ECON 4930 Spring 2011
Electricity Economics
Lecture 9

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Background

- The use of market power is a potential problem of the deregulated electricity sector
- 20 % of the world's electricity is produced by hydropower and 1/3 of the countries have more than 50%
- Hydro power is a special case due to zero variable cost, water storage, high power capacity and maximal flexible adjustment

Difference between objective functions for social planning and monopoly

- Objective function social planning
  - Area under the demand function (zero variable costs) $\sum_{t=1}^{T}\int_{z=0}^{e_t^H} p_t(z)\,dz$

- Objective function monopoly
  - Producer surplus: total revenue (zero variable costs) $\sum_{t=1}^{T} p_t(e_t^H)e_t^H$

Spilling and monopoly

- Depending on the characteristics of the demand functions it may be optimal for the monopoly to spill water
- Spilling can be regulated by prohibition
  - Making the water accumulation equation an equality constraint

$$R_t = R_{t-1} + w_t - e_t^H$$

- Zero-spilling regulation will reduce the monopoly profit

Limited reservoir and the social solution

- Reservoir dynamics: water at the end of a period = water at the end of previous period plus inflow minus release of water during the period
- Shadow prices on water and reservoir limit recursively related, solving using backward induction (Bellmann)
- Overflow is waste
- Social price may vary if reservoir constraint is binding

Limited reservoir and monopoly

- The flexibility-corrected price substitute for the social price
- Flexibility-corrected prices may differ if reservoir constraint is binding
- Social solution may be optimal if constraint is binding
- Differences with social solution depend on the demand elasticities
- Spilling may be optimal

Monopoly with reservoir constraint

$$\max \sum_{t=1}^{T} p_t(e_t^H) \cdot e_t^H$$

subject to

$$R_t \leq R_{t-1} + w_t - e_t^H$$

$$R_t \leq \overline{R}$$

$$R_t, e_t^H \geq 0 \,,\; t = 1,..,T$$

$$T, w_t, R_o, \overline{R} \text{ given}, R_T \text{ free}$$

Solving the monopoly optimisation problem

- The Lagrangian function

$$L = \sum_{t=1}^{T} p_t(e_t^H)e_t^H$$

$$-\sum_{t=1}^{T} \lambda_t (R_t - R_{t-1} - w_t + e_t^H)$$

$$-\sum_{t=1}^{T} \gamma_t (R_t - \overline{R})$$

Solving, cont.

- The Kuhn – Tucker conditions

$$\frac{\partial L}{\partial e_t^H} = p_t'(e_t^H)e_t^H + p_t(e_t^H) - \lambda_t \le 0 \; (= 0 \text{ for } e_t^H > 0)$$

$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \; (= 0 \text{ for } R_t > 0)$$

