ECON 5101/9101: Seminar exercises, spring 2011.

25 January, 2011.

Exercises to Seminar 2

1. Consider the process

$$Y_t = Y_{t-4} + \varepsilon_t \tag{1}$$

where ε_t is a white noise series.

(a) Try to find the roots of the characteristic equation associated with (1). How many "unit-roots" are there?

(b) Consider Y_t given by the following data generating process:

$$Y_t = \phi_1 Y_{t-1} + \beta_1 D_{1t} + \beta_2 D_{2t} + \beta_3 D_t + \beta_0 + \varepsilon_t \tag{2}$$

where $-1 < \phi_1 < 1$, ε_t is a white noise variable and D_{jt} $(j = 1, 2, 3)$ are seasonal dummies. D_{1t} is for the first quarter, D_{2t} is for the second and D_{3t} is for the third quarter.

In the time series literature, $(1-L^4)$ is called a "seasonal filter" (assuming quarterly data). If you apply this filter to Y_t in (2), what are the time series properties of the resulting (filtered) series?

(c) Compare the answer in b. to the result from applying $(1 - L^4)$ to Y_t if the DGP is (1) and not (2).

2. Consider

$$\begin{aligned} Y_t &= \beta_0 + \phi_1 Y_{t-1} + \varepsilon_t, \qquad (3)\\ Y_t &= \beta_0 + \phi_1 Y_{t-1} + \tau_1\ t + \varepsilon_t \qquad (4) \end{aligned}$$

for $t = 1, 2, \ldots$ and where ε_t is a white-noise variable in both models. For simplicity, set $0 \leq \phi_1 \leq 1$ and regard Y_0 as known.

(a) Explain why, in (3), the parameter β_0 represents (the slope of a) deterministic trend when $\phi_1 = 1$, and why it represents the long-run level of Y_t when $\phi_1 < 1$.

(b) Similarly, explain why, in (4) when $\phi_1 = 1$, the parameter τ_1 represents a quadratic trend and why it represents a linear trend, when $0 \leq \phi_1 < 1$.

3. In model (4) the case of $\phi_1 = 0$ is often referred to as the case of a *global trend*, while the case of $\phi_1 = 1$ in (3) is an example of a *local trend*. Use the data set in *BNPhistorisk_ Jan2011.zip* and estimate the global trend model and the local trend model for the log of the gross national product per capita ($\ln(BNPcap)$). Use the sample 1960-2007. Use the *Single Equation Dynamic Modelling* part of the program, or use the ARFIMA part (but then set "d=0" and model the differenced variable since we want to avoid fractional integration at this stage).

   (a) What is the interpretatiion of the estimated residual standard errors (`sigma` in the results, which we denote $\hat{\sigma}$) in the two models?

   (b) Comment on the (standard) residual mis-specification tests. How will you characterize the statistical properties of the estimators of the trend in the two models?

   (c) Compare forecasts from the two models. First conditional on 2007 information, then conditional on 2008 and lastly conditional on 2009. One sequence of forecasts is *adaptive* (partially responding to a structural break in 2008 and/or 2009) the other is *non-adaptive*. Explain.

   (d) Is there a premium on be adaptive as a forecaster in economics?

   (e) Test the null hypothesis of a structural break in 2008-2009 (we are aware that the tests are only informal guidance here since the residual tests suggest mis-specification).

4. (continued from seminar 1)

   Download the time series for GNP per capita in Norway (file name *BNPhistorisk_ Jan2011.zip*) from the course internet page. In the data set, N is the size of the population per 1 January each year. BNP is the gross national product in million fixed 2000 kroner. The variable *BNPcap* in the data set is BNP/N.

   As we discovered during the first seminar, it was not easy to estimate a meaningful ARMA (i.e., no fractional integration, so "$d = 0$" in PcGive ARFIMA), for example ARMA(1,1) or ARMA(2,2) with a constant and deterministic trend.

   (a) Instead we now estimate an ARIMA(1,0) model for $\ln(BNPcap)$. Note: This means to estimate an ARMA(1,0) model for the first difference of $\ln(BNPcap)$ with "d=0" in the programme). Include a constant an treat it as a regressor. Use the sample 1960-2009. Use NLS. (Experiment with "stationarity imposed". Does it make a difference here?).

   (b) Estimate an ARIMA(0,1) model on the same sample. Is the estimated model invertible and what is the importance and relevance of the invertibility property?

   (c) Estimate an ARIMA(0,2) model for $\ln(BNPcap)$, and check invertibility.

   (d) Estimate finally a combined mode, for example ARIMA(1,1). Which of the models would you use for forecasting 2010 and 2011? What about 2020?

5. Consider now the log of the gross national product (BNP) in the data set. Specify an ARMA(5,0) model for $\ln(BNP)$. Include a Trend and a Constant in the General Unrestricted Model (GUM). Set the Constant as "fixed", *Autometrics* will then not delete the Constant. Use the whole available sample. Experiment with the use of *Automatic model selection*. Tick Automatic Model Selection and choose one of the two methods of Outlier Detection. Experiment with different choices. We will compare results at the seminar. Are there any sign of one or more "smooth cycles". or is everything deterministic breaks?

   NOTE: Although *Autometrics* is available in the ARFIMA part of the programme, I suggest we use *Single equation Dynamic Modelling.*