E 4101/5101
Lecture 13: Forward-looking models

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Introduction I

- So far we have covered *stationary* and *causal* dynamic systems.
- Have also extended the analysis to non-stationary systems that can be mapped from $I(1)$ to $I(0)$, after finding the identified cointegrating relationships.
- Many interesting dynamic systems have one or more roots outside the unit-circle.
- They are stationary, but *non-causal*.
- Many examples in finance and in macroeconomics.
- The New Keynesian (DSGE) macro model contains AS and IS-curves that are non-causal.
- We are interested in solving, estimating and testing such models.
Introduction II

- The emphasis in this lecture is on *testing* the hypothesis that the data generating process (DGP) may be non-causal.
  - We will refer to the concepts from Davidson and MacKinnon Ch 15, which gives a good overview of specification and mispecification testing
  - Note also reference to tests earlier in the book (tests for overidentifying restrictions for example)

- This is a different position from the literature that develops estimation methods given the premise that the models are *not* approximations to the DGP.
  - calibration methods and
  - Bayesian estimation
  - Robust estimation
Causal and non-causal processes

- As noted a variable (or stochastic process) \( y_t \) is *causal* if the solution can be expressed as a well defined *linear filter* of a stationary input series \( \varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots \).

- For a causal process \( y_t \), the associated characteristic equation \( p(\lambda) = 0 \) in our notation) has all its roots inside the unit circle.

- This definition generalizes, as we have seen, to the case of the \( n \) dimensional vector process \( y_t \).

- The theory of cointegration extends the causal model to the case of for \( n - r \) unit-roots, or \( n - r \) *common trends*, \( 0 < r < n \), as the Engle-Granger representation theorem shows.
Causal and non-causal processes II

- $y_t$ is a *non-causal*, or *future dependent*, process if the stable solution is well defined linear filtering of $\varepsilon_{t+1}, \varepsilon_{t+2}, \ldots$.

- For a non-causal process, the associated characteristic equation has all its roots *outside* the unit circle.

- The simplest example of a covariance-stationary and non-causal model is

$$y_t = \phi_1 y_{t-1} + \varepsilon_t, \quad \phi_1 > 1,$$

where $\varepsilon_t$ is white-noise.

(1) has one root which is larger than unity. The non-causal solution is:

$$y_t = (\phi_1^{-1})^N y_{t+N} + \sum_{i=1}^{N-1} (-\phi_1^{-1})^i \varepsilon_{t+i}$$

where $y_{t+N}$ is a terminal condition.
The solution is stable since, if we look at the homogenous part,

\[ y^h_t \xrightarrow{N \to \infty} 0 \text{ if } \phi_1 > 1 \]

as we have assumed.

\( y_t \) is also stationary since it can be expressed as a well-defined linear filter of stationary variables (namely \( \varepsilon_{t+i} \)):

\[ y_t = \sum_{i=1}^{\infty} (-\phi_1^{-1})^i \varepsilon_{t+i} \]  

(3)
The model used by Hamilton in Ch 2.5 and Ch 11.2 illustrates several points.

- $P$ denotes the price of a stock
- $D$ Dividend payment
- $r$ Total real return from holding a stock in one period

Using Ch 11.2 dating:

$$r_{t+1} = \frac{P_{t+1} - P_t}{P_t} + \frac{D_{t+1}}{P_t}$$

Assume, as a theory, that $r_{t+1} = r > 0$. We then have an AR model for $P_t$:

$$P_{t+1} = (1 + r)P_t - D_{t+1}$$
Stock Price Example of Forward-Looking Solution II

- If $D_t$ is a white noise variable (remember that demeaning is allowed), $P_t$ follows a non-causal process.
- The backward-solution is explosive, often called the “bubble” solution.
- However, the dynamic equation can equivalently be written as:

  \[ P_t = (1 + r)^{-1} P_{t+1} + (1 + r)^{-1} D_{t+1} \]  

- A theory of efficient markets says that the price of the stock is determined by

  \[ P_t = (1 + r)^{-1} E_t(P_{t+1}) + (1 + r)^{-1} E_t D_{t+1} \]  

where $E_t(P_{t+1})$ and $E_t(D_{t+1})$ denotes the rational expectation for $P_{t+1}$, using information available in period $t$. 

