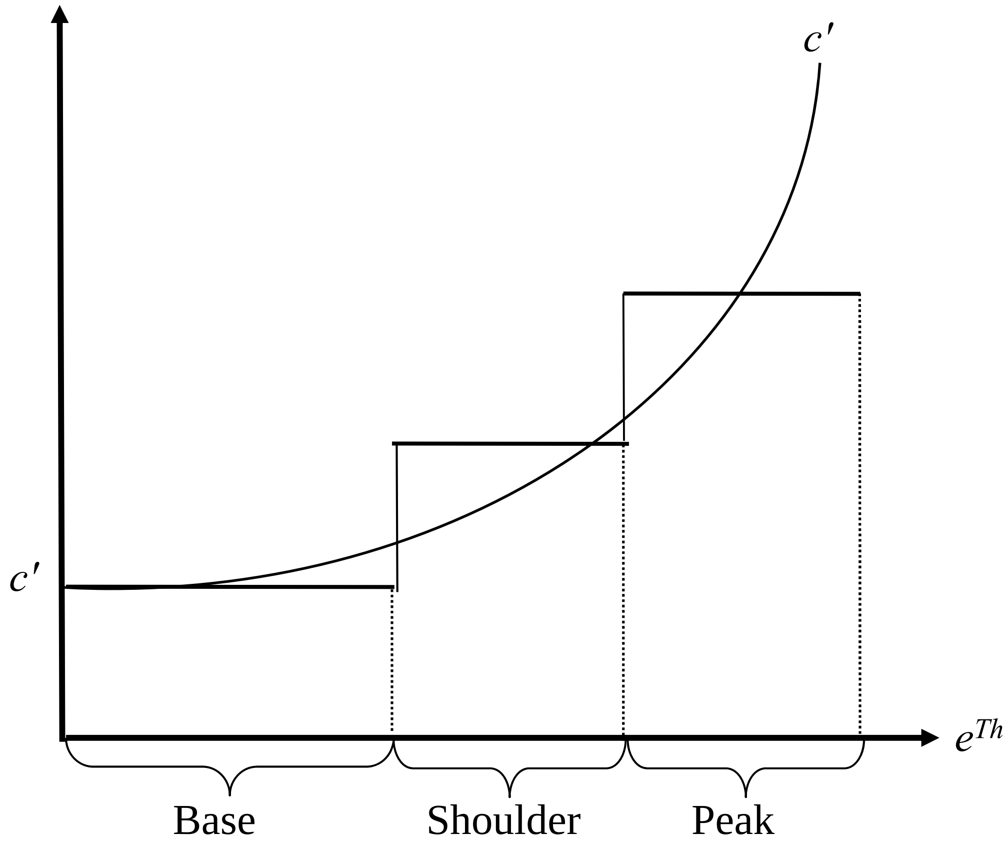
$$\overline{e}_i^{Th} = \text{capacity limit at unit } i$$

Merit-order ranking

- No intersection of marginal cost curves



Merit-order aggregation of cost curves



Thermal and hydro with a reservoir constraint

- The social planning problem

$$\max \sum_{t=1}^{T} [\int_{z=0}^{x_t} p_t(z)\,dz - c(e_t^{Th})]$$

subject to

$$x_t = e_t^H + e_t^{Th}$$

$$R_t \le R_{t-1} + w_t - e_t^H$$

$$R_t \le \overline{R}$$

$$e_t^{Th} \le \overline{e}^{Th}$$

$$x_t, e_t^H, e_t^{Th}, R_t \ge 0, \quad t = 1,..,T$$

$$T, w_t, R_o, \overline{R}, \overline{e}^{Th} \text{ given}, R_T \text{ free}$$

The Lagrangian

$$L = \sum_{t=1}^{T} [\int_{z=0}^{e_t^H + e_t^{Th}} p_t(z)\, dz - c(e_t^{Th})]$$

$$- \sum_{t=1}^{T} \lambda_t (R_t - R_{t-1} - w_t + e_t^H)$$

$$- \sum_{t=1}^{T} \gamma_t (R_t - \overline{R})$$

$$- \sum_{t=1}^{T} \theta_t (e_t^{Th} - \overline{e}^{Th})$$

The Kuhn – Tucker conditions

$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H + e_t^{Th}) - \lambda_t \le 0 \; (= 0 \text{ for } e_t^H > 0)$$

$$\frac{\partial L}{\partial e_t^{Th}} = p_t(e_t^H + e_t^{Th}) - c'(e_t^{Th}) - \theta_t \le 0 \; (= 0 \text{ for } e_t^{Th} > 0)$$

