ECON 4930 Spring 2011
Electricity Economics
Lecture 8

Lecturer:

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A system transmission network

- Two nodes with one line the simplest case
- Converting variables to energy units (kWh)
- The energy balance

$$x_t = e_t^H - e_t^L,\ t = 1,..,T$$

$$x_t = \text{consumption in kWh}$$

$$e_t^H = \text{production in kWh}$$

$$e_t^L = \text{loss in kWh}$$

A system transmission network, cont.

- Expressing loss on a line supplying a single consumer node from a single hydropower generator node

$$e_t^L = e_t^L(x_t), \ \frac{\partial e_t^L(x_t)}{\partial x_t} > 0, \ \frac{\partial^2 e_t^L(x_t)}{\partial x_t^2} > 0, \ t = 1,..,T$$

- Loss is increasing in consumption, and with a positive second-order derivative, according to Ohm's law
- A thermal capacity on the line:

$$x_t \le \overline{x}$$

The social planning problem for the two-node case for two time periods

$$\max \sum_{t=1}^{2} \int_{z=0}^{x_t} p_t(z)dz$$

subject to

$$R_t \le R_{t-1} + w_t - e_t^H$$

$$R_t \le \overline{R}$$

$$x_t + e_t^L = e_t^H$$

$$e_t^L = e_t^L(x_t)$$

$$x_t \le \overline{x}$$

$$R_t, x_t, e_t^H, e_t^L \ge 0,\ t = 1,2$$

$$R_o, \overline{R}, \overline{x} \text{ given}$$

The Lagrangian function

$$L = \sum_{t=1}^{2} \int_{z=0}^{x_t} p_t(z)dz$$

$$-\sum_{t=1}^{2} \lambda_t (R_t - R_{t-1} - w_t + e_t^H)$$

$$-\sum_{t=1}^{2} \gamma_t (R_t - \overline{R})$$

$$-\sum_{t=1}^{2} \tau_t (x_t + e_t^L(x_t) - e_t^H)$$

$$-\sum_{t=1}^{2} \mu_t (x_t - \overline{x})$$

The Kuhn – Tucker conditions

$$\frac{\partial L}{\partial x_t} = p_t(x_t) - \tau_t - \tau_t \frac{\partial e_t^L}{\partial x_t} - \mu_t \leq 0 \ (= 0 \text{ for } x_t > 0)$$

$$\frac{\partial L}{\partial e_t^H} = -\lambda_t + \tau_t \leq 0 \ (= 0 \text{ for } e_t^H > 0)$$

$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \leq 0 \ (= 0 \text{ for } R_t > 0)$$

$$\lambda_t \geq 0 (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$

$$\gamma_t \geq 0 (= 0 \text{ for} R_t < \overline{R})$$

$$\mu_t \geq 0 (= 0 \text{ for } x_t < \overline{x}) \text{ , } t = 1,2$$

Interpretation of the first-order conditions

- Assuming positive production in both periods
- The shadow prices on the energy balance will then be positive and equal to the water values

$$p_t(x_t) - \lambda_t - \lambda_t \frac{\partial e_t^L}{\partial x_t} - \mu_t = 0,\ t = 1,2$$

- The difference between the social price and the water value

$$p_t(x_t) - \lambda_t = \lambda_t \frac{\partial e_t^L}{\partial x_t} + \mu_t = 0,\ t = 1,2$$

Interpretations, cont.

