E 4101/5101
Lecture 8: Lucas critique and modelling stationary non-causal VARs

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Introduction I

- Lecture 7: VAR, and econometric models of the VAR: *Conditional plus marginal* models, *recursive* systems of equations, and *simultaneous equations models*.
- All these models included only *observable* endogenous and non-modelled variables.
- If some of the variables are expectations or other unobservables, we have measurement error-bias. Consistent estimation is then by IV/GMM.
- A special case is the Lucas-critique of conditional models that ignore rational expectations formation.
- Have posted a Lecture note on that topic, to supplement the H book

Introduction II

- So far, we have covered *stationary* and *causal* dynamic systems.
- Many interesting dynamic systems have one or more roots "outside the unit-circle".
- They are stationary, but *non-causal.* Usually called forward-looking models.
- Many examples in finance and in macroeconomics, for example rational expectations models with "$t+1$" variables
- Eventually, we are interested in solving, estimating and testing such models
- This lecture gives an introduction to this large research area.

Causal and non-causal processes I

- A time series (stochastic process) y_t is *causal* if the solution can be expressed as a well definied *linear filter* of a stationary input series ε_t, ε_{t-1}, $\varepsilon_{t-2}\ldots$.
- For a causal process y_t, the associated characteristic equation has *all* its roots inside the unit circle.
- This definition generalizes, as we have seen, to the case of the *n* dimensional vector process $\mathbf{y}_t$
- y_t is a *non-causal,* or *future dependent*, process if the stable solution is a well defined linear filtering of $\varepsilon_{t+1}, \varepsilon_{t+2}, \ldots$.
- For a non-causal process, the associated characteristic equation of one or more roots *outside* the unit circle.

Causal and non-causal processes II

- The simplest example of a covariance-stationary and non-causal model is

$$y_t = \phi_1 y_{t-1} + \varepsilon_t, \quad \phi_1 > 1, \tag{1}$$

where ε_t is white-noise.
(1) has one root which is larger than unity. The non-causal solution is:

$$y_t = (\phi_1^{-1})^N y_{t+N} + \sum_{i=1}^{N-1} (-\phi_1^{-1})^i \varepsilon_{t+i} \tag{2}$$

where y_{t+N} is a terminal condition.

Causal and non-causal processes III

- The solution is stable since, if we look at the homogenous part,

$$y_t^h \underset{N \longrightarrow \infty}{\longrightarrow} 0 \text{ if } \phi_1 > 1$$

  as we have assumed.
- y_t is also stationary since it can be express as a well defined linear filter of stationary variables (namely ε_{t+i}):

$$y_t = \sum_{i=1}^{\infty} (-\phi_1^{-1})^i \varepsilon_{t+i} \tag{3}$$

Equity Price Example of Forward-Looking Solution I

- The model used by Hamilton in Ch 2.5 and Ch 11.2 illustrates several points.
- P denotes the price of a stock
- D Dividend payment
- r Total real return from holding a stock in one period

Using Ch 11.2 dating of the variables:

$$r_{t+1} = \frac{P_{t+1} - P_t}{P_t} + \frac{D_{t+1}}{P_t} \tag{4}$$

Assume, postive and constant real total returns ($r_{t+1} = r > 0$). We then have an AR model for P_t:

$$P_{t+1} = (1 + r)P_t - D_{t+1} \tag{5}$$

Equity Price Example of Forward-Looking Solution II

- If D_t is stationary (e.g. white noise), P_t follows a non-causal process.
- The backward-solution of (5) is explosive, often called the "bubble" solution.
- However, (5) can equivalently by written as:

$$P_t = (1+r)^{-1} P_{t+1} + (1+r)^{-1} D_{t+1} \tag{6}$$

- A theory of efficient markets says that the price of the stock is determined by

$$P_t = (1+r)^{-1} E_t(P_{t+1}) + (1+r)^{-1} E_t D_{t+1} \tag{7}$$

where $E_t(P_{t+1})$ and $E_t(D_{t+1})$ denotes the rational expectation for P_{t+1}, using information available in period t.

