

# ECON 5101/9101: Seminar exercises, spring 2014.

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## Exercises to Seminar 4

1. Consider the process

$$(1) \quad Y_t = Y_{t-4} + \varepsilon_t$$

where  $\varepsilon_t$  is a white noise time series.

- (a) What are the roots of the characteristic equation associated with (1)?
- (b) Consider  $Y_t$  given by the following Data Generating Process (DGP):

$$(2) \quad Y_t = \phi_1 Y_{t-1} + \beta_1 D_{1t} + \beta_2 D_{2t} + \beta_3 D_{3t} + \beta_0 + \varepsilon_t$$

where  $-1 < \phi_1 < 1$ ,  $\varepsilon_t$  is a white noise variable and  $D_{jt}$  ( $j = 1, 2, 3$ ) are seasonal dummies.  $D_{1t}$  is for the first quarter,  $D_{2t}$  is for the second and  $D_{3t}$  is for the third quarter.

In the time series literature,  $(1 - L^4)$  is called a “seasonal filter” (assuming quarterly data). If you apply this filter to  $Y_t$  in (2), what are the time series properties of the resulting filtered series? (You do not need to give an exact formula here, but formulate an answer based on your understanding of what the  $(1 - L^4)$  filter “does to” a time series)

- (c) Compare the answer in b. to the result from applying  $(1 - L^4)$  to  $Y_t$  if the DGP is (1) and not (2).

2. Consider

$$(3) \quad Y_t = \beta_0 + \phi_1 Y_{t-1} + \varepsilon_t,$$

$$(4) \quad Y_t = \beta_0 + \phi_1 Y_{t-1} + \tau_1 t + \varepsilon_t$$

for  $t = 1, 2, \dots$  and where  $\varepsilon_t$  is a white-noise variable in both models. For simplicity, set  $0 \leq \phi_1 \leq 1$  and regard  $Y_0$  as known.

- (a) Explain why, in (3), the parameter  $\beta_0$  represents (the slope of a) deterministic trend when  $\phi_1 = 1$ , and why it represents the long-run level of  $Y_t$  when  $\phi_1 < 1$ .
- (b) Similarly, explain why, in (4) when  $\phi_1 = 1$ , the parameter  $\tau_1$  represents a quadratic trend and why it represents a linear trend, when  $0 \leq \phi_1 < 1$ .

3. In model (4) the case of  $\phi_1 = 0$  is often referred to as the case of a *global trend*, while the case of  $\phi_1 = 1$  in (3) is an example of a *local trend*. Use the data set in *BNPcap.xls* (the data are observations of Norwegian GDP in fixed 2010 prices for the period 1835-2012) and estimate the global trend model and the local trend model for the log of the gross national product per capita. Use the sample 1960-2007.

(In Oxmetrics/PcGive: Use the *Single Equation Dynamic Modelling* part of the program, or use the ARFIMA part (but then set “d=0” and model the differenced variable since we want to avoid fractional integration at this stage).

- (a) What is the interpretation of the estimated residual standard errors (**sigma** in the results, which we denote  $\hat{\sigma}$ ) in the two models?
- (b) Comment on the (standard) residual mis-specification tests. How will you characterize the statistical properties of the estimators of the trend in the two models?
- (c) Compare forecasts from the two models. First conditional on 2007 information, then conditional on 2008 and lastly conditional on 2009. Which of the forecasts are more *adaptive* (partially responding to a structural break in 2008 and/or 2009)?
- (d) Test the null hypothesis of a structural break in 2008-2009 (we are aware that the tests are only informal guidance here since the residual tests suggest mis-specification).

4. Download the data set used in Angrist et. al. (2000) from <http://people.brandeis.edu/~kgraddy/data.html>.

Use the *Daily Fulton Fish Market Data*. The facsimile at the back of the problem set provides an overview which help you identify the variables in the downloaded data file.

- (a) Formulate a relevant  $AR(p)$  model for price (using the already log transformed data) and use the estimated model to judge whether the market is characterised by immediate adjustment or not. (It is of course allowed to include one or more of the dummies). Save the last four observations for later.
- (b) Formulate a relevant  $AR(p)$  model for quantity (using the already log transformed data). Looking at the joint evidence from the two (separately estimated)  $AR(p)$  models, what do you conclude about the relationship (or not) between the two variables.
- (c) Formulate a  $VAR(p)$  model for the price and quantity variables, possibly including one or more of the dummies as non-modelled exogenous variables. Estimate your chosen model, but save the last four observations for (within sample) forecasting. Check that the properties of the residuals are not clearly indication autocorrelation, outliers, or heteroscedasticity.
- (d) Use the estimated  $VAR(p)$  for both static (1-step) and dynamic (4-step) forecasts of quantity and price, and compare with actuals.

The data used in Graddy (1995) were obtained from a single dealer who supplied his inventory sheets for the period December 2nd, 1991 through May 8th, 1992. Total price and quantity for each transaction are recorded on the inventory sheets. These data are supplemented by data that were collected by direct observation from the same dealer during the period April 13th through May 8th, 1992. For this study, the prices and quantities are aggregated by day, for the 111 days the market was open between December 2nd and May 8th. The price variable used below is the quantity-weighted average daily transaction price for the dealer observed. The quantity variable is the total quantity sold by this dealer on each day. Table 1 presents summary statistics for the data used in this paper.

Every day the demand for fish at the Fulton fish market is determined partly by which customers decide to visit the market that day, as well as by how much they buy. A number of customers visit the market every week on Mondays and Thursdays, and other customers may visit the market every day of the week. Quantities purchased by individual

TABLE 1  
*Summary statistics (111 Obs.)*

| Variable                  | mean  | s.d. | min   | max  |
|---------------------------|-------|------|-------|------|
| log (average daily price) | -0.19 | 0.38 | -1.11 | 0.66 |
| log (quantity)            | 8.52  | 0.74 | 6.19  | 9.98 |
| Stormy                    | 0.29  | 0.46 | 0     | 1    |
| Mixed                     | 0.31  | 0.46 | 0     | 1    |
| Fair                      | 0.41  | 0.49 | 0     | 1    |
| Rainy on shore            | 0.16  | 0.37 | 0     | 1    |
| Cold on shore             | 0.50  | 0.50 | 0     | 1    |
| Monday                    | 0.19  | 0.39 | 0     | 1    |
| Tuesday                   | 0.21  | 0.41 | 0     | 1    |
| Wednesday                 | 0.19  | 0.39 | 0     | 1    |
| Thursday                  | 0.21  | 0.41 | 0     | 1    |
| log (minimum daily price) | -0.57 | 0.53 | -2.30 | 0.41 |
| log (maximum daily price) | 0.06  | 0.31 | -0.51 | 0.92 |
| log (median daily price)  | -0.20 | 0.41 | -1.39 | 0.69 |
| Daily price range         | 0.48  | 0.28 | 0.00  | 2.00 |

Notes: Prices and quantities are daily observations for whiting at the Fulton fish market. Prices are dollars per pound and quantities are pounds per day.