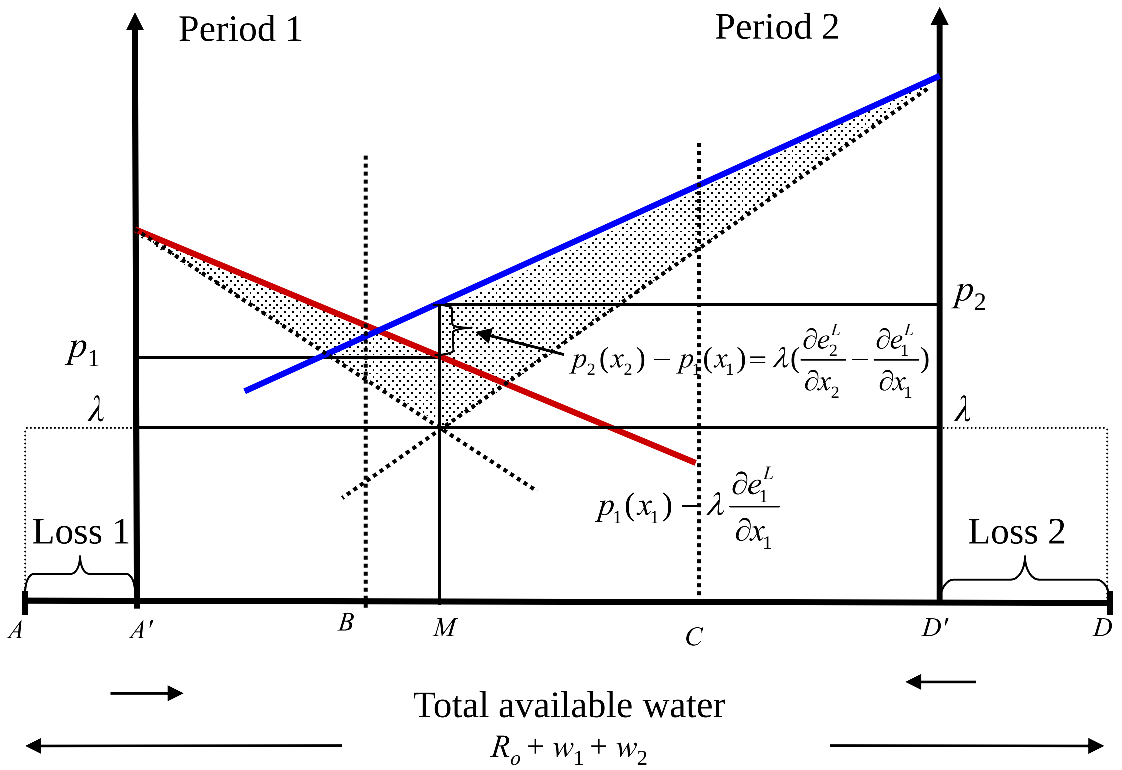
- Difference in social price between periods

$$p_2(x_2) - p_1(x_1) = \lambda_2 \frac{\partial e_2^L}{\partial x_2} - \lambda_1 \frac{\partial e_1^L}{\partial x_1} + (\mu_2 - \mu_1)$$

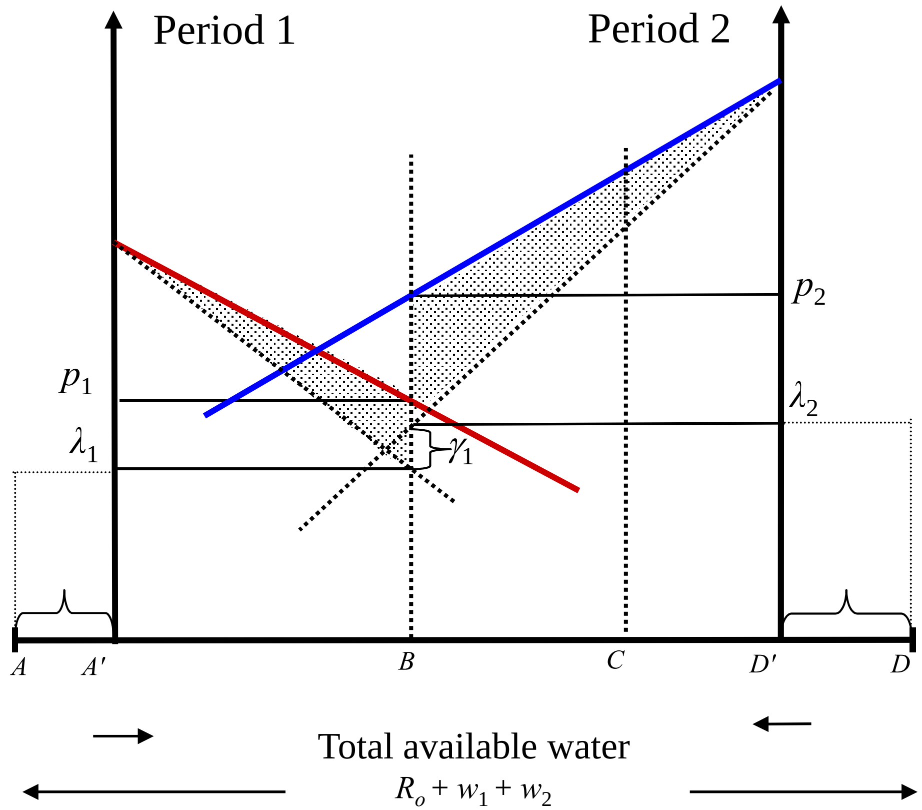
- Difference even if water values are equal (reservoir constraint not binding)