Equity Price Example of Forward-Looking Solution III

For the next period:

$$E_t P_{t+1} = (1+r)^{-1} E_t(P_{t+2}) + (1+r)^{-1} E_t D_{t+2}$$

giving:

$$\begin{aligned} P_t &= (1+r)^{-1}\left[(1+r)^{-1} E_t(P_{t+2}) + (1+r)^{-1} E_t D_{t+1}\right] + (1+r)^{-1} E_t D \\ &= (1+r)^{-2} E_t(P_{t+2}) + (1+r)^{-1} E_t D_{t+1} + (1+r)^{-2} E_t D_{t+2} \end{aligned}$$

Continuing until infinity:

$$P_t = \sum_{j=1}^{\infty} \left[\frac{1}{1+r}\right]^j E_t D_{t+j} = E_t \sum_{j=1}^{\infty} \left[\frac{1}{1+r}\right]^j D_{t+j} \qquad (8)$$

replicating H's equation (11.2.14).

Equity Price Example of Forward-Looking Solution IV

- The solution is referred to as the “market fundamental solution”, it is the solution when the price formation process is dominated by rational expectations.
- Note that it is implicit that D_t is assumed to be an exogenous “forcing variable” in this solution.
- Still, we may find that the equity price P_t *Granger causes* dividends in the VAR, and this is Hs point in Ch 11.2.

Granger Causality induced by forward-looking behaviour I

Continuing from Chapter 11.2 in Hamilton:
Assume the following MA model for dividends:

$$D_t = d + u_t + \delta u_{t-1} + v_t \tag{9}$$

where u_t and v_t are independent white-noise Gaussian series. By assumption

$$E_t D_{t+1} = d + \delta u_t$$
$$E_t D_{t+j} = d \text{ for } j > 1$$

Granger Causality induced by forward-looking behaviour II

$$
\begin{aligned}
P_t &= \sum_{j=1}^{\infty} \left[\frac{1}{1+r}\right]^j E_t D_{t+j} \\
&= (1+r)^{-1}(d+\delta u_t) + \sum_{j=2}^{\infty} \left[\frac{1}{1+r}\right]^j d \\
&= (1+r)^{-1}(d+\delta u_t) + \sum_{j=1}^{\infty} \left[\frac{1}{1+r}\right]^{j+1} d \\
&= (1+r)^{-1}\delta u_t + (1+r)^{-1} d + \left[\frac{(1+r)^{-1}}{r}\right] d
\end{aligned}
$$

Collecting terms:

$$
P_t = (1+r)^{-1}\delta u_t + r^{-1} d \qquad (10)
$$

Granger Causality induced by forward-looking behaviour III

The solution holds for all periods and it implies that

$$\delta u_{t-1} = (1+r)P_{t-1} - r^{-1}(1+r)d \qquad (11)$$

Hence we have Granger-causation from P_{t-1} to D_t

$$P_{t-1} \rightarrow u_{t-1} \rightarrow D_t$$

from the solution and (9).

- A VAR for D_t and P_t can be formulated as

$$\begin{pmatrix} D_t \\ P_t \end{pmatrix} = \begin{pmatrix} -\frac{d}{r} \\ \frac{d}{r} \end{pmatrix} + \begin{pmatrix} 0 & (1+r) \\ 0 & 0 \end{pmatrix} \begin{pmatrix} D_{t-1} \\ P_{t-1} \end{pmatrix} + \begin{pmatrix} u_t + v_t \\ \frac{\delta}{1+r} u_t \end{pmatrix} \qquad (12)$$

The second row is (10) and the first row is the result of substitution of δu_{t-1} in (9) by right hand side of (11).

Granger Causality induced by forward-looking behaviour IV

- The theory of a rational expectations price formation implies that the equity price Grangers causes dividends in the VAR (12).
- This is because P_t contains information about the expected D_{t+1}.
- A premise for this result is that D_t is autocorrelated.
- Similar implications can be established for other financial variables. For example the *term-structure of interest rates* may be a predictor of GDP growth and/or inflation if the price of bonds incoroprate "correct" expectations about these variables.