Stock Price Example of Forward-Looking Solution III

For the next period:

\[ E_t P_{t+1} = (1 + r)^{-1} E_t (P_{t+2}) + (1 + r)^{-1} E_t D_{t+2} \]

giving:

\[ P_t = (1 + r)^{-1} \left[ (1 + r)^{-1} E_t (P_{t+2}) + (1 + r)^{-1} E_t D_{t+1} \right] + (1 + r)^{-1} D_t \]
\[ = (1 + r)^{-2} E_t (P_{t+2}) + (1 + r)^{-1} E_t D_{t+1} + (1 + r)^{-2} E_t D_{t+2} \]

Continuing until infinity:

\[ P_t = \sum_{j=1}^{\infty} \left[ \frac{1}{1 + r} \right]^j E_t D_{t+j} \]  \hspace{1cm} (8)

where we have set \( E_t [D_t] = D_t \) for consistency with (7).
The solution is referred to as the “market fundamental solution”, it is the solution when the price formation process is dominated by rational expectations.

Note that it is implicit that $D_t$ is assumed to be an exogenous “forcing variable” in this solution.

Still, we may find that the stock price Granger causes dividends in the VAR.
Granger Causality induced by forward-looking behaviour I

This is from Chapter 11.2 in Hamilton:
Assume the following MA model for dividends:

\[ D_t = d + u_t + \delta u_{t-1} + \nu_t \] (9)

where \( u_t \) and \( \nu_t \) are independent white-noise Gaussian series. By assumption

\[ E_t D_{t+1} = d + \delta u_t \]
\[ E_t D_{t+j} = d \text{ for } j > 1 \]
Granger Causality induced by forward-looking behaviour II

\[ P_t = \sum_{j=1}^{\infty} \left[ \frac{1}{1+r} \right]^j E_tD_{t+j} \]

\[ = (1 + r)^{-1}(d + \delta u_t) + \sum_{j=2}^{\infty} \left[ \frac{1}{1+r} \right]^j d \]

\[ = (1 + r)^{-1}(d + \delta u_t) + \sum_{j=1}^{\infty} \left[ \frac{1}{1+r} \right]^{j+1} d \]

\[ = (1 + r)^{-1}\delta u_t + (1 + r)^{-1}d + \left[ \frac{(1+r)^{-1}}{r} \right] d \]

Collecting terms:

\[ P_t = (1 + r)^{-1}\delta u_t + r^{-1}d \]  \hspace{1cm} (10)
Granger Causality induced by forward-looking behaviour III

The solution holds for all periods. This implies that

\[ u_{t-1} = (1 + r)\delta^{-1}P_{t-1} - r^{-1}(1 + r)\delta^{-1}d \]

Hence

\[ P_{t-1} \rightarrow u_{t-1} \rightarrow D_t \]

from the solution and (9).

- The VAR can be written as

\[
\begin{pmatrix}
D_t \\
P_t
\end{pmatrix} = \begin{pmatrix}
-d \\
d \delta \\
r
\end{pmatrix} + \begin{pmatrix}
0 & (1 + r) \\
0 & 0
\end{pmatrix} \begin{pmatrix}
D_{t-1} \\
P_{t-1}
\end{pmatrix} + \begin{pmatrix}
\frac{\delta}{1+r} & 0 \\
u_t & \nu_t
\end{pmatrix} \]

(11)
Granger Causality induced by forward-looking behaviour IV

- The theory of a rational expectations driven “market fundamentals solution” predicts that the stock price Grangers causes dividends in the bivariate VAR (11).
- This is because $P_t$ contains information about the expected $D_{t+1}$.
- A premise is that $D_t$ is autocorrelated.
- Similar implications can be established for other financial variables. For example the term-structure of interest rates may be a predictor of GDP growth and/or inflation if the price of bonds incorporate “correct” expectations about these variables.
Cointegrated VAR for consumption and income I

- Define $c_t$ as the log of private consumption and $y_t$ as the log of disposable income.
- Assume that the long-run matrix $\Pi$ associated with $y_t = (c_t, y_t)'$ has rank 1, meaning cointegration.
- Consider the 1st order VAR:

\begin{align*}
    c_t &= \kappa + \phi_{cc} c_{t-1} + \phi_{cy} y_{t-1} + e_{c,t}, \\
    y_t &= \varphi + \phi_{yc} c_{t-1} + \phi_{yy} y_{t-1} + e_{y,t},
\end{align*}

where $e_{c,t}$ and $e_{y,t}$ are jointly normal. Their variances are $\sigma^2_c$ and $\sigma^2_y$ respectively, and the correlation coefficient is denoted $\rho_{cy}$.