$$p_2(x_2) - p_1(x_1) = \lambda\left(\frac{\partial e_2^L}{\partial x_2} - \frac{\partial e_1^L}{\partial x_1}\right) + (\mu_2 - \mu_1)$$

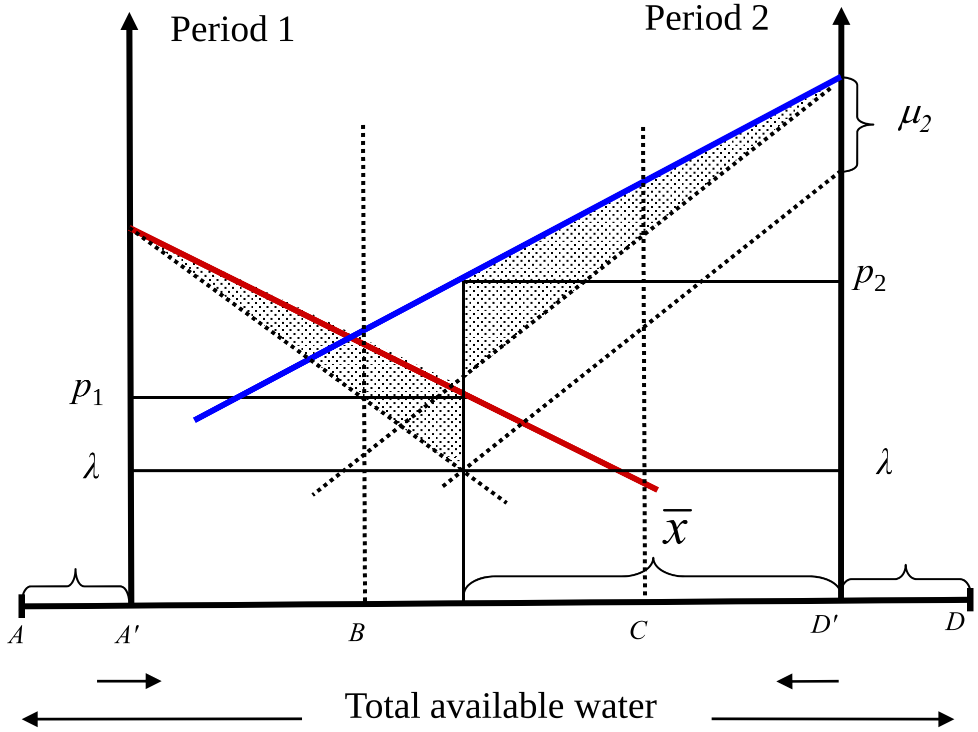
A bathtub illustration without congestion