VAR for consumption and income I

- Define c_t as the log of private consumption and y_t as the log of disposable income
- Assume the following dynamic model for Δc_t, Δy_t and a third stationary varaible x_t

$$\Delta c_t = \kappa - \alpha_c x_{t-1} + e_{c,t}, \qquad 0 \leq \alpha_c < 1, \tag{13}$$

$$\Delta y_t = \varphi + \alpha_y x_{t-1} + e_{y,t}, \qquad 0 \leq \alpha_y < 1, \tag{14}$$

$$x_t = \Delta c_t - \Delta y_t + x_{t-1} \tag{15}$$

- x_t is approximately the minus of the savings rate:

$$-x_t = -c_t + y_t \approx \text{savings rate}$$

VAR for consumption and income II

- It is often relevant to have the equilibirum of x_t as a parameter. Define
$$\mu_x = \mathsf{E}(x_t)$$
and decompose κ and φ as:
$$\kappa = \eta_c + \alpha_c \mu_x$$
$$\varphi = \eta_y - \alpha_y \mu_x$$
Thus we can rewrite this system into
$$\Delta c_t = \eta_c - \alpha_c [x_{t-1} - \mu_x] + e_{c,t}, \quad (16)$$
$$\Delta y_t = \eta_y + \alpha_y [x_{t-1} - \mu_x] + e_{y,t}, \quad (17)$$
$$x_t = \Delta c_t - \Delta y_t + x_{t-1} \quad (18)$$

VAR for consumption and income III

where the two intercdecets must have the interpretation:
$\eta_c = \mathsf{E}(\Delta c_t)$ and $\eta_y = \mathsf{E}(\Delta y_t)$.

Permanent income hypothesis (PIH) I

- A well known implication of the PIH is that the savings rate $(-x_t)$ is function of expected future income.
- The logic is very similar to the market fundamentals solution for P_t above.
- Intuitively, x_t is Granger causing income growth because the savings rate is a carrier or households expectations about future income growth—
  - the *saving for a rainy day feature of* the PIH model, see Campbell (1987).

Permanent income hypothesis (PIH) II

(16)-(17) can be written in model form:

$$\Delta c_t = \eta_c + e_{c,t}, \tag{19}$$

$$\Delta y_t = \eta_y + \gamma_y + \pi_y \Delta c_t + \alpha_y [x_{t-1} - \mu_x] + \varepsilon_{y,t}, \tag{20}$$

where (19) is the marginal model for consumption and (20) is a conditional model for real income (cf Lecture 7)

If we assume normal distrbution (e.g., Lecture 7):

$$\begin{aligned} \pi_y &= \rho_{cy} \frac{\sigma_y}{\sigma_c}, \\ \gamma_y &= -\eta_y \pi_y, \\ \varepsilon_{y,t} &= e_{y,t} - \pi_y e_{c,t}. \end{aligned} \tag{21}$$

Since $\alpha_c = 0$, cointegration implies that $0 < \alpha_y < 1$, with *rainy-day* as the economic interpretation.

Consumption function (CF) I

$$\Delta c_t = \eta_c + \gamma_c + \pi_c \Delta y_t - (\alpha_c + \pi_c \alpha_y)[x_{t-1} - \mu_x] + \varepsilon_{ct} \quad (22)$$
$$\Delta y_t = \eta_y + \alpha_y [x_{t-1} - \mu_x] + e_{y,t} \quad (23)$$

(22) is the consumption function,(23) is the marginal income equation.

$$\begin{aligned} \alpha_c &= (1 - \phi_{cc}) \\ \pi_c &= \rho_{c,y} \frac{\sigma_c}{\sigma_y}, \\ \gamma_c &= -\eta_y \pi_c, \\ \varepsilon_{c,t} &= e_{c,t} - \pi_c e_{y,t}. \end{aligned} \quad (24)$$

- Underlying the consumption function approach is $0 < \alpha_c < 1$.