$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \; (= 0 \text{ for } R_t > 0)$$

$$\lambda_t \ge 0 \; (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$

$$\gamma_t \ge 0 \; (= 0 \text{ for } R_t < \overline{R})$$

$$\theta_t \ge 0 \; (= 0 \text{ for } e_t^{Th} < \overline{e}^{Th}) \, , \quad t = 1,..,T$$

The use of hydro and thermal

- Thermal will not be used in period *t* if

$$c'(0) > p_t(x_t) = \lambda_t \ (e_t^{Th} = 0, \theta_t = 0, x_t = e_t^H)$$

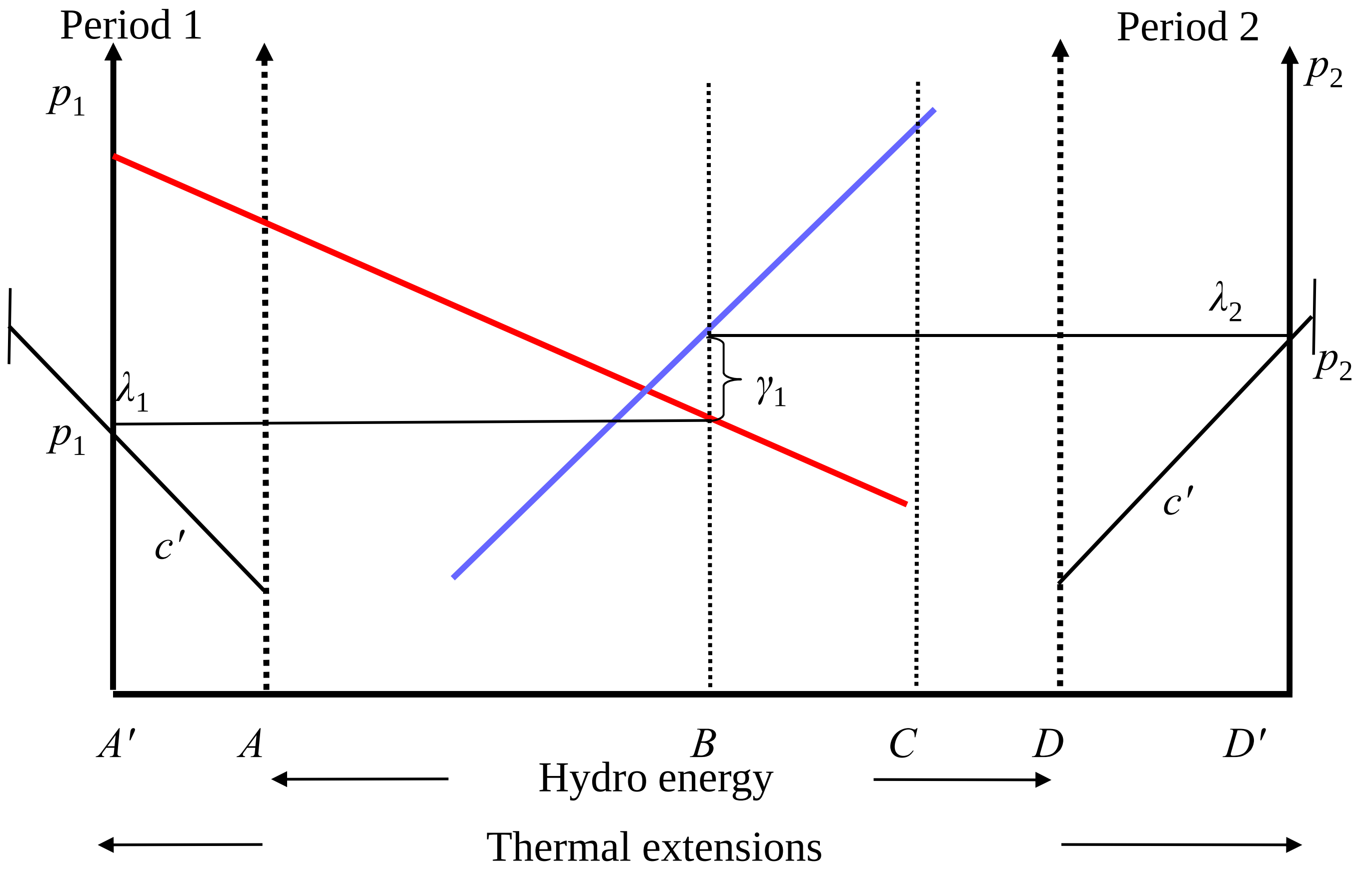
- Hydro will not be used in period *t* if

$$\lambda_t > p_t(x_t) = c'(e_t^{Th}) + \theta_t \ \ (\theta_t \geq 0, e_t^H = 0, \ x_t = e_t^{Th})$$