- $\Phi = [\phi_{ij}]$ has one unit root, and one stable root. The EqCM representation is therefore...
Cointegrated VAR for consumption and income II

\[ \Delta c_t = \kappa - \alpha_c [c_{t-1} - \beta y_{t-1}] + e_{c,t}, \quad 0 \leq \alpha_c < 1, \quad (14) \]
\[ \Delta y_t = \varphi + \alpha_y [c_{t-1} - \beta y_{t-1}] + e_{y,t}, \quad 0 \leq \alpha_y < 1, \quad (15) \]

where \( \beta \) is the cointegration coefficient and \( \alpha_c \) and \( \alpha_y \) are the adjustment coefficients.

▶ The system with mean-zero equilibrium correction term.

Define \( \eta_c = E[\Delta c_t] \), \( \eta_y = E[\Delta y_t] \) and \( \mu = E[c_t - \beta y_t] \) and consequently

\[ \kappa = \eta_c + \alpha_c \mu \]
\[ \varphi = \eta_y - \alpha_y \mu \]
Cointegrated VAR for consumption and income III

Thus we can rewrite this system into

\[
\Delta c_t = \eta_c - \alpha_c [c_{t-1} - \beta y_{t-1} - \mu] + e_{c,t}, \quad 0 \leq \alpha_c < 1,
\]

\[
\Delta y_t = \eta_y + \alpha_y [c_{t-1} - \beta y_{t-1} - \mu] + e_{y,t}, \quad 0 \leq \alpha_y < 1.
\]

In the case of \( \beta = 1 \), the savings rate

\[
s_t = -c_t + y_t
\]

is \( I(0) \), and \( -\mu \) is the long-run mean of the savings rate, \( E(s_t) = -\mu \).
Cointegrated VAR for consumption and income IV

- Co-integration represents the common ground between the
  - consumption function which assumes a causal link from income to consumption, and
  - the modern permanent-income/life-cycle theories which imply an Euler-equation for consumption—we refer to them as PIH for simplicity
Permanent income hypothesis (PIH) I

- A well known implication of the PIH is that \( s_t \) is function of expected future income.
- The logic is very similar to the market fundamentals solution for \( P_t \) above.
- Intuitively, \( s_t \) is Granger causing income growth because \( s_t \) is a carrier or households expectations about future income growth—
  - the *saving for a rainy day feature* of the PIH model, see Campbell (1987).
Permanent income hypothesis (PIH) II

(16)-(17) can be written in model form

\[ \Delta c_t = \eta_c + e_{c,t}, \]  
\[ \Delta y_t = \eta_y + \gamma_y + \pi_y \Delta c_t + \alpha_y [c_{t-1} - \beta y_{t-1} - \mu] + \varepsilon_{y,t}, \]

where (18) is the marginal model for consumption and (19) is a conditional model for real income.

From the properties of the normal distribution:

\[ \pi_y = \rho_{cy} \frac{\sigma_y}{\sigma_c}, \]
\[ \gamma_y = -\eta_y \pi_y, \]
\[ \varepsilon_{y,t} = e_{y,t} - \pi_y e_{c,t}. \]

Since \( \alpha_c = 0 \), cointegration implies that \( 0 < \alpha_y < 1 \), with the rainy-day as the economic interpretation.
Consumption function (CF) form of the VAR I

\[
\Delta c_t = \eta_c + \gamma_c + \pi_c \Delta y_t - (\alpha_c + \pi_c \alpha_y)[c_{t-1} - \beta y_{t-1} - \mu] + \varepsilon_{ct} \tag{21}
\]

\[
\Delta y_t = \eta_y + \alpha_y [c_{t-1} - \beta y_{t-1} - \mu] + e_{y,t} \tag{22}
\]

(21) is the consumption function, (22) is the marginal income equation.

\[
\begin{align*}
\alpha_c &= (1 - \phi_{cc}) \\
\pi_c &= \rho_{c,y} \frac{\sigma_c}{\sigma_y} \\
\gamma_c &= -\eta_y \pi_c, \\
\varepsilon_{c,t} &= e_{c,t} - \pi_c e_{y,t}.
\end{align*}
\tag{23}
\]
Consumption function (CF) form of the VAR II