Consumption function (CF) II

- Note that the hypothesis H_0: $\alpha_c = 0$ must be tested separately, since finding $[x_{t-1} - \mu_x]$ significant in (22) could be due to $0 < \alpha_y$.
- For the coefficient α_y there are two possibilities.
- $0 < \alpha_y < 1$ is consistent with hours worked etc. being "demand determined" and with y_t adjusting to past disequilibria. In econometric terms there is mutual (Granger) causation between income and consumption.
- The second possibility is that $\alpha_y = 0$, reflecting that income is "supply-side" determined

Consumption function (CF) III

- Of course $\alpha_y = 0$ gives the clearest contrast to the PIH: (22)-(23) simplify to

$$\Delta c_t = \eta_c + \gamma_c + \pi_c \Delta y_t - \alpha_c [x_{t-1} - \mu_x] + \varepsilon_{c,t} \quad (25)$$

$$\Delta y_t = \eta_y + e_{y,t}, \quad (26)$$

  with $\eta_y = \varphi$, since there is no equilibrium correction in income.

- The analysis of the different implications of CF and PIH carry over to the realistic case of
  - VAR(p) and
  - deterministic terms
  - Unit-root non-stationarity

Analysis of Norwegian consumption-income data I

- Quarterly data 1968(2)-1984(4)
  Refer to this as the *Before break* sample, explain in class.
- Full resuls in Eitrheim et al (2002)

Analysis of Norwegian consumption-income data II

Table: Diagnostics for the I(0) VAR

The sample is 1968(2) to 1984(4), 67 observations.		
$\hat{\sigma}_{\Delta c}$	$=$	1.63%
$\hat{\sigma}_{\Delta y}$	$=$	1.60%
$AR\ 1-5\ F(20,82)$	$=$	$0.8921[0.5973]$
$Normality\ \chi^2(4)$	$=$	$8.0609[0.0894]$
$Heteroscedasticity\ F(75,72)$	$=$	$0.5222[0.9971]$

Analysis of Norwegian consumption-income data III

Table: FIML consumption function estimates. 1968(2)-84(2)—Before Break

The consumption function

$$\widehat{\Delta c}_t = -\underset{(0.070)}{0.302}\,\Delta c_{t-1} + \underset{(0.073)}{0.227}\,\Delta c_{t-4} + \underset{(0.181)}{0.471}\,\Delta y_t - \underset{(0.048)}{0.128}\,(c-y)_{t-1}$$

$$\hat{\sigma} = 1.53\%$$

The income equation

$$\widehat{\Delta y}_t = \underset{(0.002)}{0.009} - \underset{(0.109)}{0.477}\,\Delta y_{t-1} + \underset{(0.109)}{0.311}\,\Delta y_{t-4}$$

$$\hat{\sigma} = 1.60\%$$

Analysis of Norwegian consumption-income data IV

Table: *Before break* FIML estimates for consumption function

Diagnostics		
Overidentification $\chi^2(14)$	=	15.7597[0.3283]
AR $1-5$ $F(20, 96)$	=	0.7830[0.7274]
Normality $\chi^2(4)$	=	5.057[0.2815]
Heteroscedasticity $F(75, 93)$	=	0.6530[0.9717]

FIML estimation. The sample is 1968(2) to 1984(4), 67 observations.

Analysis of Norwegian consumption-income data V

Table: *Before break* FIML estimates for PIH .

The Euler equation

$$\widehat{\Delta c}_t = \underset{(0.003)}{0.0059} + \underset{(0.089)}{0.236}\,\Delta c_{t-4}$$

$$\hat{\sigma} = 2.10\%$$

The savings equation

$$\widehat{\Delta y}_t = \underset{(0.003)}{0.0144} - \underset{(0.098)}{0.308}\,\Delta y_{t-1} + \underset{(0.01)}{0.231}\,\Delta y_{t-4}$$

$$+ \underset{(0.068)}{0.158}\,(c-y)_{t-1} + \underset{(0.013)}{0.041}\,VAT$$

$$\hat{\sigma} = 1.62\%$$

The sample is 1968(1) to 1984(4), 68 observations.

