The Lucas-critique .
Reference note to lecture 8 in ECON 5101/9101, Time Series Econometrics

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1 Introduction

Lucas (1976), together with Sims (1980), are two of the most influential and "formative" papers in the field of applied macroeconomics and econometrics. We referred to Sim's paper in the VAR lecture. Lucas'offering from 1976 contains what has become known as the *Lucas-critique*. This note contains a brief exposition, and notes the line back to Haavelmo (1944), and to the econometric coencept of super exogeneity.

2 Invariance and the Lucas critique

The essence in the Lucas-critique can be shown by starting with the simple relationship

$$y_t = \beta_0 x_t + \eta_t, \tag{2.1}$$

which we assume is estimated by OLS. η_t denotes a disturbance whose properties depend on the data generating process, which is assumed to be given by

$$y_t = \beta_0^* E(x_t \mid \mathcal{I}_{t-1}) + \varepsilon_t \tag{2.2}$$

$$x_t = a_{22} x_{t-1} + \epsilon_{xt}, \tag{2.3}$$

where $-1 < a_{22} < 1$. ε_t and ϵ_{xt} have zero means and are uncorrelated. $\mathcal{I}_{t-1}$ denotes the information set used to form rational expectations. In this simple case we can replace $E(x_t \mid \mathcal{I}_{t-1})$ by the more practical notation $E(x_t \mid x_{t-1})$. By assumption we also have

$$E(\varepsilon_t \mid x_t) = 0.$$

Without loss of generality, assume that ε_t and ϵ_{xt} are independent and normally distributed. y_t is then a normal variable with mean:

$$E(y_t \mid x_{t-1}) = \alpha_{22} \beta_0^* x_{t-1},$$

and x_t has also a normal distribution with mean

$$E(x_t \mid x_{t-1}) = a_{22} x_{t-1}.$$

The conditional mean of y_t given x_t becomes

$$E(y_t \mid x_t) \equiv \mu_{Y|X} = \beta_0^* x_t + E(\eta_t \mid x_t), \tag{2.4}$$

where $E(\eta_t \mid x_t) \neq 0$, since x_t must be correlated with η_t due to

$$\eta_t = \varepsilon_t - \beta_0^* \epsilon_{xt}$$

and the DGP (2.2) and (2.3). Specifically, we have

$$E(\eta_t \mid x_t) = -\beta_0^* E(\epsilon_{xt} \mid x_t) \tag{2.5}$$

since $E(\varepsilon_t \mid x_t) = 0$ from the assumptions of the DGP-

With reference to the assumed normality we have the regression function:

$$E(\epsilon_{xt} \mid x_t) = \delta x_t$$

where

$$\delta = \frac{E(\epsilon_{xt} x_t)}{Var(x_t)} = \frac{\sigma^2_{\epsilon_{xt}}}{Var(x_t)}$$

and where $\sigma^2_{\epsilon_{xt}}$ denotes the variance of ϵ_{xt}. Subject to stationarity we have:

$$Var(x_t) = \frac{\sigma^2_{\epsilon_{xt}}}{1 - a_{22}^2}$$

The regression coefficient δ is therefore

$$\delta = (1 - a_{22}^2). \tag{2.6}$$

Substitution into (2.4) of the expressions for $E(\eta_t \mid X_t)$, (2.5) and δ, (2.6), give the conditional expectation for y_t when the DGP is characterized by rational expectations, (2.2) and (2.3):

$$\mu_{Y|X} = \beta_0^* x_t - \beta_0^*(1 - a_{22}^2) x_t = (\beta_0^* a_{22}^2) x_t. \tag{2.7}$$

The OLS estimator $\hat{\beta}_0$ is as always consistent for the regression coefficient between y_t and x_t, but in this case that coefficient is not a single parameter but $\beta_0^* a_{22}^2$. Hence we get

$$\text{plim}(\hat{\beta}_0) = \beta_0^* a_{22}^2 < \beta_0^*, \text{ since } -1 < a_{22} < 1. \tag{2.8}$$

However, the real thrust of the Lucas-critique is that any change in the formation of expectations (changes in the a_{22} parameter) is predicted to lead to a change in plim$(\hat{\beta}_0)$. Since a main channel of policy (many would say the only) is the expectations channel, the estimator of β_0 gives the wrong conclusions about how a policy change affects y_t.

The parameter β_0 is not invariant to changes in expectations and so x_t in (2.1) is not super exogenous when the DGP is rational expectations driven as in (2.2) and (2.3).

In contrast to β_0, the parameter β_0^* is often called a "deep-structure parameter". It is invariant to interventions in the x_t process when the true DGP is (2.2) and (2.3).

3 The Lucas-critique as a possibility theorem—not a truism

The Lucas critique identifies one mechanism that can induce lack of invariance (and a low degree of autonomy) in the parameters of a conditional econometric model. In practice there are many others pitfalls as well. Omitted variables for example is a important (though mundane) cause of lack of invariance in econometric models, including those estimated by IV/GMM.

The exact set of assumptions is important when invariance. Assume for example that another theory is valid

$$y_t^p = \beta_0 x_t$$

where y_t^p is the agents' planned consumption, or price change. If $y_t = y_t^p + \varepsilon_t$ and $E(\varepsilon_t, x_t) = 0$, then $\text{plim}(\hat{\beta}_0) = \beta_0$ which is the relevant parameter for policy evaluatuion. Hence the force of the Lucas critique depends on the relevance of the underlying theory, which needs to be tested.

One apprach to testing the force of the Lucas-critique is (simply) to check whether the predicted invariance occurs after a significant structural break in the marginal part of the model. We also refer to this as tests of invarisnce (and of super exogeneity).

4 The Lucas-critique and the Haavelmo-critique

Early in the history of econometrics Frisch and Haavelmo spent a lot of time conceptualizing terms like autonomy, confluence and invariance. The dicussion in section 8 of Haavelmo (1944) an interesting read, in particuar with the Lucas-critique in mind.

About the same time as Lucas' break-through, Leif Johansen also took on board the concept of (degree of) autonomy in his theory of economic planning, see Johansen (1977, Ch 4.6).

References

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