- Both thermal and hydro in use

$$p_t(x_t) = \lambda_t = c'(e_t^{Th}) + \theta_t \, (\theta_t \geq 0, x_t = e_t^H + e_t^{Th})$$

Bathtub diagram with thermal and hydro with reservoir constraint



The formal model: technology mix

$$\max \sum_{t=1}^{T} [\int_{z=0}^{x_t} p_t(z)dz - c^C(e_t^C) - c^N(e_t^N)]$$

subject to the constraints

$$x_t = e_t^H + e_t^C + e_t^N + e_t^I$$

$$R_t \leq R_{t-1} + w_t - e_t^H$$

$$R_t \leq \overline{R}$$

$$e_t^C \leq \overline{e}^C$$

$$e_t^N \leq \overline{e}^N$$

$$e_t^I \leq \overline{e}^I$$

$$x_t, e_t^H, e_t^C, e_t^N \geq 0,\ t = 1,..,T$$

$$T, R_o, \overline{R}, \overline{e}^C, \overline{e}^N, \overline{e}^I \text{ given, } R_T \text{ free}$$

The necessary 1.order conditions

$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H + e_t^C + e_t^N + e_t^I) - \lambda_t \leq 0 \ (= 0 \text{ for } e_t^H > 0)$$

$$\frac{\partial L}{\partial e_t^C} = p_t(e_t^H + e_t^C + e_t^N + e_t^I) - c^{C\prime}(e_t^C) - \theta_t^C \leq 0 \ (= 0 \text{ for } e_t^C > 0)$$

$$\frac{\partial L}{\partial e_t^C} = p_t(e_t^H + e_t^C + e_t^N + e_t^I) - c^{N\prime}(e_t^N) - \theta_t^N \leq 0 \ (= 0 \text{ for } e_t^N > 0)$$

$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \leq 0 \ (= 0 \text{ for } R_t > 0)$$

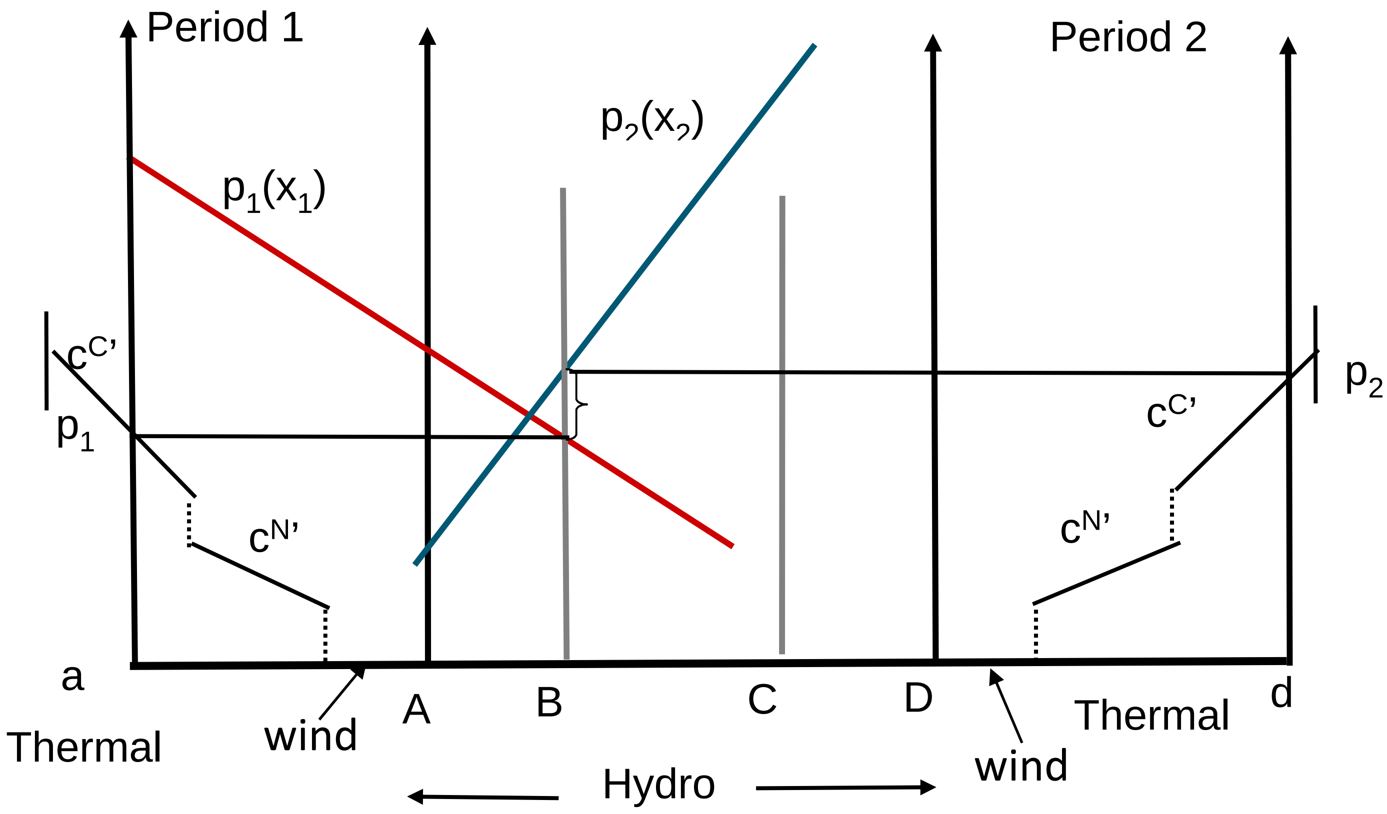
$$\lambda_t \geq 0 \ (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$