- Underlying the consumption function approach is \( 0 < \alpha_c < 1 \).
- Note that the hypothesis \( H_0: \alpha_c = 0 \) must be tested separately, since finding \( [c_{t-1} - \beta y_{t-1} - \mu] \) significant in (21) could be due to \( 0 < \alpha_y \).
- For the coefficient \( \alpha_y \) there are two possibilities.
- \( 0 < \alpha_y < 1 \) is consistent with hours worked etc. being “demand determined” and with \( y_t \) adjusting to past disequilibria. In econometric terms there is mutual (Granger) causation between income and consumption.
- The second possibility is that \( \alpha_y = 0 \), reflecting that income is “supply-side” determined.
- \( \alpha_y = 0 \) implies that income is weakly exogenous with respect to the long run elasticity \( \beta \).
Consumption function (CF) form of the VAR III

- Of course $\alpha_y = 0$ gives the clearest contrast to the PIH.
- $(21)-(22)$ simplifies to

$$
\Delta c_t = \eta_c + \gamma_c + \pi_c \Delta y_t - \alpha_c [c_{t-1} - \beta y_{t-1} - \mu] + \varepsilon_{c,t} \\
\Delta y_t = \eta_y + e_{y,t},
$$

with $\eta_y = \phi$, since there is no equilibrium correction in income.

- The analysis of the different implications of CF and PIH carry over to the realistic case of
  - VAR($p$) and
  - deterministic terms
Analysis of Norwegian consumption-income data I

- Quarterly data 1968(2)-1984(4)
  Refer to this as the Before break sample
- Congruent VAR
- Seasonal dummies and two dummies for
- VAT in 1970(1)
- Price and income freeze in 1979
- Do not show the result for the dummies here, see Eitrheim et al (2002)
**Analysis of Norwegian consumption-income data II**

Table: Diagnostics for the I(0) VAR

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The sample is 1968(2) to 1984(4), 67 observations.</td>
<td></td>
</tr>
<tr>
<td>$\hat{\sigma}_{\Delta c}$</td>
<td>1.63%</td>
</tr>
<tr>
<td>$\hat{\sigma}_{\Delta y}$</td>
<td>1.60%</td>
</tr>
<tr>
<td>$AR\ 1 - 5\ F(20,\ 82)$</td>
<td>0.8921[0.5973]</td>
</tr>
<tr>
<td>Normality $\chi^2(4)$</td>
<td>8.0609[0.0894]</td>
</tr>
<tr>
<td>Heteroscedasticity $F(75,\ 72)$</td>
<td>0.5222[0.9971]</td>
</tr>
</tbody>
</table>
Analysis of Norwegian consumption-income data III

Table: FIML consumption function estimates. 1968(2)-84(2)—Before Break

<table>
<thead>
<tr>
<th>The consumption function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\Delta}c_t = -0.302 \Delta c_{t-1} + 0.227 \Delta c_{t-4} + 0.471 \Delta y_t$</td>
</tr>
<tr>
<td>$(0.070)$ $(0.073)$ $(0.181)$</td>
</tr>
<tr>
<td>$-0.128 (c - y)_{t-1}$</td>
</tr>
<tr>
<td>$(0.048)$</td>
</tr>
<tr>
<td>$\hat{\sigma} = 1.53%$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The income equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\Delta}y_t = 0.009 - 0.477 \Delta y_{t-1} + 0.311 \Delta y_{t-4}$</td>
</tr>
<tr>
<td>$(0.002)$ $(0.109)$ $(0.109)$</td>
</tr>
<tr>
<td>$\hat{\sigma} = 1.60%$</td>
</tr>
</tbody>
</table>
Analysis of Norwegian consumption-income data IV

Table: Before break FIML estimates for consumption function

<table>
<thead>
<tr>
<th>Diagnostics</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overidentification $\chi^2(14)$</td>
<td>15.7597[0.3283]</td>
</tr>
<tr>
<td>AR 1 − 5 $F(20, 96)$</td>
<td>0.7830[0.7274]</td>
</tr>
<tr>
<td>Normality $\chi^2(4)$</td>
<td>5.057[0.2815]</td>
</tr>
<tr>
<td>Heteroscedasticity $F(75, 93)$</td>
<td>0.6530[0.9717]</td>
</tr>
</tbody>
</table>

FIML estimation. The sample is 1968(2) to 1984(4), 67 observations.
Analysis of Norwegian consumption-income data V

Table: *Before break* FIML estimates for PIH.