Analysis of Norwegian consumption-income data VI

Table: *Before break* FIML estimates for PIH

Diagnostics		
Overidentification $\chi^2(15)$	$=$	45.3776[0.0001]
AR $1-5$ $F(20, 98)$	$=$	1.5548[0.0804]
Normality $\chi^2(4)$	$=$	7.4287[0.1149]
Heteroscedasticity $F(75, 96)$	$=$	1.0269[0.4481]
The sample is 1968(1) to 1984(4), 68 observations.		

The New Keynesian Phillips curve I

The so-called hybrid NPC is given as

$$\pi_t = \underset{\geq 0}{a^f} E_t[\pi_{t+1}] + \underset{\geq 0}{a^b} \pi_{t-1} + \underset{>0}{b} s_t + \varepsilon_{\pi t} \tag{27}$$

- π_t denotes the rate of inflation
- In this model s_t denotes the logarithm of the wage-share which is the most used operational definition of real marginal costs. $\varepsilon_{\pi t}$ is a white noise disturbance.
- In many applications, notably Gali and Gertler (1999), the disturbance term is omitted, which is often referred to as the NPC holding in “exact form”

The New Keynesian Phillips curve II

- s_t is referred to as a forcing variable, in the same way as D_t in the stock price model. We assume the following model for s_t

$$s_t = c_{s1} s_{t-1} + \cdots + c_{sk} s_{t-k} + \varepsilon_{s,t} \ . \qquad (28)$$

- It can be show that $k \geq 2$ is necessary for identification of the parameters in the NPC.

NPC solution I

- Following Bårdsen et al (2005), Ch 7, we first find a partial solution for π_t as

$$\pi_t = r_1 \pi_{t-1} + \frac{b}{a^f r_2} \sum_{i=o}^{\infty} (\frac{1}{r_2})^i E_t s_{t+i} + \frac{1}{a^f r_2} \varepsilon_{\pi,t} \tag{29}$$

where $r_{1,2}$ are the two roots of

$$r^2 - (1/a^f)r + (a^b/a^f) = 0$$

A stable solution of the pure NPC, with $a^b = 0$, requires $a^f > 1$.

NPC solution II

With $a^f > 0$, $a^b > 0$ and $a^f + a^b \leq 1$ it is implied that r_1 and r_2 are real and positive.
It is usual to define r_1 as

$$r_1 = \frac{1 - \sqrt{1 - 4a^f a^b}}{2a^f} \tag{30}$$

which is $0 \leq r_1 < 1$ under the assumption of $a^f + a^b < 1$.
Under these assumption the solution is (cf Nymoen, Swensen and Tveter (2012)):

$$\pi_t = r_1 \pi_{t-1} + \frac{b}{a^f r_2} K_{s1} s_t + \frac{b}{a^f r_2} K_{s2} s_{t-1} + \frac{1}{a^f r_2} \varepsilon_{\pi,t}, \tag{31}$$

written here for the case of $k = 2$ in the forcing process (28).

NPC solution III

- K_{s1} and K_{s2} are constants that are defined when

$$\left|\frac{r_{si}}{r_2}\right| < 1 \text{ for } i = 1, 2 \qquad (32)$$

  where $r_{s1}$and r_{s2} are the roots of the characteristic equation associated with the forcing variable.
- (31) can be solved from known initial conditions, "as if" the model was causal.

Testing the NPC

Different "types" of tests:

- Robustness and strength of instruments (as we did in seminar)
- Testing the implications of $E_t[\pi_{t+1}]$ on the VAR, for example Boug et al (2010)
- Testing the encompassing implications of the NPC against existing models of the wage-price spiral—encompassing approach
- Testing invariance of NPC with respect to regime shifts.