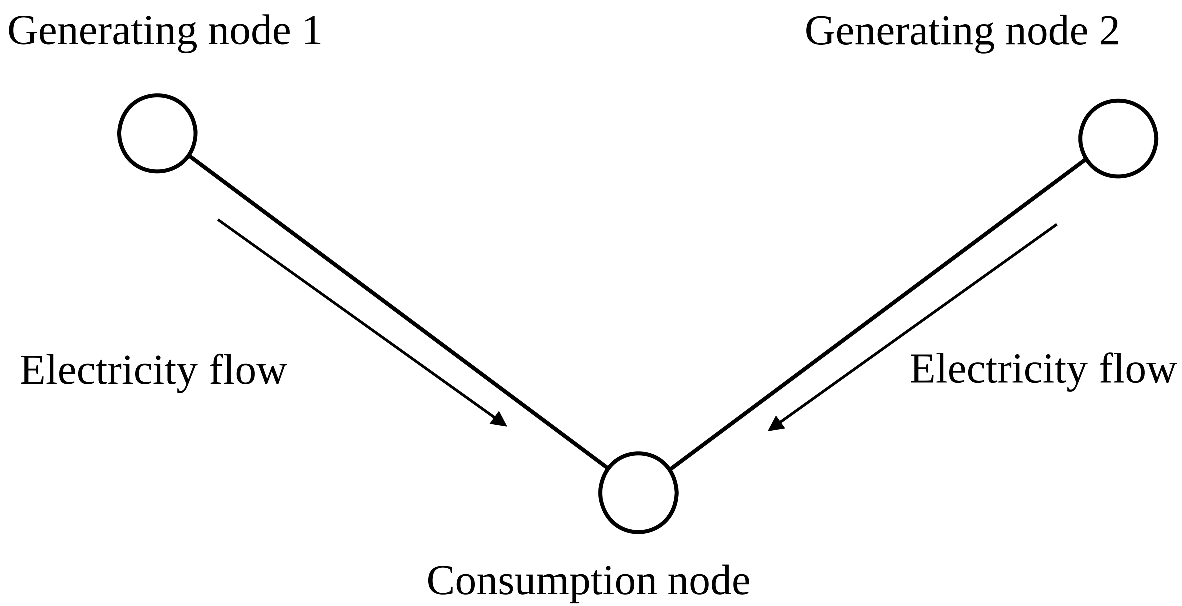
A bathtub illustration with binding reservoir constraint, but without congestion



Bathtub with congestion



Three nodes and two periods



The social planning problem

$$\max \sum_{t=1}^{2} \int_{z=0}^{x_t} p_t(z)dz$$

subject to

$$R_{jt} \le R_{j,t-1} + w_{jt} - e_{jt}^H$$

$$R_{jt} \le \overline{R}_j$$

$$x_{jt} + e_{jt}^L = e_{jt}^H$$

$$x_t = \sum_{j=1}^{2} x_{jt}$$

$$e_{jt}^L = e_{jt}^L(x_{jt})$$

$$x_{jt} \le \overline{x}_j$$

$$R_{jt}, x_t, x_{jt}, e_{jt}^H, e_{jt}^L \ge 0$$

$$w_{jt}, R_{jo}, \overline{R}_j, \overline{x}_j \text{ given}, R_{j2} \text{ free}, j = 1,2\ ,\ t = 1,2$$

The Lagrangian function

$$L = \sum_{t=1}^{2} \int_{z=0}^{\sum_{j=1}^{2} x_{jt}} p_t(z)dz$$

$$-\sum_{t=1}^{2}\sum_{j=1}^{2}\lambda_{jt}(R_{jt} - R_{j,t-1} - w_{jt} + e_{jt}^{H})$$

$$-\sum_{t=1}^{2}\sum_{j=1}^{2}\gamma_{jt}(R_{jt} - \overline{R}_{j})$$

$$-\sum_{t=1}^{2}\sum_{j=1}^{2}\tau_{jt}(x_{jt} + e_{jt}^{L}(x_{jt}) - e_{jt}^{H})$$

$$-\sum_{t=1}^{2}\sum_{j=1}^{2}\mu_{jt}(x_{jt} - \overline{x}_{j})$$

The Kuhn – Tucker conditions

$$\frac{\partial L}{\partial x_{jt}} = p_t(x_t) - \tau_{jt} - \tau_{jt}\frac{\partial e_{jt}^L}{\partial x_{jt}} - \mu_{jt} \le 0 \ (= 0 \text{ for } x_{jt} > 0)$$

$$\frac{\partial L}{\partial e_{jt}^H} = -\lambda_{jt} + \tau_{jt} \le 0 \ (= 0 \text{ for } e_{jt}^H > 0)$$

$$\frac{\partial L}{\partial R_{jt}} = -\lambda_{jt} + \lambda_{j,t+1} - \gamma_{jt} \le 0 \ (= 0 \text{ for } R_{jt} > 0)$$

$$\lambda_{jt} \ge 0 (= 0 \text{ for } R_{jt} < R_{j,t-1} + w_{jt} - e_{jt}^H)$$

$$\gamma_{jt} \ge 0 (= 0 \text{ for } R_{jt} < \overline{R}_j)$$

$$\mu_{jt} \ge 0 (= 0 \text{ for } x_{jt} < \overline{x}_j) \ , \ j = 1,2 \ , \ t = 1,2$$

