ECON 5101/9101: Seminar exercises, spring 2011.

25 January, 2011.

Exercises to seminar 1 (2 Feb 2011)

1. Consider

$$Y_t = 1.1Y_{t-1} - 0.4Y_{t-2} + \varepsilon_t \tag{1}$$

for $\{Y_t, \varepsilon_t;\ t = 0, \pm 1, \pm 2, \pm 3, \ldots\}$. Assume that ε_t is a *white-noise* process.

   (a) Give the associated characteristic equation and find the characteristic roots (eigenvalues).

   (b) Check the result by using the companion form of (1).

   (c) Give a brief characterization of the general solution of (1) as stable/unstable, causal/non-causal, and with/without cycles.

   (d) Give a characterization of the dynamic multipliers of (1). Derive the first few multipliers (by hand or with the aid of a computer).

   (e) Assume that you want to forecast Y_{t+h} based on (1) and $\varepsilon_{t+h} = 0$ for $h = 1, 2, .., H$. Assume that $Y_t = 2$ and $Y_{t-1} = 1$. Draw a graph of the forecast for $H = 12$.

2. The unit-root problem in time series econometrics is concerned with characteristic roots that are equal to unity "in magnitude" (have modulus equal to one). In order to preview the issue:

   (a) Are there one ore more unit root in any of the following:

$$
\begin{aligned}
Y_t &= 1.5Y_{t-1} - 0.5y_{t-2} + \varepsilon_t && (2)\\
Y_t &= Y_{t-2} + \varepsilon_t && (3)\\
Y_t &= 2Y_{t-1} - Y_{t-1} + \varepsilon_t && (4)\\
Y &= Y_{t-1} + 0.25Y_{t-2} - 0.25Y_{t-3} + \varepsilon_t ?
\end{aligned}
$$

   (b) Why is the backward-looking solution asymptotically globally unstable in the case of a unit-root? What about the forward-looking solution?

3. Consider the AR(1) model

$$Y_t = \phi_1 Y_t + \varepsilon_t \tag{5}$$

where ε_t is a *white-noise* process.

(a) Consider the case of a causal process for $Y_{t.}$. Show that the solution can be written as

$$Y_t = \frac{1}{1 - \phi_1 L}\varepsilon_t = \psi(L)\varepsilon_t \tag{6}$$

where L is the lag-operator and the one-sided linear filter $\psi(L) = \sum_{j=-\infty}^{\infty} \psi_j L^j$ is *well defined* (meaning that $\sum_{i-\infty}^{\infty} |\psi_j| < \infty$), if and only if $|\phi_1| < 1$. Note: A linear filter is a linear combination of L^j, $j = \pm 1, \pm 2, \pm 3, \ldots$. A linear filter can be of finite order (for example $\psi(L) = 1 - L$), or of infinite order, as in this exercise.

(b) Try also to give the corresponding argument for the non-causal case.

4. Download the time series for GNP per capita in Norway. (filename *BNPhistorisk_ Jan2011.zip*) from the course internet page. In the data set, N is the size of the population per 1 January each year. BNP is the gross national product in million fixed 2000 kroner. *BNPcap* is BNP/N. We want to specify and estimate series models for the log of the "output per capita" series.

(a) Specify an ARMA model for the full sample. Include a deterministic trend in the model

(b) Estimate models for two sub-samples: One ending in 1939 and a second for the Post WW-II period. Are there notable differences between the models?

(c) Re-do (a) and (b), but for the first difference of the log of GNP per capita. Would you include a deterministic trend in this case? Explain.

(d) Assume that the purpose of the modelling exercise is to forecast GNP per capita for the period 2010-2015. Specify a model and give your forecast.