<table>
<thead>
<tr>
<th>The Euler equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta c_t = 0.0059 + 0.236 \Delta c_{t-4} )</td>
</tr>
<tr>
<td>(0.003) (0.089)</td>
</tr>
<tr>
<td>( \hat{\sigma} = 2.10% )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The savings equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta y_t = 0.0144 - 0.308 \Delta y_{t-1} + 0.231 \Delta y_{t-4} )</td>
</tr>
<tr>
<td>(0.003) (0.098) (0.01)</td>
</tr>
<tr>
<td>+ 0.158 ((c - y)_{t-1} + 0.041 \text{ VAT} )</td>
</tr>
<tr>
<td>(0.068) (0.013)</td>
</tr>
<tr>
<td>( \hat{\sigma} = 1.62% )</td>
</tr>
</tbody>
</table>

The sample is 1968(1) to 1984(4), 68 observations.
Analysis of Norwegian consumption-income data VI

Table: *Before break* FIML estimates for PIH

<table>
<thead>
<tr>
<th>Diagnostics</th>
<th>Value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overidentification $\chi^2(15)$</td>
<td>$45.3776 [0.0001]$</td>
<td></td>
</tr>
<tr>
<td>AR 1 – 5 $F(20, 98)$</td>
<td>$1.5548 [0.0804]$</td>
<td></td>
</tr>
<tr>
<td>Normality $\chi^2(4)$</td>
<td>$7.4287 [0.1149]$</td>
<td></td>
</tr>
<tr>
<td>Heteroscedasticity $F(75, 96)$</td>
<td>$1.0269 [0.4481]$</td>
<td></td>
</tr>
</tbody>
</table>

The sample is 1968(1) to 1984(4), 68 observations.
The New Keynesian Phillips curve I

The hybrid NPC is given as

\[ \pi_t = \frac{a^f}{\geq 0} E_t[\pi_{t+1}] + \frac{a^b}{\geq 0} \pi_{t-1} + \frac{b}{> 0} s_t + \varepsilon_{\pi t} \]  

(26)

- \( \pi_t \) denotes the rate of inflation
- In this model \( s_t \) denotes the logarithm of the wage-share which is the most used operational definition of real marginal costs. \( \varepsilon_{\pi t} \) is a white noise disturbance.
- In many applications, notably Gali and Gertler (1999), the disturbance term is omitted, which is often referred to as the NPC holding in “exact form”
The New Keynesian Phillips curve II

- $s_t$ is referred to as a forcing variable, in the same way as $D_t$ in the stock price model. We assume the following model for $s_t$

\[ s_t = c_{s1}s_{t-1} + \cdots + c_{sk}s_{t-k} + \varepsilon_{s,t}. \]  

- It can be shown that $k \geq 2$ is necessary for identification of the parameters in the NPC.
NPC solution I

Following Bårdsen et al (2005), Ch 7, we first find a partial solution for $\pi_t$ as

$$\pi_t = r_1 \pi_{t-1} + \frac{b}{a^f r_2} \sum_{i=0}^{\infty} \left( \frac{1}{r_2} \right)^i E_t s_{t+i} + \frac{1}{a^f r_2} \varepsilon_{\pi, t}$$

(28)

where $r_{1,2}$ are the two roots of

$$r^2 - \left( \frac{1}{a^f} \right) r + \left( \frac{a^b}{a^f} \right) = 0$$

A stable solution of the pure NPC, with $a^b = 0$, requires $a^f > 1$. 
NPC solution II

With $a^f > 0$, $a^b > 0$ and $a^f + a^b \leq 1$ it is implied that $r_1$ and $r_2$ are real and positive.

It is usual to define $r_1$ as

$$r_1 = \frac{1 - \sqrt{1 - 4a^f a^b}}{2a^f} \quad (29)$$

which is $0 \leq r_1 < 1$ under the assumption of $a^f + a^b < 1$.

Under these assumption the solution is (see Nymoen, Swensen and Tveteter (2011)):

$$\pi_t = r_1 \pi_{t-1} + \frac{b}{a^f r_2} K_{s1} s_t + \frac{b}{a^f r_2} K_{s2} s_{t-1} + \frac{1}{a^f r_2} \epsilon_{\pi,t}, \quad (30)$$

written her for the case of $k = 2$ in the forcing process (27).
NPC solution III

- $K_{s1}$ and $K_{s2}$ are constants that are defined when

\[
\left| \frac{r_{si}}{r_2} \right| < 1 \text{ for } i = 1, 2 \tag{31}
\]

where $r_{s1}$ and $r_{s2}$ are the roots of the characteristic equation associated with the forcing variable.