$$\gamma_t \geq 0 \ (= 0 \text{ for } R_t < \overline{R})$$

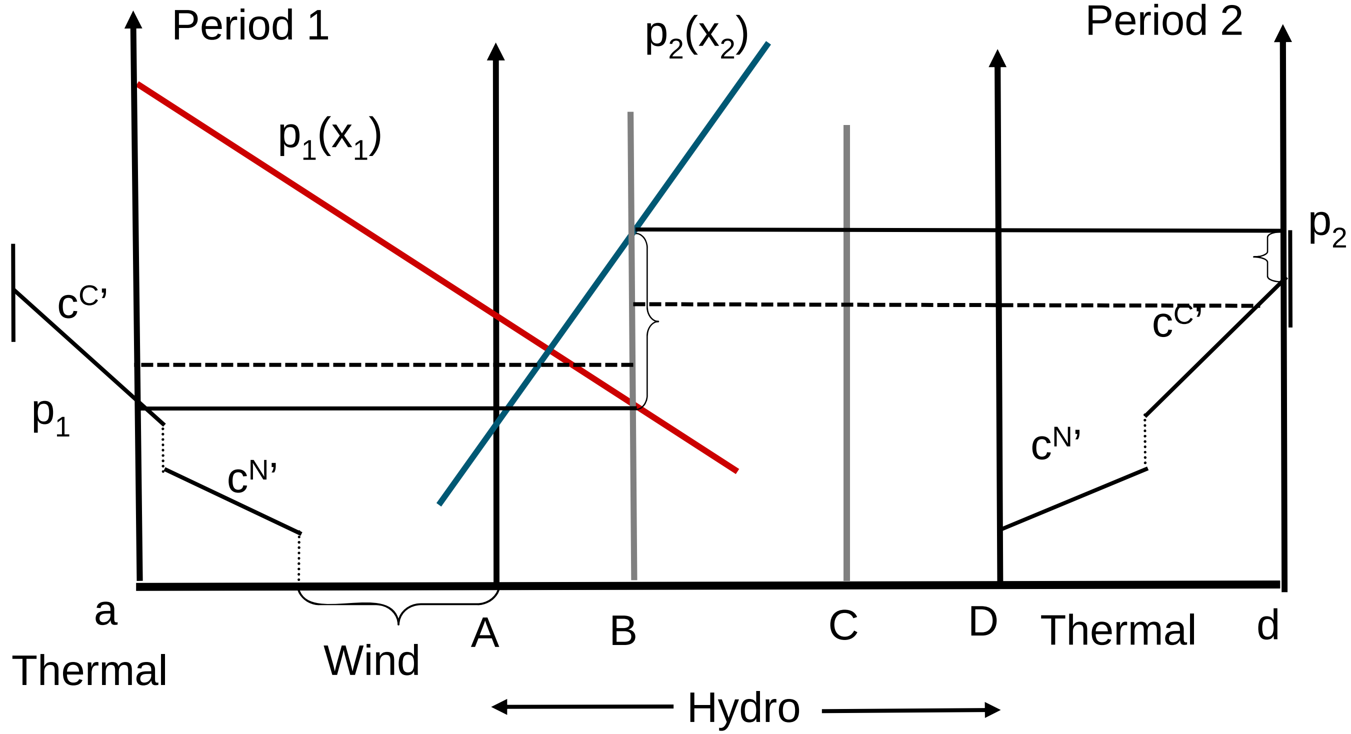
$$\theta_t^C \geq 0 \ (= 0 \text{ for } e_t^C < \overline{e}^C)$$