$$\lambda_t \ge 0 \; (0 = \text{for } R_t < R_{t-1} + w_t - e_t^H)$$

$$\gamma_t \ge 0 \; (0 = \text{for } R_t < \overline{R}) \,, \qquad t = 1,..,T$$

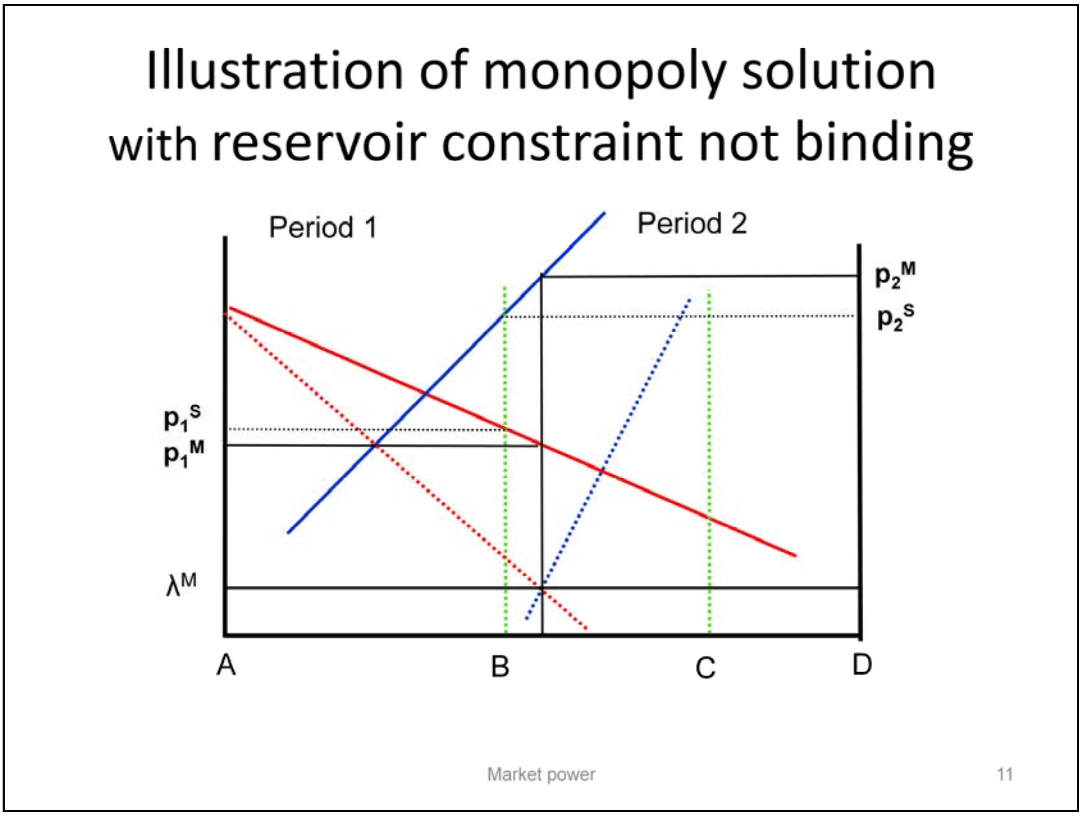
Interpretation of conditions

- Assuming positive production in all periods
- Using flexibility-corrected prices

$$p_t'(e_t^H)e_t^H + p_t(e_t^H) - \lambda_t = p_t(e_t^H)(1+\breve{\eta}_t) - \lambda_t = 0$$

$$-\lambda_t + \lambda_{t+1} - \gamma_t \leq \ (= 0 \text{ for } R_t > 0), \qquad t = 1,..,T$$

- Assuming positive flexibility-corrected prices is a stronger assumption than non-satiation
- Market prices will vary with relative elastic periods having lower prices than relative inelastic periods
- A strategy of shifting water between periods



Reservoir will not be filled, period 1 (”summer”) price will be lower, period 2 (”winter”) price will be higher than social optimum (shown by thin dotted lines), water value in period 2 will be positive, then same water value value in period 1 according to Bellman’s principle and equal to marginal revenues.