- (30) can be solved from known initial conditions, “as if” the model was causal.
Testing the NPC

Different “types” of tests:

- Robustness and strength of instruments (as we did in seminar)
- Testing the implications of $E_t[\pi_{t+1}]$ on the VAR, for example Boug et al (2010)
- Testing the encompassing implications of the NPC against existing models of the wage-price spiral—encompassing approach
- Testing invariance of NPC with respect to regime shifts.
Encompassing type test I

Start from:

\[ s_t = ulc_t - q_t, \tag{32} \]

where \( ulc \) denotes unit labour costs (in logs) and \( q \) is the log of the price level on domestic goods and services. Let \( (1 - \gamma) \) denote a constant import share, and \( p_i \) the import price index. The aggregate price level is defined as

\[ p_t = \gamma q_t + (1 - \gamma) p_i t \tag{33} \]
Encompassing type test II

Using (32) and (33), we can re-write the NPC (26), after dropping the disturbance for simplicity:

\[
\pi_t = \frac{a^f}{(1 + \frac{b}{\gamma})} E_t[\pi_{t+1}] + \frac{a^b}{(1 + \frac{b}{\gamma})} \pi_{t-1} - \frac{b}{(\gamma + b)} [p_{t-1} - \gamma u/lc_{t-1} - (1 - \gamma) pi_{t-1}] + \frac{\gamma b}{(\gamma + b)} \Delta u/lc_t + \frac{b (1 - \gamma)}{(\gamma + b)} \Delta pi_t
\]
Encompassing type test III

or

\[ \pi_t = \alpha^f \Delta E_t[\pi_{t+1}] + \alpha^b \pi_{t-1} \]
\[ + \beta (ulc_{t-1} - p_{t-1}) - \beta (1 - \gamma) (ulc_{t-1} - pi_{t-1}) \]
\[ + \beta \gamma \Delta ulc_t + \beta (1 - \gamma) \Delta pi_t \]

with \( \alpha^f, \alpha^b, \beta \) and \( \psi \) as new coefficients.

▶ This shows that the NPC has an interpretation as an EqCM for the price level.

▶ An alternative model for price and wage formation is the cointegrated imperfect competition model, ICM.
Encompassing type test IV

- The ICM the price equation with a lead-term added is

\[
\pi_t = \alpha^f \Delta E_{t[\pi_{t+1}]} + \alpha^b \Delta \pi_{t-1} \\
+ \beta_1 (ulc_{t-1} - p_{t-1}) + \beta_2 (ulc_{t-1} - p_{i_{t-1}}) \\
+ \beta_3 \Delta ulc_t + \beta_4 \Delta pi_t. \tag{35}
\]

- The NPC implies restrictions on (35)
  - \( H_0^a: \beta_3 = \beta_1 + \beta_2 \) and
  - \( H_0^b: \beta_4 = -\beta_2. \)

- For the cointegrating ICM, the only requirement is \( \beta_1 > 0 \) and \( \beta_1 > -\beta_2. \)
Encompassing type test V

- Non rejection of \( H_0^a \) and/or \( H_0^b \) would mean that the NPC encompasses the ICM:

\[
\text{NPC} \supseteq \text{ICM} \quad \text{if} \quad \beta_3 = \beta_1 + \beta_2 \text{ and } \beta_4 = -\beta_2
\]

- Conversely, if \( \beta_3 = \beta_1 + \beta_2 \) and/or \( \beta_4 = -\beta_2 \) is rejected, the ICM implies that the NPC has omitted variables bias.

  - In particular \( ulc_{t-1} - p_{t-1} \) and \( ulc_{t-1} - pi_{t-1} \) are predictors of \( \pi_{t+1} \) so omission (misrepresentation) of these variables in the NPC will affect the IV/GMM estimate of the parameter \( a^f \) in the NPC.

- Bjørnstad and Nymoen (2008)
Testing invariance of NPC I

- If the data generating process is characterized by intermittent structural break, the significance of the forward term in the NPC may be overestimated.
- Structural breaks represented by impulse indicators
- The impulse indicators are selected in the implied reduced form forecasting equation
- Then tested for significance in the NPC.
  - Under the null of correct specification, few such impulse indicators will be selected,
  - and those that are should not be significant when added to NPC;
  - moreover, parameter estimates should not alter much.
Testing invariance of NPC II

- Under the alternative that there are unmodeled outliers or breaks, there will be significant impulse indicators in the ‘forecasting’ equation, and these will remain significant when added to the NPC.
- Castle et al. (2010)
References I


References II