$$\theta_t^N \geq 0 \ (= 0 \text{ for } e_t^N < \overline{e}^N), \quad t = 1,..,T$$

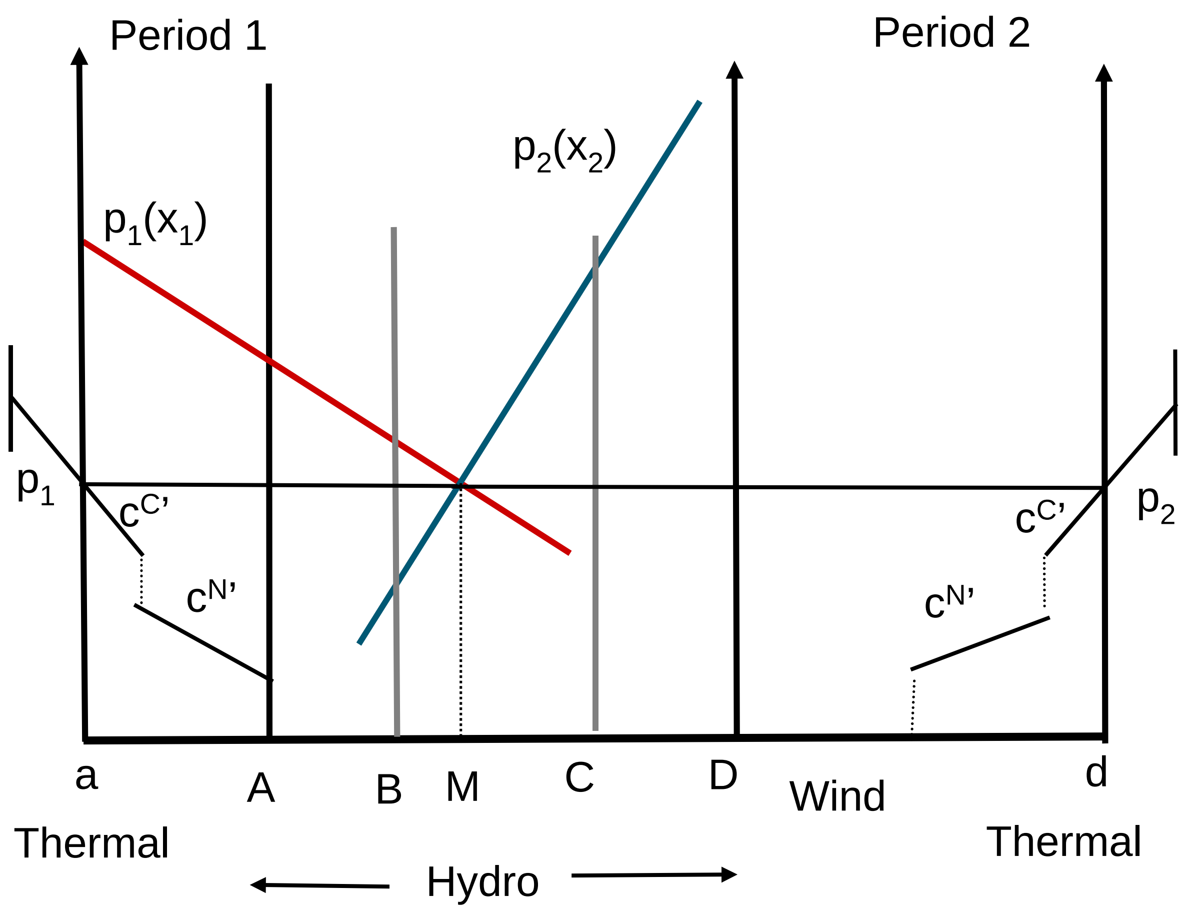
Same wind in both periods



All wind in period 1



All wind in period 2



No use of hydro in period 1, wind only blowing in period 1