Illustration of monopoly solution with reservoir constraint binding

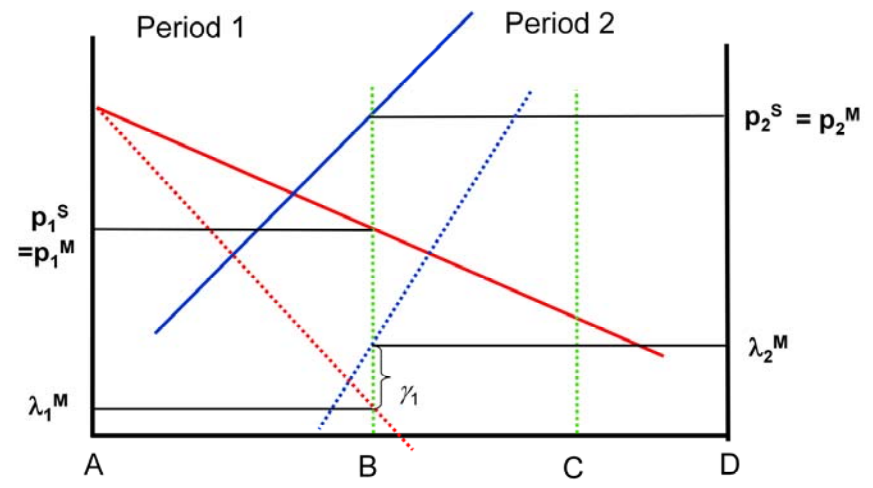
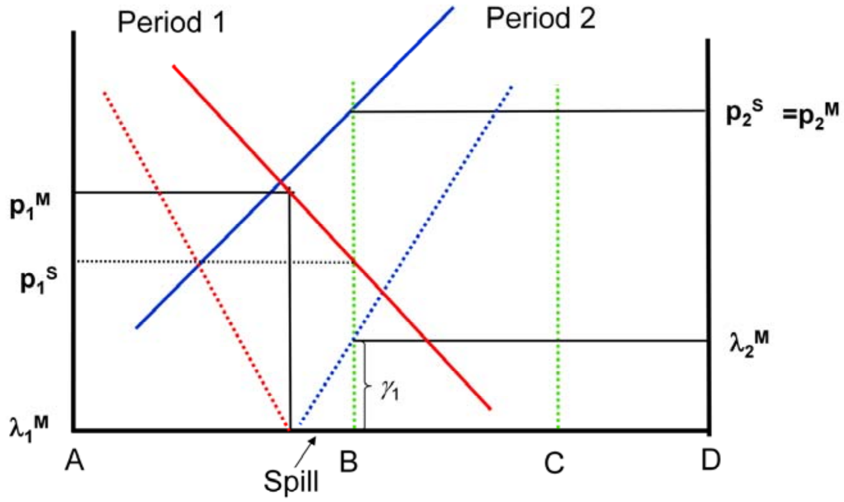


Illustration of monopoly solution with reservoir



Monopoly with thermal plants

- Optimisation problem

$$\max \sum_{t=1}^{T} p_t(x_t) \cdot x_t - c(e_t^{Th})$$

subject to

$$x_t = e_t^H + e_t^{Th}$$

$$R_t \le R_{t-1} + w_t - e_t^H$$

$$R_t \le \overline{R}$$

$$e_t^{Th} \le \overline{e}^{Th}$$

$$R_t, e_t^H, e_t^{Th} \ge 0 \,, \; t = 1,..,T$$

$$T, w_t, R_o, \overline{R}, \overline{e}^{Th} \text{ given}, R_T \text{ free}$$

- The Lagrangian

$$L = \sum_{t=1}^{T}[p_t(e_t^H + e_t^{Th})(e_t^H + e_t^{Th}) - c(e_t^{Th})]$$
$$-\sum_{t=1}^{T}\theta_t(e_t^{Th} - \overline{e}^{Th})$$
$$-\sum_{t=1}^{T}\lambda_t(R_t - R_{t-1} - w_t + e_t^H)$$
$$-\sum_{t=1}^{T}\gamma_t(R_t - \overline{R})$$

- Necessary first-order conditions

$$\frac{\partial L}{\partial e_t^H} = p_t'(e_t^H + e_t^{Th})(e_t^H + e_t^{Th}) + p_t(e_t^H + e_t^{Th}) - \lambda_t \le 0$$

$$(= 0 \text{ for } e_t^H > 0)$$

$$\frac{\partial L}{\partial e_t^{Th}} = p_t'(e_t^H + e_t^{Th})(e_t^H + e_t^{Th}) + p_t(e_t^H + e_t^{Th}) - c'(e_t^{Th}) - \theta_t \le 0$$

$$(= 0 \text{ for } e_t^{Th} > 0)$$

$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \ (= 0 \text{ for } R_t > 0)$$

$$\lambda \ge 0 \ (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$

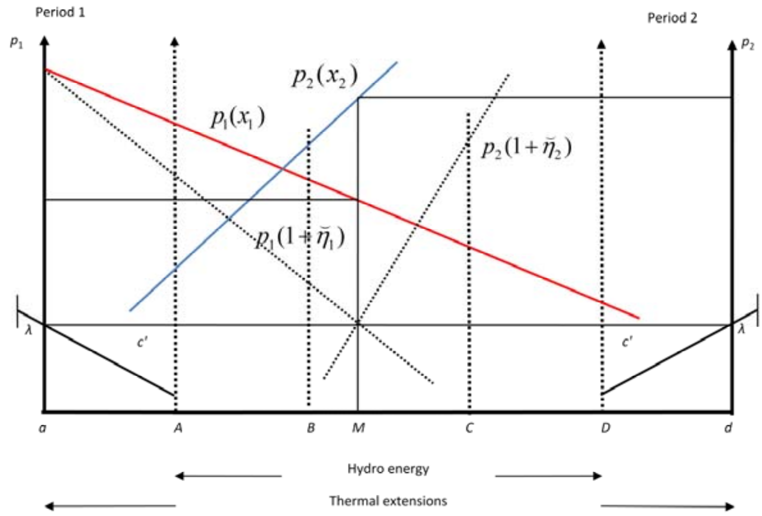
$$\theta_t \ge 0 \ (= 0 \text{ for } e_t^{Th} < \overline{e}^{Th})$$