Interpretations of the first-order conditions

- Assumptions:
  - Positive production in the first period at both plants (both empty the reservoirs in the second period)
  - No threat of overflow in the first period
    - Water values for a plant the same for both periods
- Difference between consumer price and water values

$$p_t(x_t) - \lambda_j = \lambda_j \frac{\partial e_{jt}^L}{\partial x_{jt}} + \mu_{jt}, \; j = 1,2 \; , \; t = 1,2$$

Interpretations, cont.

- Difference between water values

$$p_t(x_t) = \lambda_j + \lambda_j \frac{\partial e_{jt}^L}{\partial x_{jt}} + \mu_{jt} \Rightarrow$$

$$\lambda_1 + \lambda_1 \frac{\partial e_{1t}^L}{\partial x_{1t}} + \mu_{1t} = \lambda_2 + \lambda_2 \frac{\partial e_{2t}^L}{\partial x_{2t}} + \mu_{2t} \Rightarrow$$

$$\lambda_1 - \lambda_2 = \lambda_2 \frac{\partial e_{2t}^L}{\partial x_{2t}} + \mu_{2t} - (\lambda_1 \frac{\partial e_{1t}^L}{\partial x_{1t}} + \mu_{1t}),\ t = 1,2$$

- The plant with highest sum of marginal loss and congestion will have the lowest water value

Interpretations, cont.

- Differences between consumer prices

$$p_2(x_2) - p_1(x_1) = \lambda_j \frac{\partial e^L_{j2}}{\partial x_{j2}} + \mu_{j2} - (\lambda_j \frac{\partial e^L_{j1}}{\partial x_{j1}} + \mu_{j1}),\ j = 1, 2$$

- The highest consumer price will be in the period with the highest value of the sum of marginal loss and congestion term

Nodal prices

- Prices and water values are specific to each node
- Consumer price greater than water values for each time period
- Water values differ due to loss and congestion between plants for each time period
- Marginal loss evaluated at water values plus congestion is equal for each time period

Congestion in the two-period case

- Assumptions:
  - A line is at most congested in the high-demand period only
  - There is no lock-in of water due to congestion

    $$R_{jo} + w_{j1} + w_{j2} < \overline{x}_j, \; j = 1,2$$
  - Period 1 is the low-demand period and period 2 the high-demand period
- Immediate implication: at least one plant must produce more in the high-demand period than the low-demand -period

Production levels for the two periods

- Both plants will produce more in the high-demand period and less in the low-demand period due to marginal loss increasing in energy

$$\lambda_1 (1 + \frac{\partial e_{1t}^L}{\partial x_{1t}}) + \mu_{1t} = \lambda_2 (1 + \frac{\partial e_{2t}^L}{\partial x_{2t}}) + \mu_{2t}, \ t = 1, 2$$

- Disregarding congestion if plant 1 produces more in the high-demand period so must plant 2
- Introducing congestion does not change this situation

Implication of transmission for utilisation of the hydro plants

- Transmission causes higher price in the high-demand period 2 and leads to a relatively greater use of water in period 1 than compared to no transmission
- The plant with relatively less marginal loss will shift production from the low demand-period to the high-demand period, and opposite for the plant with relatively greater marginal loss

Implications, cont.

- The plant with relatively higher marginal loss plus congestion in one period will also have a relatively higher loss plus congestion in the other period (plant water value constant)
- The plant with the lowest water value is used relatively more in the low-demand period, and the plant with the highest water value relatively more in the high-demand period

Loop-flows (meshed network)