Encompassing type test I

Start from:

$$s_t = ulc_t - q_t, \tag{33}$$

where ulc denotes unit labour costs (in logs) and q is the log of the price level on domestic goods and services. Let $(1-\gamma)$ denote a constant import share, and pi the import price index.
The aggregate price level is defined as

$$p_t = \gamma q_t + (1-\gamma)\, pi_t \tag{34}$$

Encompassing type test II

Using (33) and (33), we can re-write the NPC (27), after dropping the disturbance for simplicity:

$$\pi_t = \frac{a^f}{\left(1+\frac{b}{\gamma}\right)} E_t[\pi_{t+1}] + \frac{a^b}{\left(1+\frac{b}{\gamma}\right)} \pi_{t-1}$$

$$- \frac{b}{(\gamma+b)} \left[p_{t-1} - \gamma ulc_{t-1} - (1-\gamma)\, pi_{t-1} \right]$$

$$+ \frac{\gamma b}{(\gamma+b)} \Delta ulc_t + \frac{b\,(1-\gamma)}{(\gamma+b)} \Delta pi_t$$

Encompassing type test III

or

$$\pi_t = \alpha^f \Delta E_t[\pi_{t+1}] + \alpha^b \pi_{t-1} + \beta(ulc_{t-1} - p_{t-1}) - \beta\,(1-\gamma)\,(ulc_{t-1} - pi_{t-1}) + \beta\;\gamma\;\Delta ulc_t + \beta\,(1-\gamma)\,\Delta pi_t \tag{35}$$

with α^f, α^b, β and ψ as new coefficients.

- This shows hat the NPC has an interpretation as an EqCM for the price level.
- An alternative model for price and wage formation is the *imperfect competition model*, ICM.

Encompassing type test IV

- The ICM the price equation with a lead-term added is

$$\begin{aligned}\pi_t = {} & \alpha^f \Delta E_t[\pi_{t+1}] + \alpha^b \Delta\pi_{t-1} \\ & + \beta_1(ulc_{t-1} - p_{t-1}) + \beta_2(ulc_{t-1} - pi_{t-1}) \\ & + \beta_3 \Delta ulc_t + \beta_4 \Delta pi_t.\end{aligned} \tag{36}$$

- The NPC implies restrictions on (36)
  - H_0^a: $\beta_3 = \beta_1 + \beta_2$ and
  - H_0^b: $\beta_4 = -\beta_2$.
- For the ICM, the only requirement is $\beta_1 > 0$ and $\beta_1 > -\beta_2$.
- Non rejection of H_0^a and/or H_0^b would mean that the NPC encompasses the ICM :

$$\text{NPC } \mathcal{E} \text{ ICM} \quad \text{if} \quad \beta_3 = \beta_1 + \beta_2 \text{ and } \beta_4 = -\beta_2$$

Encompassing type test V

- Conversely, if $\beta_3 = \beta_1 + \beta_2$ and/or $\beta_4 = -\beta_2$ is rejected, the ICM implies that the NPC has omitted variables bias.
  - In particular $ulc_{t-1} - p_{t-1}$ and $ulc_{t-1} - pi_{t-1}$ are predictors of π_{t+1} so omission (misrepresentation) of theses variables in the NPC will affect the *IV / GMM* estimate of the paramet a^f in the NPC.
- This test is formulated and apploed to panel data in Bjørnstad and Nymoen (2008)

Testing invariance of NPC I

- If the data generating process is characterized by intermittent structural break, the signficance of the forward term in the NPC may be overestimated.
- Structural breaks represented by impulse indicators
- The impulse indicators are selected in the implied reduced form forecasting equation
- Then tested for significance in the NPC.
  - Under the null of correct specification, few such impulse indicators will be selected,
  - and those that are should not be significant when added to NPC;
  - moreover, parameter estimates should not alter much.

Testing invariance of NPC II

- Under the alternative that there are unmodeled outliers or breaks, there will be significant impulse indicators in the 'forecasting' equation, and these will remain significant when added to the NPC.
- Castle et al (2014)

References (ie. additional reading) I

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