$$\gamma_t \ge 0 \ (0 = \text{for } R_t < \overline{R})$$

- Using both hydro and thermal in the same period the marginal revenue substitutes for the marginal willingness to pay in the social optimal solution

$$p_t(x_t)(1+\breve{\eta}_t) = \lambda_t = c'(e_t^{Th}) + \theta_t$$

Hydro and thermal plants



Dominant hydro with a thermal competitive fringe

- Optimisation problem

$$\max \sum_{t=1}^{T} p_t(x_t)e_t^H$$

subject to

R_T free

$$x_t = e_t^H + e_t^{Th}$$

$$R_t \le R_{t-1} + w_t - e_t^H$$

$$R_t \le \overline{R}$$

$$p_t(x_t) = c'(e_t^{Th})$$

$$x_t, e_t^H, e_t^{Th} \ge 0, \quad t = 1,..,T$$

$$T, \overline{R} \text{ given}, R_T \text{ free}$$

- The fringe response to change in hydro production
  - Differentiation of the fringe 1. order condition

$$p_t(e_t^H + e_t^{Th}) = c'(e_t^{Th})(t = 1,..,T)$$

$$p_t'(e_t^H + e_t^{Th})(de_t^H + de_t^{Th}) = c''(e_t^{Th})de_t^{Th} \Rightarrow$$

$$\frac{de_t^{Th}}{de_t^H} = \frac{-p_t'(e_t^H + e_t^{Th})}{p_t'(e_t^H + e_t^{Th}) - c''(e_t^{Th})} < 0 \quad (t = 1,..,T)$$

  - The relationship between the fringe output and hydro output

$$e_t^{Th} = f_t(e_t^H), f_t' < 0 \ (t = 1,..,T)$$

- The Lagrangian

$$L = \sum_{t=1}^{T} p_t(e_t^H + f_t(e_t^H))e_t^H$$
$$-\sum_{t=1}^{T} \lambda_t (R_t - R_{t-1} - w_t + e_t^H)$$
$$-\sum_{t=1}^{T} \gamma_t (R_t - \overline{R})$$

- First-order order conditions

$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H + e_t^{Th}) + p_t'(e_t^H + e_t^{Th})e_t^H(1 + \frac{de_t^{Th}}{de_t^H}) - \lambda_t \leq 0$$

$$(= 0 \text{ for } e_t^H > 0)$$

$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \leq 0 \ (= 0 \text{ for } R_t > 0)$$

$$\lambda_t \geq 0 \ (0 = \text{for } R_t < R_{t-1} + w_t - e_t^H)$$

$$\gamma_t \geq 0 \ (0 = \text{for } R_t < \overline{R}) \ , \qquad t = 1,..,T$$

- Inserting the fringe output reaction

$$1 + \frac{de_t^{Th}}{de_t^H} = 1 + \frac{-p_t'(e_t^H + e_t^{Th})}{p_t'(e_t^H + e_t^{Th}) - c''(e_t^{Th})} = \frac{-c''(e_t^{Th})}{p_t'(e_t^H + e_t^{Th}) - c''(e_t^{Th})} > 0$$

- The conditional marginal revenue function

$$MR_{t|p_t'=c'} = p_t(1+\breve{\eta}_t \frac{e_t^H}{e_t^H + e_t^{Th}}) + p_t' \frac{de_t^{Th}}{de_t^H} e_t^H \text{, } t = 1,..,T$$

Competitive thermal fringe